帝國大學紀要

理科

第二冊

THE

JOURNAL

OF THE

COLLEGE OF SCIENCE,

IMPERIAL UNIVERSITY,

JAPAN.

VOL. II.

WITH 27 PLATES.

帝國大學印行

明治廿二年

PUBLISHED BY THE UNIVERSITY.

TŌKYŌ, JAPAN.

1889.

Publishing Committee.

Prof. **D. Kikuchi**, *Rigakuhakushi*, **M. A.**, Director of the College (*ex officio*).

Prof. **K. Mitsukuri**, *Rigakuhakushi*, **Ph. D.**

Prof. **C. G. Knott, D. Sc., F. R. S. E.**

Prof. **S. Sekiya.**

2722

PRINTED AT THE SEISHIBUNSHA, TŌKYŌ.

CONTENTS.

Vol. II.

Ueber die

Darstellbarkeit willkürlicher Functionen durch Reihen die nach den Wurzeln einer transcendenten Gleichung fortschreiten.

von

Dr. Phil. R. Fujisawa.

Neben den trigonometrischen Reihen werden, in der mathematischen Physik, die ihnen naheverwandten, nach den Wurzeln einer gewissen transcendenten Gleichung fortschreitenden Reihen vielfach angewandt ; daher wird es wünschenswerth sein, wenigstens für alle Fälle der Natur, d. h., unter beschränkenden, jedoch für die physikalische Anwendung hinreichend allgemeinen Voraussetzungen über die Natur der Function, zu beweisen, dass die Reihe wirklich gegen den Werth der gegebenen Function convergire. Dass der von Sturm und Liouville herrührende, von Heine vervollständigte Beweis unzureichend ist, habe ich schon in einer früheren arbeit discutirt.* Es handelt sich hier darum, die Convergenz jener in Rede stehenden Reihe darzuthun, wie für die trigonometrischen Reihen durch die berühmte Arbeit Dirichlet's geschehen ist.

Die Reihe in ihrer allgemeinsten Form ist wie folgt beschaffen ; sie schreitet nach den gegebenen Functionen $\theta(x, \lambda)$, welche einen

---

* Ueber eine in der Wärmeleitungstheorie auftretende, nach den Wurzeln einer transcendenten Gleichung fortschreitende unendliche Reihe. Inaugural-Dissertation. Strassburg 1886.

Parameter λ enthält, für den man alle Wurzeln einer gegebenen transcendenten Gleichung $\phi(\lambda)=0$ zu setzen hat. In dieser Form wird aber der in Rede stehende Beweis wohl schwerlich durchzuführen sein, ohne dass man sehr beschränkende Voraussetzungen über θ und ϕ zu machen genöthigt ist; es empfiehlt sich daher, von vornherein den Beweis an einem bestimmten Beispiele durchzuführen und dadurch den Weg zu zeigen, wie man auch in andern Fällen zu verfahren hat.

Ein so directer Weg, wie der Dirichlet's ist hier der Natur der Sache nach wohl nicht möglich; es lässt sich aber dieser Fall auf einen durch das Theorem Dirichlet's erledigten Fall zurückführen wie ich in meiner eben citirten Arbeit für eine Reihe aus der Wärmeleitungstheorie dargethan habe. Es war die Reihe:

$$u=\frac{1}{r}\sum_{n=1}^{\infty} e^{-\left(\lambda_n \frac{a}{l}\right)^2 t} \sin\left(\lambda_n \frac{r}{l}\right) \frac{\int_0^l \rho f(\rho)\sin\left(\lambda_n \frac{\rho}{l}\right)d\rho}{\int_0^l \left(\sin \lambda_n \frac{\rho}{l}\right)^2 d\rho},$$

wo $\lambda_1, \lambda_2, \ldots$ die, der Grösse nach geordneten positiven Wurzeln der Gleichung

$$\phi(\lambda)=\cos\lambda+(\alpha-1)\sin\lambda=0 \qquad (\alpha>0)$$

bedeuten.

Für die Wärmeleitungsaufgabe genügt es nachzuweisen, dass u mit positiv abnehmendem t gegen $f(r)$ convergirt. Stellt die für $t=0$ formirte Reihe

$$v=\frac{1}{r}\sum_{n=1}^{\infty} \sin\left(\lambda_n \frac{r}{l}\right) \frac{\int_0^l \rho f(\rho)\sin\left(\lambda_n \frac{\rho}{l}\right)d\rho}{\int_0^l \left(\sin \lambda_n \frac{\rho}{l}\right)^2 d\rho}$$

$f(r)$ dar, so ist dies nach einem bekannten Satze über Potenzreihen hierfür ausreichend, aber nicht nothwendig. Dieser, wie ich glaube,

bisher unbeachtete Umstand spielt eine wesentliche Rolle bei meinem Beweise; die Methode des Beweises selbst lässt sich auch auf die für $t=0$ formirte Reihe v anwenden, wie ich seiner Zeit bemerkt habe. Dies zu zeigen ist der Zweck der vorliegenden Arbeit.

§. 1.

Wir beschäftigen uns mit der Reihe

$$\text{I.}\quad v = \frac{1}{r}\sum_{n=1}^{\infty} \sin\left(\lambda_n \frac{r}{l}\right) \frac{\int_0^l \rho f(\rho) \sin\left(\lambda_n \frac{\rho}{l}\right) d\rho}{\int_0^l \left(\sin \lambda_n \frac{\rho}{l}\right)^2 d\rho},$$

worin $\lambda_1, \lambda_2, \ldots\ldots$ die der Grösse nach geordneten positiven Wurzeln der Gleichung.

$$\text{II.}\quad \phi(\lambda) = \lambda \cos \lambda + (\alpha - 1) \sin \lambda = 0 \qquad (\alpha > 0)$$

bedeuten. Diese Reihe lässt sich, wenn man die Integration im Nenner ausführt, auch wie folgt schreiben:

$$\text{III.}\quad v = \frac{2}{r\,l}\sum_{n=1}^{\infty} \sin\left(\lambda_n \frac{r}{l}\right) \frac{\int_0^l \rho f(\rho) \sin\left(\lambda_n \frac{\rho}{l}\right) d\rho}{1 - \frac{\sin \lambda_n \cos \lambda_n}{\lambda_n}}.$$

Es soll nachgewiesen werden, dass diese Reihe v, unter Voraussetzung der bekannten Dirichletschen Bedingungen hinsichtlich der Function $f(\rho)$, zur Summe $f(r)$ hat.

Zu dem Zwecke sei angeführt die bekannte Ausdrucksform von λ_n:

$$\lambda_n = \frac{2n-1}{2}\pi \pm \delta_n, \qquad 0 < \delta_n < \frac{\pi}{2}, \qquad (\alpha \gtrless 1).$$

Für $\alpha = 1$ wird

$$\lambda_n = \frac{2n-1}{2}\pi;$$

und wenn wir die Reihe v für den speciellen werth von $\alpha = 1$ mit v' bezeichnen, so lautet dieselbe:

$$v' = \frac{2}{r\,l} \sum_{n=1}^{\infty} \sin\left(\frac{2n-1}{2}\,\frac{r}{l}\right). \int_0^l \rho f(\rho) \sin\left(\frac{2n-1}{2}\,\frac{\rho}{l}\right) d\rho.$$

Von dieser Reihe ist bekannt, dass sie sich aus der Entwickelung von $r\,f(r) \cos \frac{\pi r}{2l}$ nach den Sinus der *ganzen* Vielfachen von $\frac{\pi r}{l}$ ergibt, also dass, mit Ausschluss der Grenzen $r = 0$ und $r = l$, zwischen denselben $f(r)$ zur Summe hat.

Es handelt sich darum, zu zeigen dass die beiden Summen v und v' gegen die nämliche Grenze convergiren.

§. 2.

Wofern

$$\phi(\lambda_n) = \lambda_n \cos \lambda_n + (\alpha - 1) \sin \lambda_n = 0$$

ist, haben wir

$$\frac{\sin\left(\lambda_n \frac{r}{l}\right) \cdot \sin\left(\lambda_n \frac{\rho}{l}\right)}{1 - \frac{\sin \lambda_n \cos \lambda_n}{\lambda_n}} = \frac{\lambda_n \cdot \sin\left(\lambda_n \frac{r}{l}\right) \cdot \sin\left(\lambda_n \frac{\rho}{l}\right)}{\sin \lambda_n \cdot \phi'(\lambda_n)},$$

also auch

$$\frac{\sin\left(\lambda_n \frac{r}{l}\right) \cdot \sin\left(\lambda_n \frac{\rho}{l}\right)}{1 - \frac{\sin \lambda_n \cos \lambda_n}{\lambda_n}} = \frac{(\alpha - 1) \sin\left(\lambda_n \frac{r}{l}\right) \cdot \sin\left(\lambda_n \frac{\rho}{l}\right)}{\cos \lambda_n \cdot \phi'(\lambda_n)}.$$

Setzt man

$$w(z) = \frac{(\alpha - 1) \cdot \sin\left(z \frac{r}{l}\right) \cdot \sin\left(z \frac{\rho}{l}\right)}{\cos z \cdot \phi(z)},$$

so stellt w eine einwerthige Function einer complexen Variabele z dar, welche für endliche Werthe von z nur in den Punkten

$$z = \pm \lambda_1 , \quad \pm \lambda_2 , \;\ldots\ldots\ldots\ldots\ldots\ldots$$

und

$$z = \pm \frac{\pi}{2} , \quad \pm \frac{3\pi}{2} , \;\ldots\ldots\ldots\ldots\ldots\ldots$$

unstetig wird. In den beiden Punkten $z = \pm \lambda_n$ hat sie das Residuum

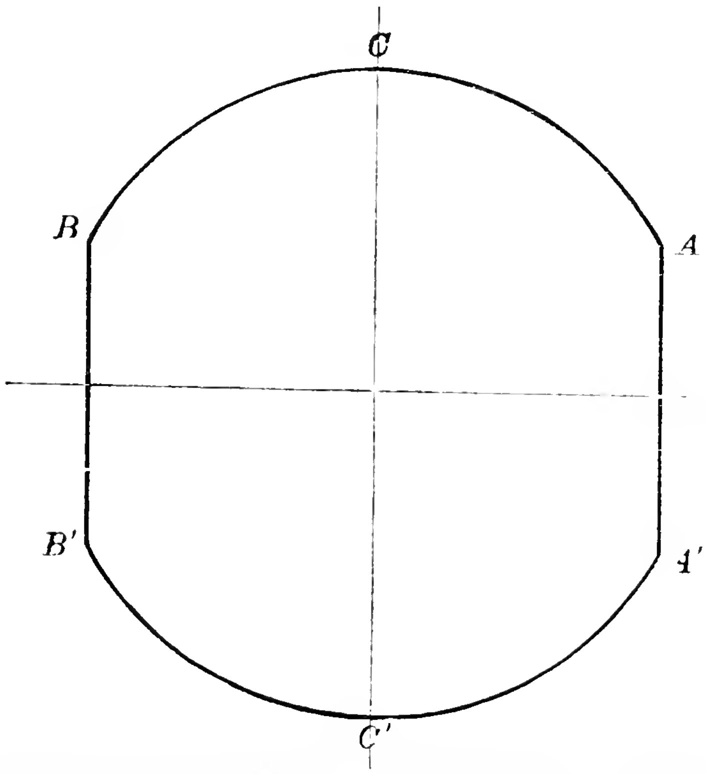
$$R\,(\pm \lambda_n) = \frac{\sin\left(\lambda_n \frac{r}{l}\right) \cdot \sin\left(\lambda_n \frac{\rho}{l}\right)}{1 - \frac{\sin \lambda_n \cos \lambda_n}{\lambda_n}} ;$$

und für $z = \pm \frac{2n-1}{2}\pi$ das folgende:

$$R\left(\pm \frac{2n-1}{2}\pi\right) = -\sin\left(\frac{2n-1}{2} \frac{\pi r}{l}\right) \cdot \sin\left(\frac{2n-1}{2} \frac{\pi \rho}{l}\right).$$

Dies vorangeschickt, sei nun in der z-ebene eine Fläche E* vorgelegt, welche wie folgt entsteht: in den Punkten $m\pi$ und $-m\pi$, unter m eine positive ganze Zahl verstanden, errichte man auf der x-axe Perpendikel AA' und BB', wo

$$\begin{aligned} A &= \quad m\pi + i\sqrt{m\pi}, \\ B &= -m\pi + i\sqrt{m\pi}, \\ A' &= \quad m\pi - i\sqrt{w\pi}, \\ B' &= -m\pi - i\sqrt{m\pi}; \end{aligned}$$



* Diese Fläcke E kommt bei Gelegenheit einer ähnlichen Untersuchung bei Heine vor. Crelle's Journal, Bd. 89.

vom Anfangspunkt 0 aus schlage man mit dem Radius $\sqrt{(m\pi)^2+m\pi}$ Kreisbogen ACB und $A'C'B'$. Diese durch die in sich zurückkehrende Linie $A'ACBB'C'A'$ eingeschlossene Fläche möge E heissen.

Alsdann haben wir nach dem Residuensatze von Cauchy

$$2\left[\sum_{n=1}^{m} R\left(\frac{2n-1}{2}\pi\right)+\sum_{n=1}^{m} R(\lambda_n)\right]=\frac{1}{2\pi i}\cdot\int_{(E)} w(z)\,dz,$$

wo die Integration über $A'ACBB'C'A$ zu erstrecken ist.

Da die Funktion w eine ungerade Function von $z=x+iy$ ist, so ist offenbar

$$\int_{A'}^{A} w\,dz=\int_{B}^{B'} w\,dz=Q$$

$$\int_{A}^{\substack{B\\C}} w\,dz=\int_{B'}^{\substack{A'\\C'}} w\,dz=P;$$

und wir haben

$$\sum_{n=1}^{m} R\left(\frac{2n-1}{2}\pi\right)+\sum_{n=1}^{m} R(\lambda_n)=\frac{1}{2\pi i}\,[Q+P].$$

Setzen wir hierin die Werthe von $R\left(\frac{2n-1}{2}\pi\right)$ und $R(\lambda_n)$ wieder ein, so ergibt sich

$$\sum_{n=1}^{m}\frac{\sin\left(\lambda_n\frac{r}{l}\right)\cdot\sin\left(\lambda_n\frac{\rho}{l}\right)}{1-\frac{\sin\lambda_n\cos\lambda_n}{\lambda_n}}-\sum_{n=1}^{m}\sin\left(\frac{2n-1}{2}\frac{\pi r}{l}\right)\cdot\sin\left(\frac{2n-1}{2}\frac{\pi\rho}{l}\right)$$
$$=\frac{1}{2\pi i}\,[Q+P].$$

Multiplicirt man die beiden Seiten dieser Gleichung mit

$$\frac{2}{r.\,l}\,\rho\cdot f(\rho)\cdot d\rho,$$

und integrirt sodann nach ρ zwischen den Grenzen 0 und l, so folgt:

$$\left.\begin{array}{l}\dfrac{2}{r.l}\displaystyle\sum_{n=1}^{m}\sin\left(\lambda_n\frac{r}{l}\right)\frac{\displaystyle\int_0^l\rho f(\rho)\sin\left(\lambda_n\frac{\rho}{l}\right)d\rho}{1-\dfrac{\sin\lambda_n\cos\lambda_n}{\lambda_n}}\\ -\dfrac{2}{r.l}\displaystyle\sum_{n=1}^{m}\sin\left(\frac{2n-1}{2}\frac{\pi r}{l}\right)\int_0^l\rho.f(\rho)\sin\left(\frac{2n-1}{2}\frac{\pi\rho}{l}\right)d\rho\end{array}\right\} = \frac{1}{r.l.\pi i}\int_0^l\rho f(\rho)[Q+P]\,d\rho.$$

Wir setzen

$$\varDelta = \frac{1}{r.l.\pi i}\int_0^l\rho.f(\rho)\,[Q+P]\,d\rho$$

und lassen m, die Reihe der ganzen Zahlen durchlaufend, über alle Grenze hinauswachsen, so haben wir, die Convergenz beiderseits vorausgesetzt,

$$v - v' = \operatorname{Lim}_{m=\infty}\varDelta.$$

Nun lässt sich aber zeigen, dass die rechte Seite dieser Gleichung in der That verschwindet.

§. 3.

Wir haben gesetzt

$$\varDelta = \frac{1}{r.l.\pi i}\int_0^l\rho.f(\rho)\,[Q+P]\,d\rho;$$

mithin

$$\operatorname{Mod}\varDelta \leqq \frac{1}{r.l.\pi}\int_0^l\rho\operatorname{Mod}f(\rho)\,[\operatorname{Mod}Q+\operatorname{Mod}P]\,d\rho.$$

Zunächst beschäftigen wir uns mit Q und P.

Es ist

$$Q = \int\Big|_{A'}^{A} w\,dz = i\int_{-\sqrt{m\pi}}^{\sqrt{m\pi}} w\,(m\pi+iy)\,dy,$$

worin

$$w(z) = \frac{(\alpha-1)\sin\left(z\,\frac{r}{l}\right).\sin\left(z\,\frac{\rho}{l}\right)}{\cos z\,.\;\phi(z)},$$

$$\phi(z) = z\cos z + (\alpha-1)\sin z$$

ist.

Bezeichnet man durch

$$Cy = \frac{1}{2}(e^{y}+e^{-y}) = 1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \cdots\cdot$$

$$Sy = \frac{1}{2}(e^{y}-e^{-y}) = y + \frac{y^3}{3!} + \frac{y^5}{5!} + \cdots\cdot,$$

so ist

$$\sin(x+iy) = \sin x\,Cy + i\cos x\,Sy,$$

$$\cos(x+iy) = \cos x\,Cy - i\sin x\,Sy.$$

Das gibt:

$$\text{Mod}\sin(x+iy) = \sqrt{Cy^2-\cos x^2} = \sqrt{Sy^2+\sin x^2},$$

$$Sy \leqq \text{Mod}\sin(x+iy) \leqq Cy,$$

und. insbesondere.

$$\text{Mod}\sin\frac{r}{l}(x+iy) \leqq C\left(\frac{r}{l}\,y\right)$$

$$\text{Mod}\sin\frac{\rho}{l}(x+iy) \leqq C\left(\frac{\rho}{l}\,y\right).$$

also

$$\text{Mod}\left|\sin\frac{r}{l}(x+iy)\cdot\sin\frac{\rho}{l}(x+iy)\right| \leqq C\left(\frac{r}{l}\,y\right)\cdot C\left(\frac{\rho}{l}\,y\right).$$

Entwickelt man $\varphi(x+iy)$,

$$\varphi(x+iy) = \left|\begin{array}{c} x \cos x \, Cy \\ + y \sin x \, Sy \\ + (\alpha - 1) \sin x \, Cy \end{array}\right| + i \left|\begin{array}{c} y \cos x \, Cy \\ - x \sin x \, Sy \\ + (\alpha - 1) \cos x \, Sy \end{array}\right|$$

und bildet alsdann den Modul, so erhält man durch einfache Reduction

$$\text{Mod}\ \varphi(x+iy) = \left|\begin{array}{c} [x \cos x + (\alpha - 1) \sin x]^2 \\ + [y \, Cy + (\alpha - 1) \, Sy]^2 \\ + x^2 \, Sy^2 \sin x^2 \end{array}\right|^{\frac{1}{2}}.$$

Hierin setze man $x = m\pi$, $\sin m\pi = 0$, $\cos m\pi = \pm 1$ ein, so erhält man

$$\text{Mod}\ \phi(m\pi + iy) = \sqrt{m\pi^2 + (y\, Cy + (\alpha-1)\, Sy)^2 + m\pi^2\, Sy^2}$$

$$= \sqrt{m^2 \pi^2 \, Cy^2 + (y \, Cy + (\alpha - 1) \, Sy)^2},$$

also

$$\text{Mod}\ \phi(m\pi + iy) \geqq m\pi \, Cy.$$

Es folgt hieraus

$$\text{Mod}\ w(m\pi + iy) \leqq \text{Mod}(\alpha - 1) \cdot \frac{1}{m\pi} \cdot \frac{C\left(\frac{r}{l} y\right) \cdot C\left(\frac{\rho}{l} y\right)}{Cy^2}$$

Es ist aber, wofern

$$0 < r < l, \quad 0 \leqq \rho \overline{\gtrless} l,$$

für jeden Werth von y

$$C\left(\frac{r}{l} y\right) \geqq Cy, \quad C\left(\frac{\rho}{l} y\right) \geqq Cy,$$

folglich

$$\frac{C\left(\frac{r}{l} y\right)}{Cy} \cdot \frac{C\left(\frac{\rho}{l} y\right)}{Cy} \leqq 1.$$

Wir haben demnach

$$\text{Mod}\ w\ (m\pi + iy) \leqq \ \text{Mod}\ (\alpha - 1) \cdot \frac{1}{m\pi}\ .$$

Wir wenden den bekannten Modulsatz an und erhalten

$$\begin{aligned}\text{Mod}\ Q &\leqq \int_{-\sqrt{m\pi}}^{\sqrt{m\pi}} \text{Mod}\ w\ (m\pi + iy)\ dy \\ &\leqq \text{Mod}\ (\alpha - 1) \cdot \frac{1}{m\pi} \int_{-\sqrt{m\pi}}^{\sqrt{m\pi}} dy \\ &\leqq \text{Mod}\ (\alpha - 1) \cdot \frac{2}{\sqrt{m\pi}}\ ,\end{aligned}$$

gültig für jeden der Ungleichheit $0 < r < l$ genügenden Werth von r und für jeden der Ungleichheit $0 \leqq \rho \gtreqqless l$ genügenden Werth von ρ.

Untersuchen wir nun

$$P = \int \begin{vmatrix} B \\ C \\ A \end{vmatrix} w\ dz = \int \begin{vmatrix} B \\ C \\ A \end{vmatrix} (z.\ w)\ \frac{dz}{z}$$

und bezeichnen zu dem Ende durch θ das Azimuth von z, so ist auf der Kreisperipherie $A\ C\ B$

$$z = \sqrt{m^2\pi^2 + m\pi} \cdot e^{i\theta}\ , \quad \frac{dz}{z} = id\theta\ ;$$

also

$$P = i \int \begin{vmatrix} B \\ C \\ A \end{vmatrix} (z.\ w)\ d\theta\ ,$$

woraus weiter folgt :

$$\text{Mod}\ P \leqq \int \begin{vmatrix} B \\ C \\ A \end{vmatrix} \text{Mod}\ (z.\ w)\ d\theta\ .$$

Zufolge der Construction des Kreisbogens $A\ C\ B$, ist auf demselben

$$y \geqq \sqrt{m\pi}$$

Es lässt sich nun leicht zeigen (Vergl. loc. cit. §. 5.) dass, wofern y von Null verscheiden ist,

$$\operatorname{Mod} \phi\ (x + iy) \geqq (x^2 + y^2)^{\frac{1}{2}} \,.\, Sy.\, \frac{1}{\Omega},$$

wo Ω eine stets von Null verscheidene für erhebliche Werth von y nahezu gleich der Einheit und mit wachsendem y schliesslich gegen die Einheit convergirende Zahl bedeutet. Unter Festhaltung dieser Bedeutung von Ω findet man weiter für die Werthe von y, die von Null verschieden sind,

$$\operatorname{Mod}\ (z.\, w) \leqq \operatorname{Mod}\ (\alpha - 1) \cdot \Omega \cdot \frac{C\left(\frac{r}{l}\, y\right) \cdot C\left(\frac{\rho}{l}\, y\right)}{Sy^2}$$

Auf dem Kreisbogen $A\ C\ B$ ist nun y positiv. Mit Rücksicht hierauf bringen wir

$$\Omega \cdot \frac{C\left(\frac{r}{l}\, y\right) \cdot C\left(\frac{\rho}{l}\, y\right)}{Sy^2}$$

in die Form:

$$\Omega \cdot \frac{\left(1 + e^{-2\frac{r}{l} y}\right)\ \left(1 + e^{-2\frac{\rho}{l} y}\right)}{(1 - e^{-2y})^2} \cdot e^{-2\left(2 - \frac{r+\rho}{l}\right) y}\ .$$

Für erhebliche positive Werthe von y ist der Faktor

$$\frac{\left(1 + e^{-2\frac{r}{l} y}\right)\ \left(1 + e^{-2\frac{\rho}{l} y}\right)}{(1 - e^{-2y})^2}$$

nur sehr wenig von der Einheit verschieden und convergirt mit wachsendem y sehr rasch gegen dieselbe; dasselbe gilt auch von Ω, also auch von ihrem Producte, welches wir der Kürze halber mit ω bezeichnen wollen, so dass

$$\omega = \Omega \cdot \frac{\left(1+e^{-2\frac{r}{l}}\right)\left(1+e^{-2\frac{\rho}{l}}\right)}{\left(1-e^{-2y}\right)^2} .$$

Der Faktor

$$e^{-2\left(2-\frac{r+\rho}{l}\right)y}$$

hat, wofern $y > 0$, $0 < r < l$ und $0 \leqq \rho \gtreqless l$, als Funktion von ρ betrachtet, seinen grössten Werth für $\rho = l$, und dieser Werth ist

$$e^{-2\left(1-\frac{r}{l}\right)y} .$$

Wir haben demnach

$$\text{Mod}\,(z.\,w) \leqq \text{Mod}\,(\alpha-1)\,w.\,e^{-2\left(1-\frac{r}{l}\right)y} ,$$

gültig für jeden der Ungleichheit $0 \leqq \rho \gtreqless l$ genügenden Werth von ρ.

Auf dem Kreisbogen $A\,C\,B$ ist

$$y = \sqrt{m^2\pi^2 + m\pi}\ \sin\theta ;$$

mithin

$$\text{Mod}\,P \leqq \text{Mod}\,(\alpha-1) \int_{A}^{B} \Big|_{C}\ \omega\, e^{2\left(1-\frac{r}{l}\right)\sqrt{m^2\pi^2+m\pi}\ \sin\theta}\, d\theta .$$

Die beiden Faktoren unter dem Integralzeichen sind positiv; es ist desshalb nach dem Mittelsatze von Cauchy

$$\text{Mod}\,P \leqq \text{Mod}\,(\alpha-1)\,\omega_0 \int_{A}^{B}\Big|_{C}\ e^{-2\left(1-\frac{r}{l}\right)\sqrt{m^2\pi^2+m\pi}\ \sin\theta}\, d\theta ,$$

wo ω_0 einen gewissen Mittelwerth von ω auf dem Kreisbogen $A\,C\,B$ bedeutet, welcher für erhebliche Werthe von m nahezu gleich Eins wird, und mit wachsendem m gegen die Einheit convergirt.

Es ist nun

$$\int_0^{\pi} e^{-C \sin\theta}\, d\theta < \frac{\pi}{C}$$

und

$$\int\begin{vmatrix} B \\ C \\ A \end{vmatrix} e^{-C \sin\theta}\, d\theta < \int_0^{\pi} e^{-C \sin\theta}\, d\theta\,.$$

Es folgt hieraus

$$\operatorname{Mod} P \leqq \operatorname{Mod}(\alpha - 1) \cdot \omega_0 \cdot \frac{\pi}{2\left(1 - \frac{r}{l}\right)\sqrt{m^2\pi^2 + m\pi}}$$

gültig für jeden der Ungleichheit $0 \leqq \rho \leqq l$ genügenden Werth von ρ.

Somit erhalten wir für jeden der Ungleichheit $0 \leqq \rho \leqq l$ genügenden Werth von ρ die Ungleichheit

$$\operatorname{Mod} Q + \operatorname{Mod} P \leqq \operatorname{Mod}(\alpha - 1) \cdot \frac{2}{\sqrt{m\pi}}$$

$$+ \operatorname{Mod}(\alpha - 1) \cdot \omega_0 \cdot \frac{\pi}{2\left(1 - \frac{r}{l}\right)\sqrt{m^2\pi^2 + m\pi}}\,,$$

mit den Zusatze, dass ω_0 eine für erhebliche Werthe von m nahezu der Einheit gleiche und mit wachsendem m gegen Eins convergirende Zahl bedeutet.

§. 4.

Wir verstärken die Ungleichheit

$$\operatorname{Mod} \varDelta \leqq \frac{1}{r.l.\pi} \int_0^l \rho \operatorname{Mod} f(\rho)\,[\operatorname{Mod} Q + \operatorname{Mod} P]\, d\rho,$$

dadurch dass wir [Mod Q + Mod P] durch die in dem vorangehenden

Paragraph nachgewiesene Zahl ersetzen, welche die Eigenschaft hat, für jeden $0 \leqq \rho \lesseqgtr l$ genügenden Werth von ρ stets grösser als $[\,\mathrm{Mod}\ Q + \mathrm{Mod}\ P\,]$ zu sein:

$$\mathrm{Mod}\ \Delta \leqq \frac{1}{r.\,l.\,\pi} \cdot \mathrm{Mod}\,(\alpha-1) \Bigg[\frac{2}{\sqrt{m\pi}} + \omega_0 \frac{\pi}{2\left(1-\frac{r}{l}\right)\sqrt{m^2\pi^2+m\pi}} \Bigg] \int_0^l \rho\ \mathrm{Mod}\, f(\rho)\, d\rho.$$

Den grössten Werth von $\mathrm{Mod} f(\rho)$ zwischen den Grenzen 0 und l, welcher der Voraussetzung nach endlich ist, bezeichnen wir mit A; so ist

$$\int_0^l \rho\ \mathrm{Mod}\, f(\rho)\, d\rho \lesseqgtr \int_0^l \rho\, A\, d\rho$$

$$\lesseqgtr \frac{A l^2}{2},$$

wo die Gleichheit stattfindet, wenn $f(\rho)$ constant und gleich A ist. Mithin haben wir endlich

$$\mathrm{Mod}\ \Delta \leqq \frac{1}{r} \cdot \mathrm{Mod}\,(\alpha-1) \cdot A\, \frac{l}{2} \Bigg[\frac{2\pi}{\sqrt{m\pi}} + \omega_0 \frac{1}{2\left(1-\frac{r}{l}\right)\sqrt{m^2\pi^2+m\pi}} \Bigg].$$

Lässt man nun hierin m, die Reihe der ganzen Zahlen durchlaufend, über alle Grenze hinauswachsen, wofern $0 < r < l$ ist; es folgt, dass $\mathrm{Mod}\ \Delta$, also auch Δ selbst verschwindet zwischen $r = 0$ und $r = l$ mit Ausschluss der Grenzen, wenn die Gliederzahl m ins Unendliche wachst.

Es ist somit bewiesen (Vergl §. 2. zu Ende) dass die Differenz der beiden Summen $v - v'$ gleich Null ist; d. h. dass, da v' für sich allein convergirt und mit Ausschluss der Grenzen $r = 0$ und $r = l$ zwischen denselben $f(r)$ zur Summe hat, dasselbe also auch von v

gilt. Wir gelangen also zu dem Satze:

Die unendliche Reihe

$$v = \frac{1}{r}\sum_{n=1}^{\infty} \sin\left(\lambda_n \frac{r}{l}\right) \frac{\int_0^l \rho f(\rho) \sin\left(\lambda_n \frac{\rho}{l}\right) d\rho}{\int_0^l \left(\sin \lambda_n \frac{\rho}{l}\right)^2 d\rho}$$

hat $f(r)$ zur Summe für $0 < r < l$ mit Ausschluss der Grenzen $r = 0$ und $r = l$.

On the Composition of Bird-lime.

by

Edward Divers, M.D., F.R.S. Prof.,

and

Michitada Kawakita, M.E., F.C.S. Asst. Prof.,

Imperial University.

Bird-lime seems never to have been examined to the extent to yield results deemed worthy of publication, until the year 1884, when J. Personne made known those of his father's and his own examination of it in the *Comptes rendus*, **98**, 1585. When that paper appeared we ourselves had been for some time occupied with the investigation of Japanese bird-lime, and had already obtained results, which proved to be in general agreement with those of Personne's examination, and yet sufficiently unlike them, and in some respects in advance of them, to lead us to continue our work, although he promised further attention to the subject. Up to the present date, however, nothing more from him has appeared, and we now offer this paper as an extension and partial confirmation of his observations.

Bird-lime, or *Tori-mochi*, is prepared in Japan, just as it is in Northern Europe, from a species of holly, by macerating and pounding its inner bark in water, afterwards picking out the fragments of crushed tissues from the viscid mass. Bird-lime exists, ready-formed in the bark, in great abundance, and is not apparently modified in any way by fermentative action during its preparation. In Europe

it is prepared from the Common or Prickly-leaved Holly, (*Ilex Aquifolium*), but in Japan it is obtained from *Mochi no ki*, the *I. integra* of Thunberg, (*Prinus integra*, H. & A.). We are not familiar with bird-lime as prepared in Europe, but judging from descriptions, Japanese bird-lime is like it, except perhaps in not having a greenish hue, though of that even we are not certain, since the Japanese product may well have it sometimes, when quite freshly prepared. Bird-lime is extensively used in Japan, as in Europe, for catching birds and insects, and with the usually attendant cruelty.

In manuals of economic botany we find enumerated as peculiar constituents of the holly, a bitter principle named *ilicine*, an aromatic resin, and bird-lime itself. In the account of bird-lime given in Ure's *Dictionary*, the true substance is not well distinguished from the viscid matter of Mistletoe, (*Viscum album*), examined by Reinsch, from which it appears to be entirely different.

Some properties of (Japanese) bird-lime.

Bird-lime is pale greyish, nearly opaque, of faint, peculiar odour, almost tasteless, soft, elastic, tenacious and very adhesive to dry surfaces, and slightly lighter than water. It can be preserved in water for an indefinite time without change, except on its upper surface. Exposed to air it very slowly turns brown outside, and becomes coated with a thin brittle skin. Heated moderately, it gives off water, and above 100° froths, through disengagement of steam. By the loss of its moisture it becomes transparent, brown, and while hot, of the consistency of cold oil. If now allowed to cool, it retains its transparency, and forms a soft solid mass, elastic, tenacious, and sticky, as before, somewhat resembling Canada balsam in appearance.

Ether, carbon bisulphide, chloroform, light petroleum, and benzene dissolve bird-lime, leaving a residue which, though of not in-

considerable volume, is of little weight. Cold alcohol scarcely dissolves it at all, and even hot alcohol, which has some solvent action at first, attacks merely the surface-portion of the mass. The alcohol solution deposits, as it cools, a nearly colourless, transparent, adhesive matter, little different from purified bird-lime itself. Ether is much to be preferred to other solvents, because it yields a clear solution, whereas carbon bisulphide and the rest give milky liquids, owing to the presence of water. The ether solution mixed with alcohol becomes turbid and deposits a tenacious mass.

Freed from water and particles of woody fibre, bird-lime undergoes, when heated, scarcely any change up to about 350°, only becoming slightly fluorescent and a little darker in colour, and acquiring a feeble waxy odour. But about the melting point of zinc, it suffers destructive distillation, in which most of it comes over as fatty acids and fluorescent hydrocarbons of waxy and mild empyreumatic odour and buttery consistence, very little permanent gas being formed, and only a small carbonaceous residue being left. Bird-lime burns in the air with a bright smoky flame.

It is not very sensitive to reagents. *Sulphuric acid* dissolves it slowly, forming a red liquid, which blackens only when heated, and which gives, when poured into water, a viscid precipitate like bird-lime, but dark coloured. Boiling *nitric acid* of moderate strength slowly dissolves it, with partial oxidation, this solution also precipitating with water. The sulphuric-acid solution poured into concentrated nitric acid yields on addition of water a precipitate of a feebly nitrated mixture of bodies. Aqueous solutions of *potassium hydroxide* only slowly and slightly emulsify bird-lime. Fusion with the hydroxide is attended with much darkening in colour and leaves a mass which emulsifies in water. Potassium hydroxide in hot spirit slowly dissolves the greater part of purified bird-lime, producing a dark-

coloured solution. In this way, that is, by continued boiling with strong alcoholic potash, bird-lime has been attacked by both Personne and ourselves, in order to determine its composition.

The Constituents of bird-lime.

Personne has found bird-lime, prepared from *I. Aquifolium*, to contain *water* 27, and *vegetable débris* and *calcareous salts* 23 parts per cent., the remaining and essential part being some *caoutchouc*, the *compound ether*, or ethers, of *a new alcohol*, and other matters undetermined. The acids or acid forming the ethers were also not determined by him. He isolated the caoutchouc by saponifying the ethers with alcoholic potash, which left it undissolved.

Japanese bird-lime is much cleaner than that described by Personne, containing only 2 per cent. of dry bark fragments, and no separate lime salts. But its water-content is larger, (probably because it is kept in stock under water), the percentage lost at 110°–120° being 38. Caoutchouc forms about 6 per cent., leaving 54 per cent. as the proportion of compound ethers and allied matters.

The bark, etc.—Of the 23 parts per cent. found in French bird-lime by Personne, some 13 parts consisted of *calcium oxalate*. On boiling out the bark fragments from Japanese bird-lime with sodium carbonate, some oxalate was dissolved out but only in small quantity. The bark burnt to ashes gave as much as 6.3 per cent. of ash, principally calcareous and largely phosphate, but with of course some carbonate. But as the whole ash was only one-eighth per cent. of the entire bird-lime, and as only a little of the calcium salts was oxalate, Japanese differs, in this respect, remarkably from French bird-lime.

The caoutchouc.—As we have stated, the caoutchouc can be separated by boiling out the purified bird-lime with alcoholic potash, and

this is the best way of proceeding. It is, however, difficult to get quite free from potash, and needs, to this end, to be repeatedly dissolved in ether and reprecipitated by alcohol. The caoutchouc can also be separated by dissolving the bird-lime in ether and precipitating the solution with 95 % spirit, but then only very imperfectly, because the main constituent of the bird-lime also precipitates partly. The caoutchouc of bird-lime is pale yellow and transparent, highly elastic, and when heated evolves the well-known penetrating odour. A combustion analysis of it gave us carbon 86.56, and hydrogen 11.31 per cent., so that oxygen to the extent of 2 per cent. was present. Before weighing it out, it had been kept for some time at 120°–130°. It left when burnt a trace of ash.

Other and principal constituents of bird-lime.—We have not fully isolated these by proximate analytical methods, but their general properties appear to be those of the partially purified bird-lime. For when a boiling spirit-solution of bird-lime is evaporated and cooled, and again when an ether-solution of bird-lime is mixed with a little alcohol, to separate the caoutchouc, and then evaporated, in both cases the solid matter obtained is quite like the partially purified bird-lime, except in being without colour when deposited from the cooling spirit solution.

Products of the saponification of bird-lime, and their isolation.—Saponification with alcoholic potash yields, besides the residual caoutchouc, firstly, the potassium salt of *palmitic acid* and a very little of that of a *semi-solid acid* which we have been unable to purify or identify; secondly, *two crystalline alcohols*; and thirdly, a small quantity of a *resinoïd body*. The separation of these bodies may be carried out in somewhat different ways, and is unavoidably tedious. The purified bird-lime is boiled for two hours with potash and 95 % spirit, in a flask fitted with a condenser; the alkaline solution,

decanted from the caoutchouc, is poured into dilute spirit, by which a voluminous, gelatinous precipitate is produced, consisting of the alcohols with some of the resinoïd body and potassium palmitate. The precipitate is well broken up by stirring, collected on a cloth filter, pressed, and washed with dilute spirit. The washing can only be very imperfectly effected. Three ways of proceeding from this point have been practised by us.

In one the precipitate is diffused through dilute spirit, and stirred well and warmed with calcium-chloride solution. The now much less voluminous precipitate is repeatedly washed with water, dried, and extracted with ether, which leaves the calcium palmitate undissolved. Spirit of 95 % may be used in place of ether, but as it dissolves out a little calcium salt its use is less satisfactory. On evaporating the ether (or spirit) the alcohols and resinoïd body are obtained. A second way of proceeding is to warm the precipitate with water and hydrochloric acid, until it has shrunken to a small volume, wash repeatedly with water, press, dry, and extract with light petroleum, which dissolves out the palmitic acid and some of the resinoïd body, and leaves behind all the alcohols and the rest of the resinoïd body. After proceeding in either way, the resinoïd body is separated by repeated extractions with warm 80 % spirit. A small quantity of the alcohols at the same time dissolves, and may be partly recovered by precipitation with a very little water and extracting the precipitate with 80 % spirit. The third way of proceeding, which is simpler in execution than the others, but much less effective, is to use 70–80 % spirit in place of the light petroleum, in the second way of work. This dissolves out the resinoïd body as well as the palmitic acid.

Personne's method of procedure is to pour the product of saponification into water, to wash the precipitate with much water, treat it

with acetic acid to neutral reaction, again wash, dry, dissolve in hot 90 % spirit, cool, and crystallise out from the solution the bird-lime alcohol. This method we have not found to work well, because of the great difficulty in washing properly the voluminous gelatinous precipitate, and in just neutralising it with acetic acid. This precipitate contains, besides the alcohols and resinoïd body, much acid potassium palmitate, to which indeed its bulky state is partly due, and we have found it far preferable to convert the potassium palmitate either into the calcium salt or into free acid, as above described. Personne seems not to have recognised the presence of any fatty salt in the precipitate of the alcohols.

Separation of the alcohols from each other, and their purification.—The crude solid alcohols can only be fully separated from each other by fractionated extraction with strong spirit, repeated until products are obtained of constant melting-point. The alcohols, already treated, as described, with 80 % spirit to remove the resinoïd body, are warmed with successive portions of spirit increasing in strength from about 85 % to over 90 %, each portion of the solvent depositing crystals of the alcohols as it cools, and each mother-liquor, by successive evaporations, yielding a series of other crystalline deposits, all similar in appearance. When the last mother-liquors are too small in quantity and too impure to yield a satisfactory product by further evaporation, they are rejected or worked up for the little resinoïd body they contain. The portions of the alcohols least soluble in spirit consist principally of the one alcohol, and those most soluble, of the other alcohol. The intermediate portions yield by renewals of the treatment with spirit other series of deposits of higher and lower degrees of solubility, the extremes of which are the two alcohols nearly free of each other. The portions of the less soluble alcohol are submitted to further fractionation, until the part undis-

solved by hot 90 °/₀ spirit, and that dissolved and deposited by it on cooling have the same melting point. It is then, finally, dissolved in hot 95 °/₀ spirit, crystallised out, and again tested as to its melting point. The crystalline deposits most soluble, and consisting principally of the more soluble alcohol, require much further fractionation, in order to separate the less soluble alcohol on the one side, and the resinoïd body on the other, and the ultimate yield of the pure alcohol becomes very small. In fractionating it out, spirit of 85 °/₀ is used, but finally this alcohol, like the other, is to be crystallised out from 95 °/₀ spirit, in order to get good crystals.

Personne observed the comparative insolubility of the solid alcohol in 80 °/₀ spirit, but making no use of this fact, he purified the cake of crude solid alcohol by repeated crystallisations from boiling 90 °/₀ spirit. During the purification he met with a body of peculiar form, visible under the microscope and less soluble than the solid alcohol in spirit, and this he found to be gradually removed by repeated crystallisations. We have met with no such substance in Japanese bird-lime.

Purification of the resinoïd body.—This is found mainly in the 80 °/₀ spirit used to wash the crude alcohols after they have been separated from palmitic acid. When this separation has been effected in the second way, the spirit contains also some fatty acids. The light petroleum used to dissolve out palmitic acid also contains some of the resinoïd body. In order, therefore, to separate palmitic acid, the residue, after evaporating the spirit or the petroleum, is dissolved in alcoholic potash, the palmitic acid precipitated with calcium chloride, water added, and the precipitate washed, dried, and extracted with ether. Evaporation of the ether leaves the resinoïd body still mixed with some of the alcohols, but free from any fatty acid. The impure product is dissolved in strong spirit, and left to evaporate slowly.

The alcohols separate as indistinctly crystalline, opaque matter, while the resin forms a translucent, gummy deposit, still containing spirit, on the bottom an l sides of the vessel. The resin is redissolved in spirit and the solution left to evaporate. Repeating these operations several times yields it in a condition in which it shows no longer any tendency to deposit crystalline matter.

Separation and purification of the fatty acids.—By far the greater part of the fatty salts remains dissolved when the saponified bird-lime solution is poured into dilute spirit. The filtrate and washings from the gelatinous precipitate of alcohols are diluted with water, mixed with hydrochloric acid, and warmed, in order to separate the fatty acids. By similar and well-known methods the portions of these acids precipitating with the bird-lime alcohols can be recovered, after separating them as calcium salts from the alcohols and resinoïd body, and added to the main quantity. The crude fatty acids which, when cold, form a soft, brown, solid msss, are dissolved in alcoholic potash and precipitated again with calcium chloride; the calcium precipitate is washed with spirit which removes chlorides and some colouring matter, as well as some of the calcium salt of the soft fatty acid; the precipitate is then washed with ether, which dissolves out, more easily than the spirit, most of the remaining colouring matter and calcium salt of the soft fatty acid; lastly, it is heated with hydrochloric acid and water, in order to get the crude palmitic acid. Repeating these operations once or twice, and finally crystallising it from its spirit-solution, gives the palmitic acid pure. The spirit and ether washings of the calcium precipitate yield by appropriate treatment the semiliquid acid, still in an impure condition.

Palmitic acid can also be prepared from bird-lime by destructive distillation. Its purification from hydrocarbons by way of saponification, presents no great difficulty, and need not be described.

The alcohols of bird-lime.

To one of the two alcohols of bird-lime we give the name *mochylic alcohol*, formed from the Japanese word *mochi* for (bird) lime or glutinous matter, and to the other we attach the name *ilicylic alcohol*, essentially the same as *ilicic alcohol* given by Personne to the single alcohol described by him, but framed more in accordance with the accepted nomenclature for alcohols. Our ilicylic alcohol differs but little from Personne's ilicic alcohol.

Both the alcohols of bird-lime are obtained in tufts of small, slender, lustrous prisms, and are distinguishable from each other only in solubility, in melting point, and in composition.

Mochylic alcohol occurs much more abundantly than ilicylic alcohol. It dissolves well in 95 – 98 % spirit, but is almost insoluble in 80 % spirit. It is very little soluble in petroleum spirit in the cold, is readily soluble in ether, and dissolves also in concentrated sulphuric acid, to which, like bird-lime itself, it imparts a red colour. It melts at 234° C and decomposes under atmospheric pressure at a little below the melting point of zinc, the principal product being a viscid matter, apparently the hydrocarbon to be described among the products of the destructive distillation of bird-lime. In a vacuum it sublimes slightly at a little above 160°, and freely and entirely near and above its melting point, without decomposing, or changing in melting point. Heated with palmitic acid in a sealed tube to 150 – 160° it yields a body indistinguishable in essential properties from bird-lime, a sticky transparent matter, readily soluble in ether, but nearly insoluble in the strongest spirit. Our attempts to form, by acetic oxide or chloride, mochyl acetate have been unsuccessful.

Ilicylic alcohol differs from mochylic alcohol in melting at 172°, and in being moderately soluble in 85 – 90 % spirit, though almost

insoluble in 80 % spirit. It begins to volatilise in a vacuum below 150°, and sublimes freely near its melting point in beautiful tufts of needles, still melting at 172°. Heated with palmitic acid it also forms a body like bird-lime. It fails apparently to yield an acetate, even after long heating at 150–170° with acetic oxide, in which when hot it, as also mochylic alcohol, readily dissolves, partly crystallising out again unchanged on cooling, and partly becoming a dark viscid matter not acetate. Personne found his ilicic alcohol to yield a crystalline acetate with acetic oxide, melting at 204°–6°. The melting point of Personne's ilicic alcohol was 175°, and its boiling point above 350°, but under the reduced pressure of 100 mms. it began to sublime at 115°. In appearance and in behaviour to spirit of different strengths, it was like our ilicylic alcohol. Both mochylic and ilicylic alcohols dissolve in a mixture of sulphuric and nitric acids, and from the solution water separates a gelatinous matter, readily soluble in spirit, and puffing only slightly, when dried and heated.

Chemical composition of the two alcohols.—Combustion of the two alcohols has given us the following results:—

Mochylic alcohol, m. p., 234°

	I.	II.	III.	$C_{26}H_{46}O$.
Carbon	83.37	83.39	83.28	83.42
Hydrogen	12.29	12.16	12.38	12.30
Oxygen				4.28
				100.00

Ilicylic alcohol, m. p., 172°

	I.	II.	$C_{22}H_{38}O$
Carbon	83.09	82.98	83.02
Hydrogen	11.93	11.92	11.95
Oxygen			5.03
			100.00

Ilicic alcohol, m. p., 175° (Personne's analyses).

	I.	II.	III.	IV.	V.	Mean	$C_{25}H_{44}O$
Carbon	83.25	83.64	83.48	83.07	83.40	83.36	83.33
Hydrogen	12.18	12.44	12.17	12.24	11.98	12.20	12,22
Oxygen							4.45
							100.00

It will be seen that Personne's numbers vary rather widely, but fall for the most part between those obtained by us for our two alcohols. It will also be seen that the formula he has proposed, as agreeing best with the mean of his analyses, is that of a homologue of our alcohols, the general expression being $C_n H_{2n-6} O$. As he worked upon bird-lime from a species of Ilex different from that which yields Japanese bird-lime, it cannot for the present be decided whether ilicic alcohol is distinct from the alcohols here described.

The resinoïd component of bird-lime.

The resinoïd body is obtained in pale-yellow fragments which are brittle, and not sticky like bird-lime. It melts at 110°, and does not volatilise when heated to 220° in a vacuum. Above 360° it darkens, boils, and distils without much apparent change. It is very soluble in spirit, even of only 80 % strength, also in ether. When its spirit solution is evaporated by heat sufficiently, it separates from its solvent as a viscid liquid still containing spirit, which evaporates by further heating below 100°. Its solubility in spirit is not increased by the presence of potassium hydroxide. Heated with the solid hydroxide barely to the melting point, it slowly combines with it, probably at the same time absorbing oxygen. The cooled mass wholly dissolves in water from which hydrochloric acid precipitates a gelatinous body very brittle when dried. We have not further examined it, for want of material.

When bird-lime is kept for a long time, a thin brittle skin forms on its surface, which is readily soluble in spirit. This skin consists probably of the resinoïd body. If it does not, then we have no evidence as to whether the resinoïd body is produced during the saponification of the bird-lime, or exists in it ready-formed, as the result of slow atmospheric oxidation.

In composition the resinoïd body differs from mochylic alcohol only in having two atoms less of hydrogen, as the following analyses and calculation show:—

	I.	II.	$C_{26}H_{44}O$
Carbon	83.79	83.66	83.87
Hydrogen	11.80	11.92	11.83
Oxygen			4.30
			100.00

The fatty acids of bird-lime.

The fatty acids of bird-lime are two, as already stated, *palmitic acid*, and, in small quantity only, a semi-liquid acid, the calcium salt of which is soluble in spirit and in ether. This acid has not been further examined. The other shows all the characters of palmitic acid. Melting point, 61.5°. Analysis (I.) of acid prepared by saponification, and (II.) of acid obtained by destructive distillation of purified bird-lime:—

	I.	II.	Palmitic acid.
Carbon	74.98	74.86	75.00
Hydrogen	12.67	12.55	12.50
Oxygen			12.50
			100.00

The potassium salt yielded 13.3 per cent. of potassium.

Products of destructive distillation of bird-lime.

These have been already enumerated, so far as their nature is known to us, and the result of analysis of the palmitic acid has just been tabulated. The principal hydrocarbon, distilling next after the palmitic acid, was prepared from the middle portion of the distillate, by treating it with hot spirit so as to leave about half undissolved. This was then washed with cold spirit. The hydrocarbon thus left was a thick oil, slightly yellow, but free from fluorescence. On analysis, it gave numbers agreeing with the formula, $C_{26}H_{44}$:—

	Found	Calculated
Carbon	87.59	87.64
Hydrogen	12.49	12.36
	100.08	100.00

Apparently the same body is obtained by distilling mochylic alcohol at the ordinary atmospheric pressure. The decomposition of the main constituent of bird-lime by heat may therefore be thus represented:—

$$\underset{\text{Mochyl palmitate.}}{C_{42}H_{76}O_2} = C_{26}H_{44} + C_{16}H_{32}O_2$$

and the decomposition of mochylic alcohol by—

$$C_{26}H_{46}O = C_{26}H_{44} + H_2O.$$

The last fractions of the distillate consisted of hydrocarbons yielding nearly 91 per cent. of carbon. No attempt was made to isolate the caoutchouc hydrocarbons present, no doubt, in the mixture.

Constitution of bird-lime.

Bird-lime is closely allied to the *waxes*, and consists principally of *mochyl* and *ilicyl palmitates*, $C_{42}H_{76}O_2$ and $C_{38}H_{68}O_2$.

On Anorthite from Miyakejima.

by

Yasushi Kikuchi.

Assistant Professor of Geology, Imperial University.

With Plate I.

The Anorthite-crystals which form the subject of the present communication were collected by Messrs. S. Ōkubo and N. Okada of the Imperial University on the Island of Miyake, forming one of the Shichitō or Seven Islands Group, situated to the south of the Izu peninsula. My best thanks are due to those gentlemen for kindly placing the specimens at my disposal.

The Shichitō Group is part of a chain of volcanoes of which Fujiyama is the highest, having its origin in Central Japan, and stretching in a SSE direction toward the Ladrone Islands in the North Pacific. The volcano on the Island of Miyake is known to have erupted occasionally within historical times. The last of these eruptions took place in the 3rd of July 1874 (7th year of Meiji) and destroyed by a lava-flow, a place called Tōgō in the village of Kamitsuki.

The Anorthite-crystals here described are found forming a primary constituent of a basaltic lava. It is however remarkable, that these crystals, besides entering into the porphyritic constituent of the lava, are found scattered on the lava-field formed by the eruption just mentioned, as well-defined separate crystals, in a manner which admits of no doubt as to their having been thrown out of the crater as such.* A. Penck† has indicated the possibility of such

* During a recent visit to Sulphur Island, I have observed the same phenomenon. Felspar crystals, the nature of which awaits further investigation, are found, covered with a black lava-crust, and forming a loose superficial layer in the vicinity of a volcano on that Island.

† Studien über lockere vulkanische Auswürflinge—Zeitschft. d. deutsch. geol. Gesellft. Bd. 30 1874. p. 124.

crystal-ejections near the crater of a volcano. A case analogous to that here described is met with in the eruptions of Vesuvius; crystals of Leucite which there play the rôle of Felspar, being sometimes ejected. Thus Leucite eruptions continued from April 1845 to January 1849, according to Scacchi,* who considers these well-defined Leucite crystals as being derived from the refusion of the crystals already formed in the older lava within the volcano.

From the basic character of the Anorthite it is to be inferred that it had early crystallized out of the magma, and that the crystals thus formed were ejected while the latter was still in a liquid condition; the heavier crystals falling near the crater, while the lighter portions of the ejected matter were carried further off. On this account, they have always a thin vesicular coating of lava, the interior however remaining uninjured.

Similar kinds of crystals have also been brought from the other islands of the Shichitō Chain, viz. Ōshima (Vries Island) and Hachijō.

Most of the specimens of lava, which I have seen are of a black colour, and have a porphyritic structure, mostly porous but sometimes very compact. The porphyritic ingredients are Anorthite-crystals of the Microtine type, and Olivine, usually in the form of rounded grains, and very rarely showing crystal-form, as S. v. Waltershausen † observed on the Olivine in the lava of Etna. A coating of red iron oxide is always found on these Olivine grains, often assuming a brilliant metallic lustre. Microscopically examined, the ground-mass is seen to consist of a microcrystalline aggregate of Felspar and Magnetite, with sometimes microcrystals of Apatite.

* Ueber den Ursprung der vulkanischen Asche.—Auszug by Rammelsberg in Zeitschft. d. deutsch. geol. Gesellschft. Bd. 24. 1872. p. 548.

† Vulkanische Gesteine in Sicilien und Island. p. 161.

Generally a brown coloured glass basis is found between these crystals. Anorthite, Olivine, and Augite constitute the essential microscopic porphyritic components. The order of crystallization of these components can pretty certainly be described as follows: — Apatite, Magnetite, Olivine, Anorthite, and Augite. A specimen of Basalt-glass or Tachylite is also known. Under the microscope, it is found to consist of a brown glass, in which well-defined crystals of Anorthite, Augite-microliths, Olivine and Magnetite are developed. The recent lava of Miyake may therefore be called Anorthite-basalt.

The volcanic rock composing the upper part of Fujiyama is, according to Lüdecke and Wada,* Anorthite-basalt. It is probable that this rock constitutes most of the lavas of comparatively recent eruptions in the volcanoes of the Shichitō Group. The chemical analysis of the Anorthite forming the porphyritic component of the rock found near Tōnosawa, situated within the extinct volcano of Hakone, was first published by Wada.* But unfortunately the crystallographic and optical characters could not be well investigated because of the nature of these crystals. Certain glassy Felspar crystals which had been brought from the Island of Ōshima and known as Sanidine, were found on examination to be completely identical with the specimens from Miyake. Dr. E. Naumann † states in his account of the Island of Ōshima, that Sanidine crystals occur very abundantly in the lava of that Island. So far, however, I have been unable to find any Sanidine crystal in any of the collections brought from Ōshima, which have been to my hand.

The Anorthite-crystals from Miyakejima are $1-4_{\mathrm{cm.}}$ in their longest direction, and are always covered with a black or sometimes

* Notes on Fujiyama—Transactions of the Seism. Soc. of Japan. Vol. IV 1882.

† Die Vulkaninsel Ooshima u. ihre jungste Eruption—Zeitschft. d. deutsch. geol. Gesellft. Bd. 29. 1877. p. 378.

reddish coloured thin spongy crust of lava. This thin coating examined under the microscope, is found to consist of a brown amorphous glass, sometimes with microcrystals of Plagioclase, a few Augite grains and Magnetite. The interior of the crystal in fresh specimens is transparent, and has a hyaline lustre. The cleavage is perfect parallel to P, and M. The latter is less perfect than P, and the cleavage-face always exhibits twin-striæ. A slightly pearly lustre is observed on both of these cleavage-faces; on one portion of the brachypinacoidal cleavage-plane, there has been observed in one specimen, a play of colour somewhat like that of Labradorite. Grains of Olivine are always found as enclosures, sometimes arranged in distinct zones. In some portions glass-enclosures are found often arranged parallel to a crystallographic direction. Some specimens show the beginning of decomposition, becoming hazy in appearance. Single individuals are rather rare; they are mostly found as twins, or several individuals are grouped together in a most irregular manner, assuming a globular mass.

Crystal-form.—The thin coating of lava which always covered the crystals, rendered the determination of the crystal-faces peculiarly difficult. The reflecting goniometer could not be used, except in the cleavage-faces. By measuring the angle between cleavage-faces $P:M$, and by finding the position of the face y. which is always developed, and the position of the Pericline twin-lamellæ on M, I was able generally to fix the position of the crystal, so that the other faces could be determined by approximate measurements with a contact goniometer and by zonal relations. It was not easy to get a perfectly clear cleavage-piece for goniometric and optical purposes, as the twin-striæ, Olivine-enclosures, and numerous fissures which make the crystals very brittle, were always present. The cleavage-face parallel to P usually gave a good reflection, but that parallel to M, a dis-

turbed or diffused reflection. Hence the measurement of this facial angle did not give always a satisfactory result. With the clearest piece which I could get, the angle of $P : M$ (001 : 010) was found to be 94° 8′ and the angle $P : M'$ (001 : 0$\bar{1}$0) 85° 51′, while the corresponding angles of the Anorthite of Vesuvius are 94° 10′ and 85° 50′.

The following faces have been identified :—

P	$= {}_{o}P$	(001)	e	$= 2P'\breve{\infty}$	(021)
M	$= \infty P\breve{\infty}$	(010)	n	$= 2'P\breve{\infty}$	(021)
T	$= \infty{,}'P$	(1$\bar{1}$0)	m	$= P'$	(111)
l	$= \infty P'_{,}$	(110)	o	$= P_{,}$	(11$\bar{1}$)
z	$= \infty{,}'P\breve{3}$	(1$\bar{3}$0)	p	$= {}_{,}P$	(11$\bar{1}$)
f	$= \infty P'_{,}\breve{3}$	(130)	b	$= 4'P\breve{2}$	(241)
t	$= 2'P'\bar{\infty}$	(20$\bar{1}$)	v	$= 4P_{,}\breve{2}$	(24$\bar{1}$)
y	$= 2{,}P_{,}\bar{\infty}$	(201)	w	$= 4{,}P\breve{2}$	(24$\bar{1}$)
x	$= {}_{,}P_{,}\bar{\infty}$	(101)	? u	$= 2P_{,}$	(22$\bar{1}$)

Some of the approximate measurements of the important angles are given below, compared with the angles calculated by v. Kokscharow* for the Vesuvian crystal :—

		Kokscharow.		
$y : P'$ =	98½°	98°	46′	8″
$y : M'$ =	89	89	27	26
$y : M$ =	90½	90	32	34
$y : o$ =	142¼	142	13	6
$y : p$ =	139½	139	48	34
$y : n$ =	83¼	83	8	36
$P : n$ =	133½	133	14	12

The faces t, m, b, v, w, u, are very rare, x has been observed in one specimen only as a very narrow band.

* Materialien zur Mineralogie Russlands. Bd. IV.

The most predominating faces are P, M, y, e, n, to which T, l, are sometimes combined. The most primitive form is a parallelopiped formed by the faces P, M, y. Usually the hemidome e or n truncates the edge of $P : M$, or $P : M'$, to which prisms T, l, and quarter-pyramids o, p, are added. (fig. 1, 2). The different modes of facial development give rise to different forms, which may conveniently be distinguished into two types.

First Type—The crystal assumes a tabular shape by the predominance of the Base P, while the Brachypinacoid M appears as a narrow band extending in the direction of the Brachyaxis $\breve{a}$ (fig. 1, 2). Fig. 3. represents in projection this type of crystal, in which the development of the faces is most perfect. Some of these crystals show an irregularity arising from parallel intergrowth. (fig. 4).

Second Type—In this type, the form is also tabular, but the predominating face is M, while P appears as a narrow band extending along the Brachyaxis. The faces forming this type are the same as those of the first type (fig. 5).

Simple crystals of the first type are more numerous than those of the second ; the latter however appear more frequently as twins. In both types, the form becomes sometimes irregular by the unequal development of the faces on the back and front sides. Generally o, p, appear on one side as narrow bands, while their opposite faces o', p', become much broader and intersect in a line, which would be parallel to the face x, did this face occur, as it generally does not. But owing to the imperfection of many specimens it is often difficult to say whether the faces o, p, have greater extension on the back or on the front side.

The predominance of the faces P, M, y, o, p, and the absence or rarity of prisms T, l, resemble to some extent the case of the Anorthite-crystals first described by G. vom Rath,* found as a second-

* Beitraege zur Petrographie—Zeitschft. d. deutsch. geol. Gesellft. Bd. 27. 1875 p. 392.

ary mineral in the contact-rock of Pesmeda-Alps, in Monzoni, Tyrol. But the difference between the two lies in the fact that the Pesmeda-Anorthite is elongated along the Macroaxis $\bar{b}$, while the Miyake-crystal has greater elongation along the Brachyaxis $\breve{a}$.

Twins—Simple crystals are rare; most of the crystals being found as penetration-twins. The twin law is analogous to that of the Carlsbad type in Orthoclase. The twinning-axis is the Vertical axis $\dot{c}$, the plane of composition the Brachypinacoid M ($\infty \breve{P} \breve{\infty}$). But as the crystals have often no prismatic edge, the two individuals have apparently no common direction. An exact measurement of the angles made by the faces of the two individuals is impossible. The angular divergence of the Brachyaxis, as measured along the edges of $P : M$ and $\underline{P} : \underline{M}$ is approximately 128°. Fig. 6 represents a typical form of the twin, the faces consisting of P, M, y, e, n, o, p; the last two faces being usually developed unequally on the two sides of the Brachyaxis. It is well-known that in this twin, the face x (${,}P{,}\bar{\infty}$) of one individual comes very near the Base $\underline{P}$ of the other individual. The faces o, p, M, lying in one zone with x, their intersection-edges become very nearly parallel to the zone of $\underline{P}$, $\underline{n}$, $\underline{M}$. According to vom Rath* the angular difference of these two directions is calculated to be 0° 24′ 4″ from the axial elements of the Vesuvian Anorthite, and this value is found to be more constant than in the case of the Orthoclase of Elba, in which the two faces $\underline{P}$, x tend to coincide. In the crystals of Miyakejima, this angular difference can not be detected at least by the eye, and the intersection-edges of P, M, e, n, become apparently parallel to those of $\underline{M}$, $\underline{o}$, $\underline{p}$. On account of this fact, this kind of twin crystal very often assumes a rhombic outline, when the faces o, p, attain their maximum develop-

* Ein Beitrag zur Kenntniss des Anorthits—Poggendf. Annalen der Physik u. Chemie. Bd. 147. 1872. p. 55.

ment. Fig. 7 shows such a crystal. The general outline of this modified type reminds us of a tabulate Anorthite-crystal (Cyclopite of v. Waltershausen) described by A. v. Lasaulx* from Cyclopean Island, near Etna, where the crystals are found in a druse within the doleritic rock. In the crystal shown in fig. 7, several subindividuals are found making parallel intergrowth. In another modification of this twin, one of the individuals becomes much smaller than the other, and the smaller individual is attached on one side or at the middle of the larger one. Very rarely the halves of the two individuals are attached by *M*, without making penetration (fig. 8). Of the two modifications arising in this twin, according as the one individual is attached to the right or the left side of the brachypinacoidal face of the other, it is almost impossible to say anything certain.

Another kind of twin, found only as twin-lamellæ, is that known as the Pericline-type; the twin-axis being the Macroaxis *b*. The trace of the "Rhombische Schnitt" on the cleavage-face of *M*, is indicated by very fine striæ making an angle of $-15°$ to $-17°$ with the edge of *P* : *M* (fig. 1). These twin-lamellæ are extremely fine, but they become comparatively much broader in the crystals imbedded in the lava, sometimes attaining a breadth of $^1/_4$ mm. Fig. 11, 12, show these broader twin-lamellæ as observed in sections of porphyritic crystals, cut nearly perpendicular to the edge *P* : *M*.

Optical characters—The direction of extinction has been examined on the cleavage-faces of *P*, and of *M* under parallel polarised light. As the mean of several observations, the direction of maximum extinction on the face *P* was found to be $-38°$ to $-40°$, while that of the face *M* was $-40°$ to $-41°$; the latter does not completely extinguish the light, owing to the interposition of the Pericline twin-lamellæ.

---

* Sartorius-Lasaulx, Der Aetna. Bd. II. p. 505. also Zeitschft. f. Krystallographie. Bd. 5. 1881 p. 326.

In convergent polarised light, the cleavage piece parallel to P shows one of the axial rings at the margin of the field, while that parallel to M shows another, but in the latter the centre of the ring is much nearer to the margin of the field. These observations agree completely with Max Schuster's* on the Anorthite of Vesuvius and Pesmeda. The position of the positive Bisectrix would hence be nearly perpendicular to the face e ($2P'\breve{\infty}$). The section cut as nearly as possible parallel to this face, shows in a convergent polarised ray, a biaxial interference-figure of a wide angle. Further optical characters can not at present be given.

The porphyritic crystals which are found within the basaltic lava, show somewhat different habitus, as compared with the crystals already described. The following observations may serve to indicate the characters of the porphyritic crystals as seen under microscopic slides of the lava.

The section cut nearly parallel to P (oP) presents an almost rectangular outline formed by the traces of the faces y and M. Fig. 10 represents a typical form of the section; sometimes the corners being differently cut off as in fig. 9. Generally a few lamellæ of the Albite type are found parallel to the cleavage-trace of M, the direction of extinction making 40° on both sides of the boundary. A noteworthy feature which is often observed in this section, is the presence of fine diagonal fissures (fig. 9, 10), intersecting at angles of nearly 85° and 95°, the acuter angle being nearly bisected by the cleavage-trace of M. Such fissures are also to be seen on the cleavage-face of P of separate crystals with the effect of making them very brittle, but the larger porphyritic crystals imbedded within the lava, exhibit the structure much more conspicuously. Such crystals when taken from the matrix fall into fine splinters along the fissures. Usually one of

* Die optische Orientirung der Plagioklase.—Tschermak's. Min. u. Petro. Mitth. Bd. 3. 1881. p. 213.

these sets of fissures is more perfect and distinct than the other, as in fig. 10, where the crystal has been broken along one of the directions of the fissures. G. Tschermak* has observed and photographed an analogous series of fissures found in an Anorthite-crystal within the tufaceous Meteorite (Eucrite) of Stannern. But in the crystals from Miyake, they are more numerous and distinct. A third series of fissures parallel to $\infty P\bar{\infty}$, shown in Tschermak's figure has not been observed in the Miyake Anorthite. An anomalous optical phenomenon is observed in the crystals showing this fissured structure. When a slide of such a crystal is examined under crossed Nicols, the extinction of light takes place in some parts, as usual at a direction making $-38°$ to $-40°$ with the trace of *M*, but the entire field never becomes dark; the dark portion being distinctly marked off from the luminous part. The boundary of the two parts is formed by zigzag lines which correspond to the directions of the fissures and to the cleavage-trace of *M*. Fig. 9 represents the section of a crystal formed by the parallel intergrowth of two individuals. The upper part, almost devoid of fissures, is separated by an irregular boundary from the lower half, in which very characterestic series of fissures are to be seen. The crystal is twinned according to the Albite-type, consisting of two principal twin-individuals *aa* and *bb*, which extinguish the light symmetrically on both sides of the trace of *M* at an angle of 38°. The part marked *cc* never becomes dark in any position, only showing a light bluish-violet tint when the portion *aa* becomes entirely dark. Sometimes patches of such luminous parts are found scattered within the dark field, assuming a damasked appearance. The phenomena just described may probably be due to a strain resulting from the sudden contraction experienced at the moment of eruption. In fig. 9, the mode of attachment of the upper and the lower halves shows that the upper went on growing in the direction of the lower

* Mikroskopische Beschaffenheit der Meteoriten. Taf. II, fig. 1.

half, after the latter had experienced a sudden contraction, the magma still retaining a certain degree of fluidity.

The section parallel to M ($\infty P \breve{\infty}$), is generally a parallelogram with angles of 98° and 82°, but sometimes becomes rhombic in outline by the development of the faces o, p, combined with P, the angles of the rhomb measuring 52° and 128° (compare fig. 7). Pericline-lamellæ making an angle approaching to –16 with the trace of P, are often found.

The section perpendicular to the zone of $P : M$, presents a rectangular form formed by the traces of P and M, the corners being usually cut off by the faces e, n. Fig. 11. 12, show the usual forms of this section; fig. 11 representing the first type, fig. 12 the second type, according to the classification of separate crystals already given. In this section, broader Pericline-lamellæ are inserted, which would be parallel to the trace of P, if the section falls just parallel to the Macroaxis. Some of these lamellæ thin out at the extremities. The broad band arranged parallel to M is probably twinned according to the Albite type (fig. 12). This band itself is divided into two halves a, b in the manner of a penetrating twin, with different optical orientation, ($a/b = 40°/25°$), so that these halves may be twinned according to the Carlsbad law.

The sections parallel to M and perpendicular to the Brachyaxis are traversed by irregular fissures, some of which run nearly in the direction of the Vertical axis. The substance of these felspars is always fresh, and nearly homogeneous; some variation in chemical composition being sometimes indicated by zonal structures as in fig. 12, when examined under polarised light. The enclosures are Olivine-grains, glass, and gas pores which are sometimes arranged parallel to a crystallographic direction. Olivine-grains are generally found as enclosures within the Anorthite, but occasionally we observe the inclusion of Anorthite within the Olivine.

Corrosion-figures.—Some experiments have been tried in regard to the character of corrosion-figures (Aetzfiguren) on cleavage-faces. Clear cleavage-pieces were selected, and acted upon by dilute fluoric acid, and by hydrochloric acid. In some natural cleavage-faces which have been long exposed to the action of atmospheric agencies, corrosion-figures are found similar in character to those produced artificially. These figures are most distinct under a magnifying power of 200–300 diameters.

On the face $P=001$ triangular impressions are formed (fig. 13). These triangles are asymmetric, rounded on one side, while the other sides are nearly straight; the angle formed by the latter being directed backward, as observed by Wiik.*

On the face $P'=00\bar{1}$, the figures are reversed as shown in fig. 14, the angle formed by the straight sides being pointed forward. On observing the different figures, we find some of them formed of different faces apparently of a hemidome and quarter-pyramids (fig. 13 *a*, *b*, *c*), different stages of such development being seen. The figures produced by hydrochloric acid on these faces are smaller and the angle formed by the straight sides more acute than those produced by fluoric acid. Wiik considers the two straight sides as being formed of $f\,(\infty P'_{,}\breve{3})$ and $z\,(\infty{,}'P\breve{3})$.

On the face $M=010$ are produced the elliptical figures observed by Wiik. The one end of the longer axis of the ellipse is pointed, and directed upward and backward (fig. 5). The direction of the longer axis is inclined in the same sense as that of the Pericline-lamellæ, making an angle of nearly 53° with the edge of $P : M$.

The figures which are produced on the face $M'=0\bar{1}0$ are of the same form, but reversed in position, as shown in fig. 16, the pointed end of the figure being directed downward and forward, and arranged in the same direction as those on M.

* Referat in Zeitschft. f. Krystallographie. Bd. 7. p. 188.

The figures produced by fluoric acid are nearly all elliptical (fig. 15, 16, *b*) while those produced by hydrochloric acid are round-sided asymmetric triangles (fig. 15, 16, *a*), one of the sides being apparently parallel to the direction of the Pericline-lamellæ. Fig. 17 is intended to show the relative position of these figures with reference to a simple primitive form consisting of the faces *P*, *M*, and *y*; the figures on *P'* and *M'* being supposed to be looked at through the face *y*.

For the chemical analysis of the mineral, I am indebted to Mr. Y. Kitamura, of the Geological Survey. The sample taken for the analysis was obtained from fresh specimens of separate crystals. The lava-coating was removed, and transparent portions of the interior carefully selected. The small grains of Olivine attached to the powdered sample were removed by means of Thoulet's solution, until the microscope showed the ideal purity of the sample. The result of the analysis is shown below in Column I. For the sake of comparison, we quote here in Column II, the analysis of a similar kind of Anorthite as that of Miyake, from a porphyritic volcanic rock of Tōnosawa in Hakone, given by Wada.*

	I. Miyake (Anal. Y. Kitamura)	II. Tōnosawa (Anal. M. Hida)
SiO_2	44·03 %	44·16 %
Al_2O_3	36·80	31·87
Fe_2O_3	....	1·33
CaO	19·29	20·90
MgO	·20	·53
Na_2O	·23	·32
K_2O	....	·55
H_2O	·12	·60
	100·67	100·26

* l. c. p. 33.

Specific gravity = 2·761 as determined by Thoulet's solution. Before the blowpipe, somewhat difficult to fuse. Very fine splinters which can sometimes be obtained from the fissured variety within the lava, fuse comparatively easily to a clear glass, glowing intensely, and imparting a reddish-yellow tint to the flame. In hydrochloric acid, the powder is entirely decomposed, leaving gelatinous silica.

Explanation of the Plate I.

Fig. 1.—A simple crystal of the first type in the position adopted by Des Cloizeaux and vom Rath. On the face $M=010$, the direction of the Pericline-lamellæ, or the trace of the "Rhombische Schnitt" is indicated by fine lines, which make an angle of about -16 with the edge $P:M$.

Fig. 2.—Same as fig. 1. A very common form of the first type.

Fig. 3.—A horizontal projection of a crystal of the first type; the plane of projection perpendicular to the Brachyaxis.

Fig. 4.—Same as fig. 3, showing parallel intergrowth; the crystal is imperfect at the middle portion and an irregular attachment of smaller individuals is observed in the specimen $\breve{a}$.

Fig. 5.—A horizontal projection as in fig. 3, 4. Some imperfection is observed in the lower portion.

Fig. 6.—A common form of the penetration-twin analogous to the Carlsbad type. In this form it often happens that the faces o, p, are unequally developed on different sides as in the figure. The intersection-edges of $\underline{o}'$, $\underline{p}'$, $\underline{M}'$ become very nearly parallel to those of P, e, M.

Fig. 7.—A rhomb-shaped modification of the twin, the same as in fig. 6. The two individuals which are distinguished by the thicker and the lighter lines, are composed of several subindividuals. The faces o, p, are developed on both sides very extensively, the edge P, e, M becoming very nearly parallel to the edge $\underline{o}'$, $\underline{p}'$, $\underline{M}'$. The projection is made on a plane perpendicular to the edge P, x, y, of one individual, so that the edge $\underline{P}$, $\underline{x}$, $\underline{y}$ does not coincide exactly with the first. The face t becomes here relatively large, and very nearly parallel to $\underline{y}'$.

Fig. 8.—Horizontal projection of the twin, similar to those of

fig. 6, 7, but only the halves of the two individuals attached by *M* without penetrating.

Fig. 9.—Section of a porphyritic crystal within the lava, magnified about 20 times, cut nearly parallel to the face *P*, consisting of the nonfissured upper, and the fissured lower, halves in a parallel position. These individuals consist of two principal twin-individuals *a*, *a*, and *b*, *b*, of the Albite type, the part *c*, *c*, showing an optical anomaly.

Fig. 10.—Same as fig. 9, magnified 10 times, the rectangular outline being formed by *M*, *y*. Very characteristic diagonal fissures are here developed, similar to those of the lower half of fig. 9, the upper right and the lower left corners being removed by the one set of fissures which is more perfect than the other. A single lamella of the Albite type traverses the middle portion.

Fig. 11.—Section of a porphyritic crystal, cut nearly perpendicular to the zone *P* : *M*, magnified 5 times, showing the broader Pericline-lamellæ.

Fig. 12.—Same as fig. 11, magnified 10 times, showing the fine transversal Pericline-lamellæ, and the broader vertical lamellæ *a*, *b*, probably of Albite and of Carlsbad types. Zonal structure is well exhibited in this section under polarised light.

Fig. 13.—Corrosion-figures on the cleavage-face *P*; the acute angles of the triangular impressions are pointed backward. They are shown magnified in *a*, *b*, *c*.

Fig. 14.—The same as seen on the cleavage of *P'*, of the same form as those on *P*, but the acute angles are pointed forward. Magnified in *a*, *b*.

Fig. 15.—The same of elliptical forms, on the cleavage-face of *M*. The fine lines represent the direction of the trace of the "Rhombische schnitt" *a*, figure produced by hydrochloric acid; *b*, by fluoric acid, magnified.

Fig. 16.—The same as seen on the cleavage-face *M'*; *a*, *b* as in fig. 15.

Fig. 17.—A diagrammatic representation of the relative positions of corrosion-figures on the faces P, P', M, M', in a primitive form consisting of P, M, y; the figures on P' and M' being supposed to be looked at through the face y.

The Source of Bothriocephalus latus in Japan.

by

Isao Ijima, Ph. D.

Bothriocephalus latus, which was formerly thought to be restricted to Europe in its distribution, is the commonest tape-worm found in Japan. It is met with everywhere in the country, of course with local differences as to its frequency. The identity of the species with the European form admits in my opinion of no doubt. I would not have made this remark, had not Küchenmeister of late erroneously assumed that in Europe more than one species is included under the name of B. latus.

As far as my experiences go, Tænia mediocanellata occurs in Japan much more rarely than B. latus ; among dozens of tape-worms that I have examined, only a single specimen of the former species was found. That fish, the recognized source of Bothriocephalus, is used as food much more generally in Japan than beef (the source of Tænia mediocanellata), sufficiently explains the above-stated fact. Tænia solium, if ever it occurs in Japan, must be exceedingly rare. I have indeed, never as yet met with it in Tōkyō. This is undoubtedly due to the fact that pork as an article of food is even less used than beef. Unfortunately means are not yet at my disposal for determining the occurrence of human tape-worms at any locality on statistical grounds.

As is well known, Prof. M. Braun[1] was the first to show that in the Baltic region (Dorpat) the plerocercoid larvæ of B. latus infest the viscera as well as the flesh of the pike (Esox lucius) and of the burbot (Lota vulgaris). He proved that the above-mentioned tape-worm, so common in Dorpat and vicinity, is derived from using these infected fishes, and the pike especially, as food. Küchenmeister[2] alone stood out against considering the pike as the intermediate host of B. latus, substituting certain Salmonidæ (Salmo salar) in its stead. His opinion has however suffered a well-grounded refutation by Braun and Leuckart,[3] so that the former's discovery stands as an established fact. According to Leuckart, Grassi in Catania also succeeded in raising B. latus in himself from larvæ taken from the pike. And Parona[4] in Lombardy found larvæ, which were experimentally proved to belong to B. latus, not only in the pike but also in the perch (Perca fluviatilis). Further it turned out from Zschokke's and my observations that some Salmonidæ are also to be counted among the intermediate hosts of B. latus.

Zschokke's researches[5] at Geneva, one of the places noted for the occurrence of B. latus, have proved that there its principal source is Lota vulgaris and after it Perca fluviatilis, while infection from pike and salmon is considered as occurring only in exceptional cases. Of the latter, Salmo umbla was found almost regularly infested by Bothriocephalus-larvæ.

---

1) Zur Entwicklungsgeschichte des breiten Bandwurmes. Würzburg 1883.

2) "Wie steckt sich der Mensch mit Bothriocephalus an?" Berlin. klin. Wochenschrift. Nr. 32, 33. 1885.—"Die Finne des Bothriocephalus und ihre Uebertragung auf den Menschen." Leipzig 1886.—"Weitere Bestätigung meiner Behauptung, dass die Finne des Hechtes nichts mit Both. latus zu thun hat." Deutsche medic. Wochenschrift. Nr. 32. 1886.

3) See Leuckart "Zur Bothriocephalus-Frage." Centralbl. f. Bacteriologie u. Parasitenkunde. Nr. 1, 2. 1887.

4) In "Estratto dei Rendiconti del R. Istituto Lombardo. Ser. II. Vol. 19. 1886.

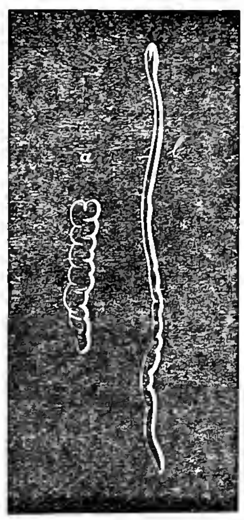
5) "Der Both. latus in Genf." Centralb. f. Bact. u. Parasstenk. Nr. 13, 14. 1887.

From what has been said it is apparent that the (last) intermediate host of B. latus includes several species of fish which may belong to different families and that the principal source of that tapeworm may vary according to localities.

In Japan, where as already said, B. latus is abundant, none of the above-mentioned fishes are known to occur, although the pike is said to exist in Saghalien. What fish is then the source of B. latus in Japan?

There has been among the Japanese a popular belief that tapeworms develope from eating certain fishes. Onchorhynchus Perryi Hilgd. (*Masu*) and Onchorhynchus Haberi Hilgd. (*Sake*) were the most suspected, a belief which was certainly without any scientific basis. Guided by this suspicion, however, I examined in May 1886 a specimen of Onchorhynchus Perryi, and was not disappointed. Seven Bothriocephalus-larvæ, unmistakeable by the configuration of their head, were found imbedded in the trunk-muscles. In form, size and motions, they corresponded exactly with Braun's description and figures of the larva of B. latus.

Concerning the appearance of the larvæ I have nothing of importance to add to what is already known. Nevertheless a few words about them might be useful, since zoological literature is not accessible to many in Japan. The larva of B. latus is a slender elongated worm of white color. The body is properly speaking not flat but rather thick. Its length varies from 8 to 30 mm, its breadth from 1 to 3 mm. When contracted, the length is reduced by more than one-half, but the breadth relatively increases so that it acquires a thick wrinkled form. The head is then involuted and shows a cleft-like depression at the end. If such a larva be put into



Larvæ of Bothriocephalus latus from Onchorhynchus Perryi. *a*, in contracted; *b*, in extended state. About 2 × magnified.

luke-warm water or salt-solution, it begins to move energetically. The head is alternately thrown in and out, the body at the same time bending and stretching and exhibiting also peristaltic motions which travel from end to end. The head, when extended, is club-shaped, hardly 1 mm in length and bears two shallow longitudinal grooves, thus essentially agreeing in form with the head of the fully grown Bothriocephalus latus. The body is solid as in the mature worm but without any trace of strobilization or of sexual organs. Examined under the microscope, it clearly shows some of the excretory vessels and calcareous bodies, of which there are comparatively many. The larva, as it lies in the flesh, is tolerably extended but with the head always drawn in. It rarely lies straight but usually irregularly coiled. No sort of capsular membrane invests it. Within the body of the host, it by no means makes such active movements as described above; but it is exceedingly probable that it may at any time shift its position within the host. At least, there can be no doubt about this much, that the larvæ found in various parts of fishes have wandered in from the intestinal canal. Braun found them not only in the muscles of the pike, but also in the various organs of its body-cavity, some hanging on the intestinal wall, others free in that cavity. Moreover he found on the liver-surface tracks of their wanderings. In Onchorhynchus Perryi, I have as yet never found them in any other place than in the trunk-muscles.

To prove that the Bothriocephalus larvæ obtained from Onchorhynchus Perryi belonged to B. latus, it was necessary to experiment with them. So I swallowed two larvæ, one of which was however injured and my assistant, Mr. M. Kikuchi, kindly offered his services and swallowed three. This was done on May 10th 1886.

From the 2nd or the 3rd day, I began to experience now and then slight pains in the duodenal region. This lasted for some days and

then entirely stopped, so that I felt myself in my usual health until the 28th of the same month, when slight diarrhœa began to inconvenience me. On June 1st a piece of B. latus, 22.5 cm long, was discharged. Since it had the characteristic terminal proglottis, it was certain that no segments had been previously lost. On the following day, viz. 22 days after the beginning of the experiment, the remainder of the tape-worm body including the head was obtained by means of anthelmintic. It seemed that only one (probably the uninjured) of the two larvæ swallowed had developed into the tape-worm.

In Mr. Kikuchi's case, intestinal complaint commenced a little later than in mine. In both cases, fœces were subjected to microscopical examination from time to time, but Bothriocephalus-eggs were never met with. It seems that it requires a considerable length of time before the worm is ripe enough to let some of its eggs escape from the uterine opening. On June 27th anthelmintic was given to Mr. Kikuchi. Unfortunately however no worm was obtained. Apparently it became lost, proper precautions not having been taken to secure it. After this his complaint entirely ceased.

The total length of the worm that had grown in me was 315 cm (over ten feet!) and the number of proglottides, as far as could be counted, 1467. Of these the last 617 had their uterus already filled with eggs. Considering that a larva of insignificant size had acquired the above-stated respectable dimension during a period of only 22 days, the rate of growth is really wonderful and one might doubt the genetic connection between the larva voluntarily swallowed and the tape-worm produced, were it not for the similar results arrived at by Braun and Zschokke. The former experimented not only on cats and dogs but also on three of his students, who had voluntarily offered themselves for the purpose. They were first ascertained to have no tape-worm in the body and then each swallowed 3 or 4

larvæ. Symptoms appeared in 3 weeks and in a month every one of them began to discharge Bothriocephalus eggs. At this, anthelminthic was given and in all more than 6 specimens of B. latus were obtained, averaging about 335 cm in length. Zschokke also obtained positive results in several experiments on man. In 4 cases symptoms appeared in 16–22 days after swallowing the larvæ and the tape-worms obtained in 20–26 days measured about 129 cm on an average.

Some specimens of Bothriocephalus-larvæ from Onchorhynchus Perryi were sent to Prof. Leuckart for examination. He did not hesitate to pronounce them as identical with those found in the pike of Europe.

Thus, the fact is established that Onchorhynchus Perryi does harbour the larva of B. latus; and I believe I am right in claiming that fish to be at least one of the chief sources of B. latus in Japan. Subsequently I have had occasion to examine 7 examples of that fish at different periods of the year and found Bothriocephalus-larvæ in all but one. The single negative case was a young O. Perryi from the Hokkaidō. It was in a state of putrefaction, making the task of search so unpleasant, that a thorough examination was abandoned. Another Hokkaidō specimen, said to have been sent in ice contained at least 4 larvæ. The other specimens were brought to the Tōkyō market from neighboring districts, probably from the Tonegawa. The number of larvæ lodged in one fish was not large, 7 having been the maximum. In one case, only a single larva could be found after a laborious search. Even then it would be unsafe to assert that no chance was left for some to escape unnoticed. At all events, the number of the B. larvæ in O. Perryi is much less than that found by Braun or Zschokke in the pike, burbot, etc. But to judge from my somewhat limited experience, their occurrence in O. Perryi seems to

be tolerably constant. Another reason for regarding O. Perryi as at least one of the chief sources of B. latus, is the undeniable fact that where that fish abounds, the tape-worm is also abundant. Toyama in the province of Ecchiu is such a place. In Yezo, especially among the Ainos, tape-worms are said to be common. Considering that O. Perryi occurs there in plenty, these tape-worms presumably belong to the species Bothriocephalus latus.

O. Perryi is in some parts of Japan often eaten raw (*Sashimi*) like many other fishes. Under such circumstances, infection with Bothriocephalus is highly probable and all the more so since its larvæ in the flesh might easily be mistaken for a piece of fat, tendon or nerve. Once a friend told me, soon after his return from a short visit to Saikyō, that he had there tried the "*sashimi*" of the fish in question. I warned him of what change might perhaps occur in his health within 3 weeks' time, little suspecting that he would, as he really did, present me with a fine specimen of B. latus at the end of a month. In Tōkyō, where O. Perryi is altogether scarce, its "*sashimi*" is not generally indulged in, a fact which probably explains the comparative scarcity of B. latus in this city.

The life-tenacity of Bothriocephalus-larvæ seems to be of no small degree. They may retain vitality for a week outside their host and Braun even found them alive not only in weakly salted pike but also in such that had been frozen. It is however certain that the heat of thorough cooking or roasting suffices to render them harmless. At the same time it must be admitted that ordinary methods of cooking do not exclude all chances of infection, as is clear from the way we obtain our Tænia from beef or pork. A naturalist friend in Tōkyō once discharged a B. latus, which he is inclined to regard as having been received from the flesh of O. Perryi, which he had eaten *roasted* about one month previously. The fish had been

sent from Yezo preserved in *Sake*-dreg (*Kasu*). A few other enquiries were made of persons who had been tape-worm patients and many of them recollected having eaten O. Perryi in some form or other.

To all appearances then O. Perryi is the chief source of B. latus. I do not wish however to emphasize this too strongly, since my examination of other river-fishes is still very incomplete.

Onchorhynchus Haberi, another species of salmon common in Japan, closely resembling O. Perryi in habits and other respects, is peculiarly open to suspicion. Nevertheless, three specimens of this fish, obtained in the Tōkyō market and carefully searched by Mr. Kikuchi and myself, showed no trace of Bothriocephalus-larvæ. Carassius auratus (*Funa*), Cyprinus carpio (*Koi*), Plecoglossus altivelis (*Ayu*) and a species of Salmo (*Yamabe*) were also searched but with negative results. However the number of individuals examined was, as in the case of Onchorhynchus Haberi, too small to allow of much weight being attached to the conclusions. A more complete investigation of our fresh-water fishes is very much to be desired.

Earthquake Measurements of recent years especially relating to Vertical Motion.

By

S. Sekiya

Professor of Seismology, Imperial University.

This paper contains the records of earthquake observations made during the two years from September 1885 to September 1887. Severe shocks as well as feeble tremors are arranged in the accompanying table, and in some shocks separate notes are added by way of fuller description. The measurements were made at two places in Tōkyō; one set at Hitotsubashi where the ground is soft and marshy, and the other set at Hongō where the soil is hardened alluvial mud.

Vertical motion which forms the principal subject in this paper has not hitherto been so much studied as horizontal motion. This is on account of its comparatively rare occurrence, and when it occurs its smallness makes it of secondary importance to the more prominent horizontal movement. Reference will be made to Plates IX, XI and XXVI, Vol. I. of this Journal, in which the characteristic features of vertical motion occurring in conjunction with horizontal motion can be seen.

In this country absolute vertical motion[1] was first measured by Mr. E. Knipping between 1878 and 1880. During that period

1 Mittheilungen der Deutschen Gesellschaft, etc. Ostasiens Vol. 17, May 1879 and Transactions of the Seismological Society of Japan Vol. I. p. 72.

eight observations were made; in four cases the vertical motion reached 0.02 m.m. The largest value 0.56 m.m. was observed in the severe earthquake of February 22nd, 1880. In 1883 Prof. J. Milne, in conjunction with Prof. T. Gray, made experiments on artificial earthquakes.[1] Vibrations then were caused by letting fall a heavy weight from various heights or exploding dynamite in holes made in the ground. His results were principally as follows. (1) In the soft ground vertical motion appears to be a free surface wave which outraces the horizontal components of motion. (2) Vertical motion commences with small rapid vibrations and ends with vibrations which are long and slow. (3) High velocities of transit of seismic waves may be obtained by the observation of this component of motion. It is possibly an explanation of the preliminary tremors of an earthquake and the sound phenomena.

In the table are given the following quantities.

1.—Maximum Motion ($2r$) or the largest range of the displacement of the ground in each shock.

2.—Complete Period (t) of the maximum motion or the time taken to make a complete for-and-back motion of the ground.

3.—Maximum Velocity (v) of the ground, or $v = \frac{2r\pi}{t}$.

4.—Maximum Acceleration $= \frac{v^2}{r}$.

The last two quantities were calculated by assuming for convenience sake the motion of the ground to be harmonic though it is not exactly so in actual cases.

5.—Direction of the maximum horizontal motion of the ground.

6.—Duration of the earthquake, *i. e.*, the interval of time from the commencement to the end of the disturbance. It is almost

1 Transactions of the Seismological Society Vol. VIII.

impossible to measure the absolute duration of earthquakes as they usually begin with exceedingly feeble tremors and end with very slow undulations.

7.—The distance and direction of the origin of each earthquake from Tōkyō, and its area. These were kindly supplied by the Imperial Meteorological Observatory. Existence of vertical motion, the range and the direction of horizontal motion, etc., may be examined in reference to the position of the origin of shocks and their area.

By *Tremors* are meant feeble shocks whose range of motion is less than one-tenth of a millimeter.

Local shocks marked *Local* are small earthquakes shaking only limited regions of the country usually from five to fifteen miles around.

Prof. J. A. Ewing's Horizontal Pendulum and Vertical-motion Seismographs were mainly used in making these measurements. They are automatically started by the earthquake motion when it attains one-fifteenth part of a millimeter. By increasing the sensitiveness of the instruments the number of records may be proportionally increased.

The records, unless otherwise stated, are those obtained at Hitotsubashi.

No.	Date	Max. Horiz. Motion in m.m.	Complete Period of Max. Horiz. Motion in Sec.	Max. Velocity, in m.m. per Sec.	Accel. in m.m. per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion.	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion.	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
							min sec.			min. sec.		
1	Sept. 2, 8.36 0 p.m.	0·3	0·8	1·2	9·6	NNE	2. 30	0·05	0·70	0. 46	46 miles due North inland.	54 m.
2	Sept. 26, 0 30. 0. p.m.	6·5	2·2	9·3	26·6	N76°W	3. 35	0·14	0·56	1. 42	110 miles S 30 W in the ocean.	146 m.
3	Sept. 28, 5 28. 0. a.m.	3·8	1·7	7	25·8	EW	3. 00	Trace			The same in general features as No. 2.	
4	Sept. 29, 8 36.16. a.m.	0·1	1	0·3	2·	SN	1. 45				Local	
5	Oct. 1, 1. 9. 0 p.m.	1	0·7	4·5	40·5	SN	2. 00	0·01	0·04	0. 09	59 miles N 35 E on the sea shore.	90 m.
6	Oct. 3.	0·1	1	0·31	2·	SN	0. 50				Local	
7	Oct. 7, 7.34 45. a.m.	0·4	0·7	1·8	16·2	N15°E	0. 18				Local	
8	Oct. 9, 7.54 0. p.m.	0·1	0·8	0·4	3·2	EW	2. 00				Local	
9	Oct. 11, 5.28.18. a.m.	1·1	1	3·5	22·3	WNW	1. 03	0.02		0. 06	71 miles N 60 E in the sea.	83 m.
10	Oct. 15, 9. 2.29. a.m.	0·3	0·8	1·2	9·6	W15°N	1. 10				178 miles W S W	14,333 sq. m.
11	Oct. 15, 8.18.43. p.m.	1·0	0·7	4·5	10·5	ESE	2. 28	0.03		0.07	15 miles S S E in Tokyo Bay.	33 m.
12	Oct. 18, a.m.	Tremors				EW	0. 20				Local	
13	Oct. 18, 0.15. 0. p.m.	0·3	9 0	1·6	17·1	NS	1. 00				N 66°E 59 miles	61 m.
14	Oct. 21, 1.15. 0. a.m.	0·4	0·9	1·4	9·8	NW	2. 02				98 miles N 73° W	19,500 sq. m.
15	Oct. 24, 5.12.18. p.m.	0·7	0·9	2·4	16·0	S60 W	1. 15				81 miles nearly E in the sea.	93 m.
16	Oct. 26, 10 41 11. p.m.	2·2	1·4	4·9	21·8	EW	3. 20				112 miles S in the sea.	139 m.
17	Oct. 30, 8.31 16 p.m.	0·3	1·0	0·9	5·4	WNW	2. 30	Trace			415 miles N W in the ocean.	34,700 sq. m.
18	Nov. 16, 1.53.36. p.m.	Tremors					0. 30				15 miles S S E	22 m.
19	Nov. 18.	Tremors				NS					Local	
20	Dec. 3, 6. 1 42. a.m.	0·2	0·6	1·0	10·	NS	1. 00				20 miles N N E inland.	29 m.
21	Dec. 7, 1. 2. 0. a.m.	2·1	1·6	4·1	16 0	NNW	5. 02				9[illegible] miles E 13° N in the sea.	17,120 sq. m.
22	Dec. 19, 2.12. 0. a.m.	Tremors				NS	0. 43				Local	
23	Dec. 19, 6.28. 0. p.m.	2·8	1·7	5·2	19·4	S 55° W and then to N 30 W	1. 46	0·22	0·05	0. 45	37 miles E 35° N inland.	160 m.
24	Dec. 25, 1.13 30. p.m.	0·2	0·7	0·9	8·1	EW		Trace		0. 10	Local	
25	Dec. 28, 10. 6 30 p.m.	3 5	1 4	7·9	35·7	EW	3. 30	1·00	0·06	1. 62	29 miles N N E inland	98 m.

No.	Date	Max. Horiz. Motion in m.m.	Complete Period of Max. Horiz. Motion in Sec.	Max. Velocity, in m.m. per Sec.	Accel. in m.m per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion. (m'n sec.)	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion (min sec.)	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
26	Jan. 4, 8.31.30. p. m.	0·4	0·7	1·8	16·2	EW	0. 54				59 miles N 60° E in the sea.	61 m.
27	Jan. 5, 4.26.42. p. m.	0·8	1	2·5	15·8	EW	0. 75	Trace			66 miles due E on the sea shore.	73 m.
28	Jan. 9, 6.48. 0. a. m.	0·1	1·2	0·3	1.8	SN	2. 00				Local	
29	Jan. 18, 9.15. 0. p. m	0·3	0·8	1·2	9·6	NS	0. 65				Local	
30	Feb. 18, 3. 0. 0. a. m.	0·2	0·7	0·9	8·1	EW	0. 22				Local	
31	Feb. 19, 9.51.11. a. m.	0·1	1	0·3	1·8	EW	0. 15				Local	
32	Feb. 22,	0·1	0·7	0·4	3·2	EW	0. 40				Local	
33	Feb. 24, 7.34. 0. a. m.	0·5	0·6	2·6	27·0	NW	1. 45	0·08	0·3	0.56	29 miles. due N inland.	73 m.
34	Feb. 24, 3.36.25. p. m.	0·3	1	0·9	5·0	EW	1. 20				Local	
35	March 2, 5. 3.49. a. m	0·6	0·8	2·4	19·2	NNE	2. 10				73 miles due E in the sea shore.	78 m.
36	March 3, 3.36. 0. p. m.	Tremors				EW	0. 15				Local	
37	March 13, 6.25. 0. p. m	0·7	0·9	2·4	16·2	WNW	2. 00				68 miles nearly E on the sea shore.	73 m.
38	March 26, 6. 6. 0. p. m.	0·1	0·8	0·4	3·2	SSE	0. 65				17 miles S S E in the bay.	24 m.
39	April 1.	0·1	1	0·3	1·8	EW	0. 65	Trace		0.04	Local	
40	April 4, 1. 0. 0 a. m.	0·4	1	1·3	8·0	EW	2. 10	Trace		0.15	134 miles N N E away in the sea.	146 m.
41	April 13, 4.45. 0. a. m	1·2	0·9	4·2	29·2	S54E	4. 02	0·05	0·4	0.10	312 miles S E in the ocean.	Disturbance 33,000 sq. m.
42	April 14, a. m	0·8	0·9	2·8	19·6	EW	0. 40				Local	
43	April 23, 4.22.22. a. m.	0·7	0·8	2·7	20·8	ENE	1. 26				171 miles N W on the other side of the island	16,400 sq. m.
44	May 2, p m.	0·1	0·9	0·4	3·2	WE	0. 40				Local	
45	May 3, 0. 0 0. p. m.	0·2	0·7	0·9	8·1	ESE	2. 00				Local	
46	May 5.	0·2	0·9	0·7	4·9	EW	0. 54				29 miles N N E inland.	32 m.
47	May 8, 10.14. 0. p. m.	2·1	0·8	8·2	64·0	N17°E	2. 24	0·33	0·5	0.44	32 miles W N W inland.	85 m.
48	May 9, 3. 0. 0. p. m.	0·5	1·2	1·3	6·8	E30°N	2. 50				Local	
49	May 11, 2.31.58. p. m.	0·5	0·9	1·7	11·6	E10°S	1. 25				Local	
50	May 16, 9. 7.16. a. m	3·3	1·2	8·6	14·8	N50°W	3. 05	0·4	0·8	1.25	53 miles nearly N W inland.	18,530 sq. m.

1886.

No.	Date	Max. Horiz. Motion in m.m	Complete Period of Max. Horiz. Motion in Sec.	Max. Velocity in m.m. per Sec.	Accel. in m.m. per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion. min. sec.	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion. min. sec.	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
51	May 18, 8.12.51. p.m.	0·5	0·9	1·7	11·6	S50 E	2. 15	0· 1	0· 3	0. 36	20 miles N N E inland.	85 m.
52	May 30, 8.38.18 p.m.	0·4	0·9	1·4	9·8	EW (mainly)	1. 30				57 miles N N E inland.	57 m.
53	June 3, 3. 6.37. p.m.	0·4	1·1	1·1	6·4	EW	1. 31	Trace			N E 76 miles in sea shore.	76 m.
54	June 6, 6. 0. 0. p.m.	0·5	0·7	2·2	19·4	EW	1. 06	Trace			25 miles.	
55	June 11, 1.45.44. a.m.	Hongo. 0·1	0·7	0·4	3·2	NW	1. 02				E S E in Tokyo Bay 15 miles.	35 m.
		Hitotsu-bashi. 0·4	0·6	2.1	22·1	EW	1. 15					
56	June 12.	0·5	0·7	2·2	19·4	WE	1. 15				Local	
57	June. 13.	0·3	0·9	1·0	6·7	EW	0. 36				Local	
58	June 14, 6.25.19. p.m.	0·6	0·8	2·4	19·2	ESW	1. 10	0. 1	0· 5	0. 30	In or near Tokyo.	35 m.
59	June 22.	0·3	0·8	1·2	9·6	EW	0. 52				Local	
60	June 28.	0·2	0·8	0·8	6·4	EW	0. 31				Local	
61	July 2, 0.33. 6. p.m	Hongo. 0·7	1.0	2·2	13·8	ESE WNW	1. 27	0. 3	1. 3	1. 30	N E in the Pacific ocean	Extensive earthquake shaking the whole of North Japan, Tokyo on its edge.
		Hitotsu-bashi. 1·8	0·9	6.2	12·7	WNW ESE	2. 24	0. 3	0. 8	1. 22		
62	July 23, 0.57. 0. a.m.	0·6	0·9	2·1	14.7	NS	1. 24				N W 110 miles on shore line.	Extensive shock Tokyo on its edge.
63	August 3, 2.11.40 a.m.	Tremors				EW	0. 20				N 30 miles inlnad.	30 m.
64	August 9, 11.24. 0. a.m.	Hongo. 0·5	0·9	1·7	11·6	NE SW	1. 44				The same as No. 62 in general features.	
		Hitotsu-bashi. 0·5	0·8	2·0	16·0	NE	1. 10					
65	August 29, 8.34.54. p.m.	0·6	0·8	2·4	19.2	ESE	1. 10					
66	Sept. 6, 0.38.53. p.m.	0·2	0·7	0·9	8·1	EW	0. 35				Local	
67	Sept. 12, 8.43.22. p.m.	0·8	1·2	2·4	11·0	S30°W	0. 50	0. 1	0. 1	0. 16	In or near Tokyo.	65 m.
68	Sept. 15, 3. 9.23. a.m.	0·2	1·1	0·6	3·6	S30 W	1. 13	Trace			Local	
69	Sept. 16, 1. 2.57. p.m.	0·1	0·4	0·8	12·8	SW	0. 50	Trace			N N W 40 miles inland. Tokyo on edge.	50 m.
70	Sept. 21, 8.17. 0. p.m.	Hongo. 0·2	0.9	0·7	4·9	EW	0. 53					
		Hitotsu-bashi. 0·1	0.8	1·6	12·8	NS	1. 31					
71	Sept. 30.	0·8	0.8	3.1	21·0	SN	0. 58				Local	

No.	Date		Max. Horiz. Motion in mm.	Complete Period of Max. Horiz. Motion in Sec.	Max. Velocity in m.m. per Sec.	Accel. in m.m. per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion.	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion.	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
								min. sec.			min. sec.		
72	Oct. 4, 2.35.25. p.m.	Hongo.	0·2	0·6	1·0	10·0	SW	24	0.1	0.4	0.15	N N E 25 miles inland.	40 m.
		Hitotsu-bashi.	0·3	0·9	1·0	6·7	WNW	1. 25	0.1	0.5	0.24		
73	Oct. 22, 3.49.14. a.m.		0·5	0·8	2·0	16·0	NS	1. 05				E N E 34 miles inland.	70 m.
74	Oct. 25, 10.11.18. p.m.		0·4	1·2	1·1	6·1	SE	1. 40	0.1	0.5	0.34	The same in genaral features as No. 72.	
75	Nov. 1, 5.13. 5. a.m.	Hongo.	0·3	1.9	0·9	5·4	NS	1. 15	Trace				
		Hitotsu-bashi.	0·5	0·8	2·0	16·0	NS	0. 59	0.1	0.5	0.16		
76	Nov. 2, 8.21.46. p.m.		0·2	1·1	0·6	3·6	WNW	1. 40	0.1	0.6	0.33	N [illegible]0 miles inland N E in the Pacific ocean.	35 miles Moderate sized shock.
77	Dec. 4, 2. 0.39. p.m.	Hongo.	0·3	0·8	1·2	9·6	SW	1. 50	0.1	0.3	1.12	N E 120 miles in the Pacific ocean.	140 m.
78	Dec. 6, 0.40. 0. p.m.		Tremors					0. 29				Local	
79	Dec. 8, 11.58.16. a.m.		0·3	0·9	1·0	6·7	SN	2. 02				N 50 miles inland.	35 m.
80	Dec. 11, 10.16.25. p.m.		0·4	0·9	1·3	8·5	SN	1. 16	Trace			S E 150 miles in ocean near the shore.	120 m.
81	Dec 12, 10.11.55. p.m.		Lost.										
82	Dec. 21, 3. 7. 2. a.m.		Tremors										
83	Dec. 26, 5.48. 5. p.m.	Hongo.	0·3	0·5	1·9	24·1	S50°W	0. 37	0.1	0.4	0.26	In or very near Tokyo.	65 m.
		Hitotsu-bashi	0·8	0·6	4·2	44·1	NNW	0. 51	0.3	0.6	0.21		
84	Dec. 29, 11. 5.43. a.m		0·5	0·6	2·6	27·0	SE	1. 0	0.2	0.5	0.18	N 22 miles inland.	43 m.
85	Jan. 15, 6.52. 0. p.m.	Hongo.	7·3	2·0	11·5	36·2	SSW	6. 24 (Principal motion 2 min.)	1.3	1.0	1.12	35 miles.	2[illegible]0 m.
		Hitotsu-bashi.	21	2·5	26	6[illegible]	S65°W	6. 35 („)	1.8	0.9	1.38		
86	Jan. 16, 10.16.19. p.m.		0·6	0·7	2·7	24·3	variable	1. 55				The same in general features as No. 85.	
87	Jan. 24, 10.40.50. p.m.		0·1	0·7	1·8	16·2	EW	2. 06				Local	
88	Jan. 28, 3.54. 8. p.m.		0.4	0·8	1·6	12·8	ENE	1. 05				N W in the Pacific ocean.	Extended along the coasts of North-Japan
89	Feb. 2, 2. 8.14 p.m.		0·8	0·9	2·8	22·1	S30W	1. 58				178 W S W	180 m.
90	March 2, 5.33.21. p.m.		Tremors									Origin inland.	Small earthquake.
91	March 20, 11.32.55. p.m.		Tremors									Local	
92	March 23.		0·3	0.8	1·2	9·6	WE	0. 45				Local	

1887.

No.	Date	Max. Horiz. Motion in m.m.	Complete Period of Max. Horiz. Motion in Sec.	Max Velocity in m.m. per Sec.	Accel. in m.m per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion.	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion.	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
							min sec.			min sec.		
93	April 4, 8.46. 0. a.m.	Tremors									N. 58 miles inland, Tokyo on edge.	65 m.
94	April 9, 11.49.54. a.m.	Tremors									The same as N. 93.	
95	April 16, 3.35. 0. a.m.	Hongo 0·2	0·7	0·9	8·1	variable	0. 10	Trace			E N E 35 miles.	37 m.
		Hitotsu bashi 0·4	0·8	1·6	12·8	SW NE	1. 52	0·1	0·8	0.28		
96	April 20, 2.35. 0. a.m.	0·3	1	0·9	5·4	EW	0. 10				Local	
97	April 23, 6.30. 0. p.m	Tremors									Local	
98	April 27, 9.30.38. p.m	0·5	2·7	0·6	1·4	SN	2. 20				S S E in Tokyo Bay 17 miles.	33 m.
99	April 29, 11.12.10. a.m.	1·4	Lost					(occurred)				
100	May 2, 11.25.40. a.m.	Hongo 0·4	0·6	2·1	22·1	NE	0. 57	0·1	0·5	0·17	In or near Tokyo.	
101	May 4	0·8	1	2·5	15·6	ENE	1. 12				Local	
102	May 5, 2.35.10. a.m.	0·1	0·9	0·3	1·8	EW	0. 35				Local	
103	May 6, 3.49.50 p.m.	Tremors									Local	
104	May 7, 7.13.12. a.m.	0·2	0·7	0·9	8·1	NS	1. 27				The same in general features as No. 88, but smaller in extent.	
105	May 9, 0. 9.14. a.m	0·8	0·7	3·6	31·9	S65°W	2. 11				Local	
106	May 17, 4.19.44. p.m	Hongo 0·2	0·3	2·1	44.1	NS	0. 20				N 30 miles inland, Tokyo on the edge of the disturbance.	10 m.
107	May 21, 9.46.20. p.m.	Hongo 0·4	0·6	2·1	22·1	NS	0. 35				N 20 miles, Tokyo near origin.	50 m.
108	May 29, 0.50.52. a.m.	Hongo 0·6	1·8	1·0	3·3	NS	4. 02	0·2	1·1	2 22	N N E 1 0 miles on shore-lines, Tokyo in the middle of disturbed area.	Extensive shock.
		Hitotsu bashi 2·1	1·1	6·0	34.3	W10°N	4. 30	Lost				
109	June 1.	0·1	0·7	0·4	3·2	EW	0. 51				Local	
110	June 17, 1.41.41. a.m.	Tremors									Local	
111	June 20, 8.38.30. a.m.	Hongo 0·2	0·8	0·8	6·4	EW	1. 20				N E 70 miles on shore-line, Tokyo on the edge of d. a.	100 miles. Moderately extensive earthquake.
		Hitotsu bashi 0·4	0·9	1·4	9·8	variable	1. 35					
112	June 21, 2. 2.35. p.m.	Tremors					1. 05				Local	
113	June 22, 7.42.30. a.m.	Hongo 0·2	0·6	1·0	10·0	SN	0. 46				E 48 miles on shore-line, Tokyo on edge.	53 m.
		Hitotsu bashi 0·3	0·7	1·3	11·2	NW	1. 38					

No.	Date	Max. Horiz. Motion in m.m.	Complete Period of Max. Horiz. Motion in Sec.	Max. Velocity in m.m. per Sec.	Accel. in m.m. per Sec. per Sec.	Direction of Max. Horiz. Motion.	Duration of Horiz. Motion.	Max. Vertical Motion in m.m.	Complete Period of Vertical Motion in Sec.	Duration of Vertical Motion.	Distance and direction of origin from the observatory in miles.	Radius of propagation of seismic waves in miles, or area of disturbance in sq. miles.
							min sec.			min. sec.		
114	June 30, 8. 0.35. a.m.	Tremors										
115	July 2, 3.16.24. p.m.	Hongo. 0·6	0·6	3·1	32·0	S20°E	1. 30	Trace			W 45 miles, Tokyo on edge.	55 m.
		Hitotsu-bashi. 0·6	0·9	2·1	14·7	NNE	1. 01	Trace				
116	July 4.	Hongo, 0·2	0·9	0·7	4·9	EW	0. 32				Local	
117	July 11, 3. 7.42. p.m.	Hongo, 0·2	0·5	3·1	9·61	SN	0. 24					
118	July 22, 8.27. 0. p.m.	Hongo, 1.0	1·4	2.3	10·6	EW	1. 38				The same in general feature as No. 62 and No 61 Extensive earthquake.	
		Hitotsu-bashi. 1·9	1·8	3·3	12·1	N30W	3. 37					
119	August 15, 0.59.15. a.m.	1·9	2	3·0	9·5	SN	3. 05	0. 1	0. 6	1. 01	N E 105 miles on shore-line, Tokyo on the edge.	120 m.

Notes.

For reference see corresponding numbers in the tables and in notes.

No. 1.—This earthquake was moderate in its size being enclosed within the radius of 47 miles. It affords a good example of both horizontal and vertical motions. The maximum horizontal motion occurred at the third second from the commencement of the shock; at this time the vertical motion was still exceedingly feeble although it was recognazible from the beginning. It reached its maximum 3 seconds later than the horizontal motion which had been then much reduced in its amplitude. The vertical motion was smaller than the horizontal motion in the ratio of 1 to 6; its period was quicker in the ratio of 7 to 8 and its duration of motion was shorter in the ratio of 1 to 3·3. The direction of the maximum horizontal motion was NNE and SSW while the origin of the earthquake lay in due N from the observing station.

No. 2.—This shock gave the second largest motion recorded in the Table. The horizontal motion was comparatively feeble during the first 20 seconds, but gradually augmented and remained active during 80 seconds. The vertical motion appeared from the beginning but was very small notwithstanding the large horizontal movement that accompanied it. The ratio of the former to the latter was 1 to 46 in amplitude, 1 to 4 in period and 1 to 3·8 in duration.

In this and in the following shocks it will be observed that the direction of the local movement of the ground at the observing station and the direction of the origin of the shock from the city did not generally coincide.

No. 3.—This earthquake disturbed the same portion of the country as *No. 2*, but with less force. The ground moved almost

equally in all directions. More than 120 complete waves whose periods varied from 0·7 seconds to 3 seconds were registered. Notwithstanding the existence of the considerable horizontal motion no vertical motion appeared.

No. 4.—This was one of the local shocks which frequently occur in this and in other parts of the country. Its area of disturbance is often not more than a few square miles. The motion is generally feeble in those local shocks.

No. 5.—The ratio of the vertical motion to the horizontal motion was 1 to 10 in amplitude and 1 to 13 in duration.

No. 13.—More than 50 distinct waves of small amplitudes were counted.

No. 14.—This extensive earthquake originated among the mountain district of Shinano which is one of the highest portions of the country 2,000 ft. above the sea level. There are one active and many extinct volcanoes. The seismic waves were not propagated much beyond Tōkyō.

No. 21.—Tōkyō was in the middle of the shaken district.

No. 23.—Both horizontal and vertical tremors were visible from the beginning ; but at the fifth second there suddenly appeared a large horizontal motion (maximum). Distinct vertical waves came a few seconds later. See Plate XI, Vol. I of this Journal.

No. 25.—The motion commenced slowly and was not preceded by quick tremors as is usually the case. The Observatory was comparatively near the origin of the disturbance.

No. 33.—This was a middle sized earthquake in which the observing station was near its origin. The maximum horizontal

motion occurred 6 seconds from the commencement and the maximum vertical motion 2 seconds later. Several distinct vertical waves of the average period of 0·3 seconds were registered.

No. 39.—Trace of the vertical motion was visible though the shock was only local and the motion small.

No. 41.—The whole of North Japan was disturbed by this shock, the observing station being near the southern extremity of the disturbed district.

No. 47.—The origin which was inland was comparatively near the city. There were hardly any vertical tremors during the first few seconds while there were considerable horizontal tremors. A decided horizontal motion occurred at the beginning of the sixth second; more pronounced vertical motion began one and half seconds later and its maximum ocurred several seconds after. See Plate IX, Vol. I of this Journal.

No. 50.—This was another large earthquake in which the seismic waves were propagated from the origin some 120 miles both north and south, and 61 miles toward the west, where they were stopped by the mountain. On the east they reached the Pacific Ocean. The observing station was comparatively near the origin.

This shock was preceded by tremors of quick period during the first eight seconds, then there suddenly appeared the maximum horizontal motion; at this time the vertical motion, which was visible from the beginning was yet very small—0·08 m.m. with a period of 0·4 second; after 6 seconds it reaching the maximum, and continued for eighty-five seconds with decreasing amplitudes and with lengthening periods. The ratio of the vertical motion to that of horizontal motion was 1 to 8·3 in amplitude, 1 to 1·5 in period and 1 to 2·2 in duration.

No. 51.—This earthquake, although it was quite extensive and its origin was comparatively near the observing station, produced small motions. The vertical motion was visible from the commencement, and exhibited its maximum at the seventh second when the horizontal motion was also largest.

No. 61.—This extensive shock disturbed the whole of North Japan, Tōkyō being near the edge of the disturbed area. The peculiarity in this shock was the unusually large vertical motion with its slow period.

The ratio of vertical motion to horizontal motion in Hongō	**1:2·5**
,, ,, ,, ,, ,, ,, ,, ,, ,, Hitotsubashi	**1:6**
,, ,, ,, horizontal ,, in Hongō to that of ,,	**1:2·5**
,, ,, ,, vertical ,, ,, ,, ,, ,, ,, ,,	**1:1**

No. 62.—Originating on the shores of the Japan Sea, the shock crossed the whole breadth of the main island. Nearer the origin the motions were very violent and somewhat destructive: it stopped the flow of springs and shattered houses.

No. 64.—In general features this shock resembled that of *No. 62.* It disturbed the same parts of the country and likewise caused considerable damage though in less degree.

No. 67.—Vertical and horizontal motions began at the same moment, but the maximum of the latter preceded that of the former by several seconds.

No. 72.—Tōkyō was comparatively near the origin.

No. 75.—Tōkyō was near the edge of the disturbed area. The maximum horizontal and vertical motions were simultaneous.

No. 76.—Tōkyō was near the outskirt of the affected district.

No. 77.—It was quite a strong shock nearer its origin which was in the sea not far from the shore. The maximum vertical motion arrived several seconds before the maximum horizontal motion.

No. 83.—Vertical motion was comparatively large considering the smallness of the horizontal motion ; moreover it was clearly pronounced exhibiting eight distinct waves. Its maximum appeared a few seconds after the horizontal maximum.

No. 85.—This is one of the two largest earthquakes in 1887. The origin of the shock was in SW about 35 miles from the Observatory. The seismic waves propagated nearly 200 miles to the west and north-east along the Pacific sea-board. On the north-west they approached to the shores of the Japan Sea. They shook, in all, about 32,000 square miles of land area.

At Hitotsubashi, after few seconds from the commencement of the shock the ground moved suddenly 3 m.m. toward the west. At the thirtieth second the maximum horizontal motion recorded in the Table was observed, which apparently corresponded with the maximum horizontal motion in Hongō. More than sixty distinct shocks were recorded.

At Hongō, the earthquake commenced with quick tremors. During the third second there appeared for the first time a vigorous horizontal motion in NW and SE (i. e. at right angles to the line joining the origin of the disturbance and the Observatory) accompanied by a considerable vertical displacement. The maximum vertical motion occurred at the ninth second. The maximum horizontal motion occurred at the thirty-third second.

Decided vertical and horizontal motions simultaneously occurred at the second second. See Plate XXVI, Vol. I of this Journal.

The ratio of the max. h. m. in	Hongō to that of Hitotsubashi	1:3
,, ,, ,, ,, ,, v. m. ,,	,, ,, ,, ,, ..	13:18
,, ,, ,, ,, ,,	v. m. to max. h. m. in Hitotsubashi	1:12
,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,	Hongō	1:6

No. 95.—Tōkyō was on the edge of the disturbed area. There were a few distinct vertical waves at Hitotsubashi.

Summary of Results.

The results obtained from a study of the preceding table may be summarised as follows.

In most of the earthquakes vertical motion did not appear, that is, the ground moved entirely in a horizontal plane. This was on account of the origin of the disturbance being far away from the observing station. In the above table, if we reject the slight shocks marked *tremor*, and the doubtful cases, we see that out of 100 earthquakes vertical motion occurred in 28 only, *i. e.*, once in 3·6 shocks.

Taking averages of all the quantities involved in these 28 cases, we find :—

Maximum horizontal motion	1·2 m.m.
Complete period of maximum horizontal motion	1 sec.
Duration of horizontal motion	124 sec.
Maximum vertical motion....	0·18 m.m.
Complete period of maximum vertical motion	0·56 sec.
Duration of vertical motion	42 sec.

Treating similarly the horizontal motions in 95 earthquakes recorded at Hitotsubashi (soft soil). we find :—

Maximum horizontal motion	0·73 m.m.
Complete period of maximum horizontal motion ...	0·76 sec.
Duration of horizontal motion	117 sec.

Also from a like treatment of the horizontal motions in 18 earthquakes recorded at Hongō (hard ground), we get :—

Maximum horizontal motion	0·37 m.m.
Complete period of maximum horizontal motion ...	0·76 sec.
Duration of horizontal motion	74 sec.

The values of the horizontal motion given in the first tabulated set of averages are larger than those in the second and third. The reason is, clearly, that shocks containing vertical motion are generally larger than those without, so that the second and third sets, which include shocks both with and without vertical motion, naturally give smaller averages.

Although the second and the third sets, referring as they do to different shocks, are not strictly comparable, they nevertheless show in a general way that in hard ground the motion is smaller, the period quicker, and the duration shorter, than in soft soil. Their ratios are 1 to 2, 1 to 1·3 and 1 to 1·5 respectively.

In the above sets of averages the records of the somewhat destructive earthquake of January 15th, 1887, were not included as that was much larger than the ordinary shocks we are dealing with. For the sake of comparison the characteristics of that shock as registered at Hitotsubashi are now given.

Maximum horizontal motion	21 m.m.
Complete period of maximum horizontal motion ...	2·5 sec.
Duration of horizontal motion	6 min. 34 sec. (Principal motion 2 min.)
Maximum vertical motion	1·8 min.
Complete period of maximum vertical motion	0·9 sec.
Duration of vertical motion...	98 sec.

For details see *No. 85*. In the earthquake of October 15th, 1884,

a maximum horizontal motion of 42 m.m. with a complete period of 2 seconds was recorded at the above named place.

When vertical motion occurred it was invariably smaller than the horizontal motion as is obvious from the first set of averages. The average ratio of the two components of the motion was 1 to 6, or the former was only one-sixth of the latter.

The period of the vertical motion was shorter than that of the horizontal motion. Their average ratio being 1 to 1·8. That is when the ground made one to-and-fro motion it also performed during the same time nearly two complete up-and-down oscillations. In all the preceding tables the periods of the *maximum* vertical motion are given, but the periods in the other parts of the disturbances were much shorter. They varied from 0·2 seconds to 0·5 seconds. Exceedingly feeble tremors of a few seconds duration generally preceded the principal motions as in the case of the horizontal motions.

The duration of the vertical motion was much shorter than that of the horizontal motion. It occurred invariably during the early stages of the earthquake and generally ended before the horizontal components. The average ratio of the two durations was 1 to 3, or the horizontal motion continued three times longer than the vertical.

The vertical motion almost invariably appeared when the horizontal motion had reached 1 m.m. which was more than the average amplitude in ordinary earthquakes. Out of 100 shocks there were 18 cases in which the ground moved more than 1 m.m. Out of these 18 earthquakes vertical motion occurred in 14 cases, or 78 per cent. and did not appear in the remaining 4 cases.

But on the other hand vertical motion also appeared in certain cases when the horizontal motion was less than 1 m.m. Out of the

28 shocks already specified as showing vertical motion, 14 showed a horizontal motion of more than 1 m.m., while in the other 14 cases the horizontal motion was less than 1 m.m.

Again when we analyze the 14 cases which had vertical motion with less than 1 m.m., we see that in 9 shocks the observing station was in, or comparatively near to the centre of the disturbed districts, and in the remaining 5 cases it was at a considerable distance from the origin. In other words vertical motion generally appeared when the origin of the disturbance was near the observing station. In such a case the vertical motion might have come directly through the earth-crust from the origin, and not in the form of free surface waves. It must be, however, noted that there were 4 cases in which the observing station was comparatively near the centre of disturbance, but in which vertical motion did not occur. They were all small shocks, with maximum motions less than 1 m.m.

In earthquakes showing both horizontal and vertical motions, feeble but quick-period tremors of both types simultaneously preceded the principal movements. The more decided and pronounced motion usually appeared first in the horizontal component, and then came the large vertical movements. For the diagrams of motion see Plate IX and Plate XI, Vol. I of this Journal.

Tremors, or quick-period minor waves generally precede earthquake proper and their probable connection with sound phenomena has been discussed in this country by Profs. Ewing[1], Milne[2] and Knott.[3] The vertical motions which have just been considered possess in great measure the characteristics of these minor tremors.

It is a well-known fact that the movement of the ground at the time of earthquakes is very complex, and that the ground moves in

1 Memoirs on Earthquake Measurements, p. 11.

2 Earthquake Notes— Sound Phenomena, Trans: Seis: Soc: Vol. XII.

3 Earthquakes and Earthquake Sound, etc., to be published in Trans: Seis: Soc: Vol. XIII.

all azimuths during a prolonged shaking. In the earthquakes discussed in the present paper the direction and the distance of the origin of disturbance from the observing station were known, as also the direction of the maximum movement of the ground. No definite relation between these directions can be established. Seismic waves indeed must suffer much reflection, refraction and diffusion as they progress, and the co-existence of normal and transverse waves is a distinct element of confusion.

Out of 119 earthquakes recorded in the tables 42 were local shocks, which extended only over a small tract of land from a few miles to 10 or 15 miles around. They caused the ground only slightly to tremble and the horizontal motion of the ground was from 0·1 m.m. to 0·3 m.m., or even less. There were four cases in which the motion reached 0.8 m.m. No vertical motion occurred in these local shocks.

There were 119 shocks during the two years, which means more than one shaking per week.

As the continuation of these observations will be published in the coming numbers of this Journal further generalizations or conclusions, that might be deduced from the facts now given, will be reserved for a future paper.

...Schists of Chichibu.

Errata.

Page 33, 6th line, for "Tachylite" read "Tachylyte."
" 39, 9th line, add "The dispersion along the axis $\mathfrak{c}$ is $\rho > v$."
" 45, 11th line, for "specimen ă" read "specimen."

and Southern ...

conflexure (Schaarung), as Suess[2] and ...

1) Ueber den Bau und die Entstehung der japanischen Inseln, Berlin, 1885, p. 79.

2) loc. cit. p. 50.

3) E. Naumann, Die Erscheinungen des Erdmagnetismus in ihrer Abhängigkeit vom Bau der Erdrinde, Stuttgart, 1887, p. 17.

4) A letter from T. Harada read in the 'Sitzung der mathematisch-naturwissenschaftlichen Classe vom 7. Juli, 1887.' Vide 'Akademischer Anzeiger,' No. XVII, Wien.

On the so-called Crystalline Schists of Chichibu.

(The Sambagawan Series.)

by

Dr. Phil. Bundjirō Kotō.

With plates II, III, IV, & V.

Introduction.

The district of Chichibu is in the Main Island (Honsiū) of Japan lying to the north-west of our metropolitan city of Tōkyō within a day's march. From considerations geological rather than political, Dr. E. Naumann[1] has very appropriately called this region the "alte Bergland von Kwantō"—all the mountainous districts surrounding Chichibu proper being comprehended under this designation. The district now under consideration lies to the east of the so-called great geologic ditch or '*fossa magna*' of E. Naumann,[2] which intersects the Main Island from south-east to north-west through the provinces of Izu, Suruga, Kai, and Sinano, thus separating Northern and Southern Japan. The '*fossa magna*,' or the Japanese mountain-conflexure (Schaarung), as Suess[3] and Harada[4] call it, sends

1) Ueber den Bau und die Entstehung der japanischen Inseln, Berlin, 1885, p. 79.

2) loc. cit. p. 50.

3) E. Naumann, Die Erscheinungen des Erdmagnetismus in ihrer Abhängigkeit vom Bau der Erdrinde, Stuttgart, 1887, p. 17.

4) A letter from T. Harada read in the 'Sitzung der mathematisch-naturwissenschaftlichen Classe vom 7. Juli, 1887.' Vide 'Akademischer Anzeiger,' No. XVII, Wien.

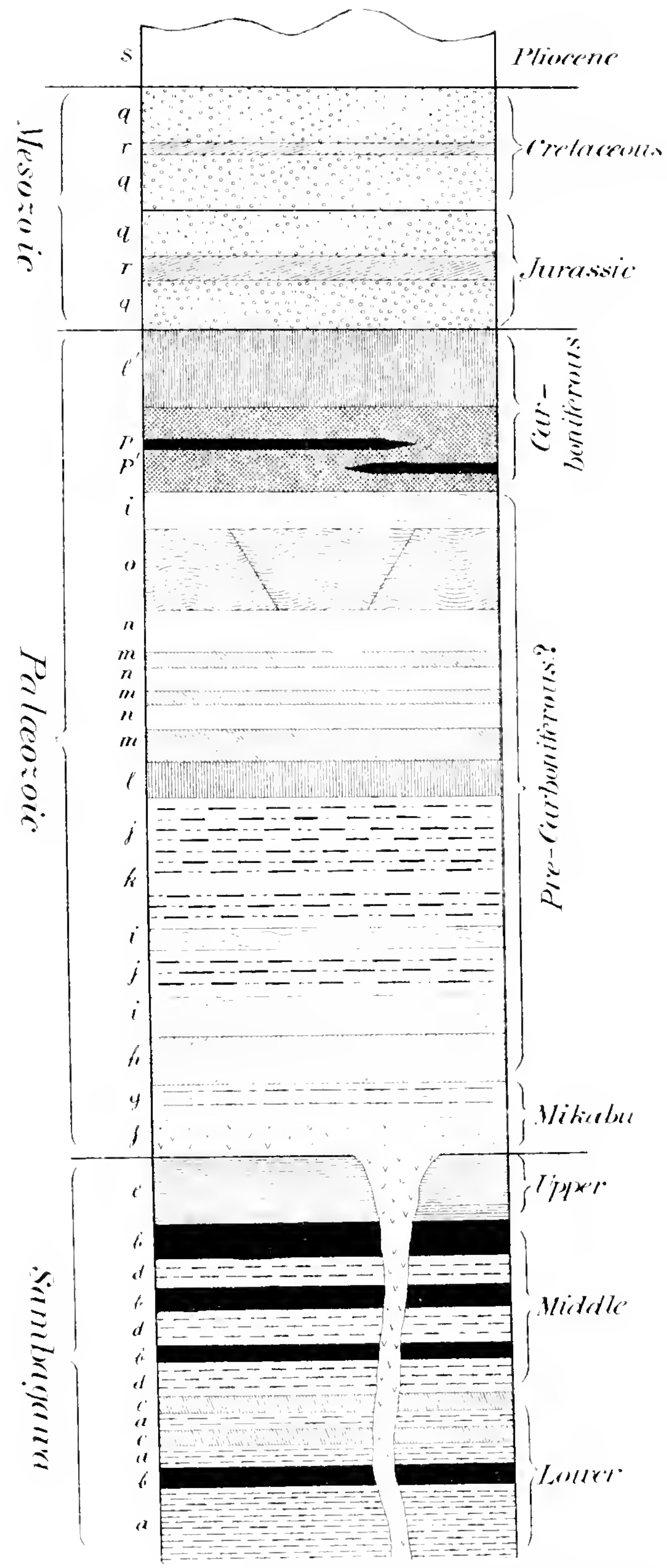


Diagram I. *a.*—Normal sericite schist. *b.*—Green spotted schist. *c.*—Piedmontite schist. *d.*—Black spotted schist. *e.*—Epidote-sericite gneiss. *f.*—Gabbro and gabbro-diorite. *g.*—Pyroxenite, amphibolite and serpentine. *h.*—Red and white, platy quartzite. *i.*—Adinole slate. *j.* Lower schalstein. *k.*—Limestone (lens-shaped bed). *l.*—Coralline limestone. *m.*—Slate. *n.*—Graywacke-sandstone. *o.*—Common hornstone. *p.*—Upper schalstein. *p'*—Diabase sheets. *l'.*—Fusulina limestone. *q.*—Sandstone. *r.*—Shale. *s.*—Tufaceous sandstone.

out a wing which continues further northward into the Abukuma and Kitakami mountains. This north-easterly wing of the remarkable mountain-conflexure of Japan suffered many, considerable interruptions in its way, especially in the plain of Musasi and Kōzuké; and the district in question is only a part of this wing.

Geologically speaking, this exten ive region of Chichibu is in itself a complete one, being bounded on the west and south by high volcanic chains of the well-known Fuji and Yatsugadaké, together with the granite-massives of Kimpōzan and the Mikuni-yama; while the remaining parts are open, and covered by Tertiary and still younger series of the Tōkyō basin, of which a brief account was lately given by Prof. D. Brauns in his "Geology of the Environs of Tōkiō."

Isolated as it is within a

limited area, still the building materials of its geologic edifice are, to the writer's view, those composing nearly all the systems that occur elsewhere in Japan, and therefore the earth's history of Chichibu may indeed serve as a type of our geologic formations.

Mr. Ōtsuka, a graduate of our university, took up, at the writer's request, the district in question for the subject of his thesis, in order to explore thoroughly its geognostic condition. As the result of his study in field and laboratory, he has eventually succeeded in recognizing a certain regular order of sequence of strata younger than the so-called crystalline schists. From various observations taken as far as possible at different points of the Chichibu mountains by the writer and Mr. Ōtsuka, and from the data ascertained by Mr. Ōtsuka,[1] with which the writer's scheme is also incorporated, the latter chiefly referring to the ? pre-Carboniferous and Sambagawan series, we were eventually enabled to recognize the following stratigraphic order of rock series, counting from the very lowest that is ever developed in this region ; it must at the same time be remarked that the names of fossils here given should be taken with reserve, and considered as only of an approximate value, as we can not pretend to any thing more than this at the present state of our knowledge :—

(Compare Diagram I. page 78.)

The Sambagawan series.

1 Normal sericite-schist together with the piedmontite-schist in its upper horizon.
2 Green, and black spotted schist.
3 Lamellar epidote-sericite-gneiss.

4 Amphibole-pyroxene schist, pyroxene-amphibole schist, pyroxene-epidote, and amphibole-epidote schist, together with serpentines, gabbros and the gabbro-diorites.

1) On the Geology of the Mountain-districts in Chichibu and Kanra, 1887. (Manuscript).

? The pre-Carboniferous.

5 Red, and white, platy quartzite.

6 Adinole-slate overlying the preceding.

7 The lower schalstein, intercalated with the adinole-slate, and also the limestone with crinoidal stems and corals.

8 Graywacke-sandstone and slate, intercalated in their lower horizon with the adinole slate.

9 Siliceous slate or common hornstone overlaid by another series of the adinole-slate.

The Carboniferous.

10 The upper schalstein and the siliceous *Radiolarian* slate.

11 Diabase-sheet sometimes occurs in this horizon.

12 The *Fusulina*-limestone, etc.

The Mesozoic group.

13 The Jurassic subgroup composed of the medium-grained, grayish sandstones and shales.

(*a*) The above-mentioned sandstones afford rich remains of *Cyrena*, *Melania*, *Myacites*; while (*b*) in shales we have found the remains of plants, viz., *Thyrsopteris*, *Nilsonia*, *Dicksonia*, *Podozamites*, *Pecopteris* and *Zamites*.

14 The Cretaceous subgroup consists of thick sandstones and shales with *Trigoniæ* belonging to the section *Scabræ*, *Peteroperna*, *Ostrea (Alectryonia)*, *Patella*, *Exogyra*, an evolute form of *Ammonites*, *Inoceramus*, *Alaria*, *Echinospatangus*, *Arca*, *Cucullaea*, *Solen*, *Anisocardia*, *Nucula*, *Pholadomya*, *Montlivaultia* and *Lucina*.

The Neogene Tertiary.

15 Lastly, the Tertiary basin of Chichibu, being made up of hard sandstones with marly layers, has furnished many fossils, the descriptions of which have lately been given by Dr. D. Brauns in his valuable contribution to the geology of Japan, entitled: 'Geology of the Environs of Tokio.' Mr. Ōtsuka made an addition of other forms to the list of Brauns, viz., *Arca*, *Turritella*, *Voluta*, *Nucula*, and an *Echinoid*.

The same rule holds good with regard to the regular successive order of the geologic series in the other parts of the empire; the Trias, however, which is represented by the *Pseudomonotis*-bearing strata extensively developed in Isadomae in Rikuzen,[1] Nariwa in Bitchiū, and Sakawa in Tosa has not yet been observed in our region.[2]

As to the literature relating to this district, there exists absolutely nothing worthy of a scientific value. A few years ago, the writer travelled through the present geologic terrane, and embodied his preliminary result in a paper "On the Geology of the Southwestern Districts of Kōzuke (manuscript);" this is all we have up to the present. The detailed geological map, Section Maebasi, covers a part of our region; but to his great disappointment, the writer could not derive any profit from it, as the system adopted in the map is too primitive.

During the last winter-holidays, the writer made a fortnight's trip, with the hope of ascertaining, if possible, a certain regular succession of the oldest rock-complexes known in this district, as heretofore no one had ever attempted to follow up the stratigraphical order in detail.

The field of our researches is only a small part of the rather extensive region of the 'Bergland von Kwantō,' being especially confined to the boundary-district of Musasi and Kōzuké provinces, where

1) E. Naumann; Ueber das Vorkommen von Triasbildungen im nördlichen Japan, in the 'Jahrbuch der geologischen Reichsanstalt' in Wien, XXXI Band, 1881, ps. 519–528. Vide also 'Mittheilungen der deutchen Gesselschaft für Natur-und Völkerkunde Ostasiens. Vol. III, p. 205.

2) The Japanese fossil described as *Monotis salinaria* by E. Naumann seems now to be included among the group of *Pseudomonotis ochotica* which, according to Teller, has a wide distribution in the western States, U. S, in New Zealand, New Caledonia, in the Himalaya, and Japan.—M. Neumayr, 'Erdgeschichte,' II Band, p. 266. E. Mojsisovics, 'Arktische Triasfaunen.' Mém. Ac. imp. de St. Petersbourg, Tome XXXIII, No. 6. p. 123.

the so-called crystalline schists[1] are fully exposed to view, and the study of their geognostic condition forms the subject of the present paper. The following gives only a compendium of the writer's notes, and from the nature of the circumstances, much more cannot be expected.

The writer now proposes to treat the subject in the following order:

A.—Petrography of the Sambagawan series.

(*a*) General remarks.
(*b*) The lower division or the normal sericite-schist.
(*c*) The middle division or the spotted graphite-schist, and spotted chlorite-amphibolite-schist.
(*d*) The upper division or epidote-sericite-gneiss.

B.—Architectonics of the Sambagawan series.

(*e*) General remarks.
(*f*) The Lower Sambagawan.
(*g*) The Middle Sambagawan.
(*h*) The Upper Sambagawan.
(*i*) Profiles.
(*j*) Relation of the topography and geology of the Sambagawan terrain.
(*k*) Massive rocks.

C.—Conclusion.

---

1) It should be here expressly remarked that, when speaking of *crystalline schists* of Chichibu in the present paper, the writer takes them only in the petrographical sense, and does not necessarily imply those of the Archaean group.

A. Petrography of the Crystalline Schists.

(The Sambagawan Series.)

(a) General Remarks.

However simple and monotonous the structure may appear when we travel through a territory of schists, still there are sufficient diversities to be seen even by a casual observer. It is to be deeply regretted that crystalline schists have been regarded with utter indifference by our geologists, so that they have been mapped barely as such and taken heed of no more; but in reality, these old schists can tell more of the history of the past than others, although the events lie hidden by the veil of time. Dissimilarity in structure, in colour, and the fluctuation of relative quantities of mineral components are sufficiently considerable to allow of brief characterization.

The rocks, of which the writer is now speaking, represent those complexes which form the very foundation of the geologic architecture in the Chichibu region, upon which the later formations were built up. E. Naumann[1] says, when speaking of the Archæan group in general, "that the crystalline-schist system in Japan consists of mica-schist, talc-schist, and chlorite-schist etc., together with subordinate layers of serpentines and marbles............Those schists of the 'Bergland von Kwantō' are also characterized by a very complicated mode of occurrence............They describe a deformed arc." His statements evidently refer to the region which is now under consideration. But the rocks here developed show many peculiar characters of their own, not observed in the normal types of crystalline schists, and this abnormality

1) loc. cit. p. 9.

has lately induced the writer to make a somewhat closer study of these ancient schists.

The rock-components of the Chichibu schists are of a crystalline nature as in those of normal types ; the texture of the rocks is, however, less perfectly developed, some being compact, others thick-banded, while the third is wrinkled and corrugated by mechanical action due to the earth's movement. All our schists in common present a *phyllitic* aspect ; but the use of that name should here be guarded against, for to the term phyllite, geologists attribute sometimes a geochronic meaning on account of its usual occurrence in the crystalline schist system ; sometimes we understand it petrographically as having green or colourless fibrous scales—the phyllitic constituent, and also as having a schistose structure.

The most characteristic components of our schists are sericite, epidote, and calcite. The presence of these already bespeaks the nature of the rocks in question ; for in the genuine crystalline schists, these minerals are often totally absent.

Taken in general, our rocks have a close relation to the so-called 'Casanna Schiefer,' or Studer's 'aeltere graue Schiefer' of the Alps, or to the sericite-gneisses, and sericite-schists of the districts of Nassau, and Taunus, in Germany.

The whole series of the rocks in question may be found along the Sambagawa valley,[1] north-west of a small town Onisi,[2] Kanra district, where it is developed in its full proportions, and where interesting exposures may be seen. The writer proposes now the name of the *Sambagawa series* to the metamorphic schists of this district. It is a name which does not involve any theory, and may be used by any party in the case of a controversy respecting its age. This designation may not be of a mere local value as the same series recurs in other parts of Japan, especially in the Island of Sikoku.

---

1) 上野國南甘樂郡三波川村 2) 鬼石町

(b) The Lower Division.

Normal Sericite-schist.

It is a grayish-white, thick-foliated rock with a wavy sweep on the sheen surface. Slabs show numerous prominences of a yellowish tinge, owing to the presence of epidote crystals. The cleaved face has a silky lustre, due to the parallel arrangement of fibrous scales of sericite. The rock-ingredients are *quartz*, some *felspars*, *sericite*, *calcite*, a *yellowish-green epidote*, *iron-glance*, *iron-mica*, and lastly, *rutile* ; apatite, the most common accessory component in the massive rocks and schists is remarkable by its absence in the rock-series now in consideration.

The greater part of the rock-mass is made up of *quartz-grains*. Their external boundaries are quite irregular, and appear as if one overlapping the other. By a simple macroscopical examination, the part occupied by quartz presents almost an homogeneous aspect, and gives an impression as if it were occupied by solidified masses of the colloid silica of the diagenetic origin. By the use of an upper Nicol, the state of things is found to be quite otherwise ; the apparently homogeneous mass resolves itself into an aggregate of quartz grains, each having optically different orientations, and in many cases showing a marked undulatory extinction. The granulation of the quartz is particularly pronounced where the rock-masses have been subjected to minute foldings and flexures, whereby the dismembered parts are more or less pushed further on one side or the other. This complicated condition may be best seen in *fig. 1. Pl. II.* which is drawn from a transverse section of the glaucophane schist from Ōtakisan, Awa,—the rock probably belonging to the same geologic age as the Lower Sambagawan schists. On looking at the figure, it is evident that the sericite (hydromica) was originally arranged in

regular order, separated by bands of a homogeneous (but not amorphous) quartz. The so-formed rock has been then subjected to the act of crushing by mountain-making, and that trituration, as it were, must have been secondarily induced, may be proved from the discontinuity of irregular fissures which become very numerous on the upper and lower margins of the quartz-bands, but stop short at the centre. These fissures are particularly numerous at the turning points of the plicature of the rocks, and indeed, the quartz appears there perfectly granular.[1] F. Becke has, in rocks of a similar nature, encountered the quartz-grains arranged in a fan-shaped form; this being shown by the manner in which the extinction-direction of polarized light varies as the grains are examined in succession around the radial centre. He has rightly compared this phenomenon with that in the twisted smoky quartz from the Kreutzlipass in Switzerland.[2] This peculiar fact of which the cause is apparently unknown to him may, as it seems to the writer, be ascribed to the special circumstances under which the rock containing a homogeneous quartz has been granulated. R. Küch,[3] while studying the rocks from Mamanyamatali, Western Africa, seems to have noticed a similar fact, and from this, he formed a hypothesis regarding the clastic nature of the epidote-mica-schist, in which he saw the structure. Some of these grains (apparent) may be a product of secondary infiltrations, and they are distinguished from the rest by weak polarization-colours. Such grains are generally free from interpositions and inclosures, whatever the kind may be, as this quartz is the latest among the rock-components. Slow, insensible, undulatory extinction of light invariably takes place in this variety.

In a schist from Inatsuka, in the Sambagawa valley, we noticed

1) Michel-Lévy calls such quartz masses "Quartz granulitique." Bull. géol. (3) VII. p. 846. 1879.

2) Tschermak; Min. u. petr. Mitth. Band VI. 'Gesteine von Griechenland,' p. 66.

3) ibid. Band VI, p. 102.

the very interesting fact that the quartz here behaves just like an isotropic mineral. As this variety fills up the interstitial spaces of the matrix, it is highly probable that this quartz may be an infiltration-product, and serves as a cement for the other ingredients. Without the use of an upper Nicol, it is hardly recognizable as such, and a slide presents a perfectly homogeneous appearance; but by the help of an upper Nicol, it is soon discovered that the portion apparently homogeneous resolves itself into numerous crystalline grains of quartz cemented by an amorphous variety.

Taken as a whole, the quartz in all varieties of rocks of the Sambagawa series is poor in both liquid-inclosures and gas-pores, and such as are found are exceedingly minute in size. These inclosures are arranged in chains, and run approximately parallel to each other, sometimes terminating at the margin of the grains, or sometimes being continued on to adjacent grains of different optical orientation; sometimes again, these chains begin from contiguous points of the neighbouring grains, especially where the margins end in protuberances and indentations. Bearing in mind the facts just stated, these inclosures seem probably to be of a secondary nature.

The rock now under consideration has possibly assumed the present state through siliceous induration of a graywacke or a volcanic tuff; hence the colloid aspect of quartz in it and the paucity of inclosures. The silica may have been derived from the original rock by molecular rearrangement or in part may have come from other sources as a solution, and after the solidification the quartz may have been granulated by mechanical force due to movements of the earth's crust.

The *felspar* is very variable in quantity. In the normal sericite schist from Sueno near Yorii, it is rare; while in that of Ōboké along the Yosino-gawa, in Awa, it constitutes the main part of the rock at

the expense of quartz. The felspar is usually larger in size than the quartz-grains, and never shows any indication of crystallographic faces ; on the contrary, the periphery is exceedingly irregular having deep indentations and sharp projecting points.

It appears to the writer that the felspar grew externally by secondary enlargement, consequently it has tendency to assume more or less a porphyritic habitus. While the exterior of a larger felspar was still undergoing the process of accretion, the imbibed silica, or in part a silicate of sodium or potassium which had served as a solvent, seems to have just begun hardening. Such being the case, the margin of the porphyritic felspar assumes a *quasi*-granophyritic or micropegmatic structure, though by no means a perfect one, when compared with the typical development in granophyres, *fig. 2, Pl. II.*

The felspars often show a corroded appearance. They are of a remarkably vitreous aspect, quite fresh, and often bear evident traces of the basal cleavage ; parallel stripes are few, and there is no polysynthetic lamellar structure. The oblique extinction takes place at about 27° with reference to the vertical stripes. It may be inferred from these facts that the felspars are mainly orthoclase, while some may probably belong to an acidic plagioclase.

The most prominent feature in the felspar is the interposition of opaque, highly lustrous iron-glance; the latter by reflected light shows a slight bluish tinge, especially on the surface of thick lamellæ into which, according to Mügge,[1] the mineral is said to be easily parted, the plane of the lamellæ corresponding to $R = \pi$ $(10\bar{1}1)$ of iron-glance. Thin plates of blood-red iron-mica together with epidote-grains and rutile-needles are also mixed up with the opaque iron-glance in the felspar-substance as in *fig. 3, Pl. II.* All these are localized in the centre, and the periphery of the felspar is fringed with greenish

1) Neues Jahrbuch f. Min. etc. 1884, I Band p. 216.

fibrous lamellæ of sericite. The same structure also occurs in all the rocks belonging to the Sambagawan series, and may be considered as characteristic of these rocks. The same is also typically developed in the graphite-sericite schist, to which we shall have occasion to refer afterwards.

Quartz-grains are frequently found as enclosures in the felspars without any definite crystallographic relation to the latter. In massive rocks, felspars belong to an earlier generation in crystallizing out from the rock-magma, but here a portion of quartz has solidified prior to that of alkali-alumina silicates, so that the physical condition during the formation of the rock must have been a different one.[1)]

The *sericite* is of a yellowish-white, or light grass-green colour with silky lustre. In the latter case, it is pleochroic, showing strong absorptions parallel to the axis of a plication. In some places as in Azuhata in the province of Hitachi, a rock similar to the 'pierre ollaire' or Giltstein of the Alps is found, being essentially made up of sericite with a few admixtures of quartz. This affords us a good material for the study of sericite. It is a light greenish-white mineral with a perfectly smooth surface of cleavage and is extremely thin-lamellar. Its stauroscopic figure shows a tolerably wide axial-angle like that of muscovite. The sericite was isolated by means of the Thoulet solution from a glaucophane-bearing rock occurring in Ōtakisan near Tokusima in Awa, and Mr. Takayama, of the Geological Survey of Nippon, kindly undertook a chemical analysis of the mineral, the result of which is as follows:—

$Si\,O_2$	53.01
$Al_2\,O_3$	34.70

1) Michel-Lévy calls those round quartz-grains irregularly distributed in orthoclase "Quartz de corrosion," and considers them to be of a secondary origin. J. Roth, Allgemeine u. Chemische Geologie, II Band, III Heft, p. 393. 1887.

Fe_2O_3	trace
CaO	0.27
MgO	0.50
K_2O	6.05
Na_2O	1.01
H_2O	4.67
	100.21

Our sericite differs from the normal type in containing an excess of SiO_2, but less of Al_2O_3 and K_2O.

Bořický's sample also contained potassium, and a small quantity of calcium and sodium.

The flakes of sericite show a minutely folded texture; their appearance closely resembles wrinkled bamboo-paper, and this structure is very characteristic of this mineral. The sericite is commonly heaped around the periphery of felspars and epidote after the manner of a fringe, as if it were produced from both minerals. (*fig. 3, Pl. II*).

A greenish-yellow *epidote* occurs in irregular plates, traversed as usual by numerous, approximately parallel, curved fissures in the direction of the vertical axis. As these transversal rents are probably the contraction-fissures, their course is at right angles to the cleavage-planes ($oP, \infty \bar{P}\infty$) of this mineral. The size varies from $^1/_{10}$–1 centim., and seldom sinks into microscopic dimensions; so that the mineral is macroscopically discernible as yellow spots on the cleaved plane of the rocks.

A characteristic feature of the epidote in this rock is the abundance of microscopic interpositions of iron-glance, and rutile-needles. The former is sometimes heaped together in such enormous quantity within the crystal as to give to the epidote almost an opaque appearance, while the periphery is comparatively free from it. Crystals are torn asunder into various parts, each part being joined to the other

by fibrous-scaly lamellae of a greenish sericite. Pleochroism is distinct and intense nearly in the direction of the vertical axis. Epidote crystals arrange themselves with the orthopinacoid parallel to the plane of the schistosity of the rock, consequently its transversal section exhibits a vicinal clinopinacoid of epidote as an imperfect rhomboid, tapering at both ends.

Our epidote belongs to a common type in its crystal-form by having M and r or oP and $P\bar{\infty}$ as the predominating faces; consequently the clinopinacoidal section presents a flat rhomboid (M : r=116°); while the manganese-epidote which also makes its appearance in this variety of rock, is of a nearly rectangular rhomboid, when viewed in a similar section as the preceding. This is due to the fact that in the latter, T and i constitute the predominating faces which make with each other an angle of 99° or 81°.

Irregularities of outline mainly result from the neighbouring quartz grains, which push inwards into the substance of the crystals, and when the interior become full of such large, vitreous quartz-crystalloids, the epidote appears not unlike a melilite in basalts in regard to the structure. No evidence of twins can be found.

The rhombohedra of *garnet* occur, without exception, of a light green colour (in the garnet-amphibolite the colour is deep brownish-red, and the crystal becomes a magnetic bead before the blow-pipe). Sometimes the crystals merely make up the external part, while the centre is filled with quartz, and then the garnet-substance is full of fissures proceeding from the contact point with the quartz; rutile-needles are found within the crystal in most confused accumulations, suggestive at times of whirlwinds, and showers of needles occur within the garnet-substance as in *fig. 4. Pl. II.* The garnet is equally distributed throughout the whole of the rock, so differing,

in the lack of any special arrangement, from the other mineral components. Optical anomalies are not observed in pure individuals.

Some occurrences are particularly rich in *Calcite* in irregular patches, occasionally, however, in the idiomorphic form of perfect rhombohedral crystals. The latter case seems usually not common in the gneiss of the Archaean group. Calcite-patches often contain quartz- and felspar-grains. Traces of the rhombohedral cleavage appear in bent and zigzag lines, partaking of the plicature of the rock-mass itself. It is highly probable that the calcite in this rock is of a primary nature.

The opaque *iron-glance* and the blood-red *iron-mica* are constant components in this rock as well as in those of the two upper divisions. The opaque iron-oxide has a lamellar structure and exhibits a bluish tinge on the smooth surface, when observed by reflected light. Both varieties of hematite are usually irregular in their distribution, being massed together in one part and remarkably absent in others, and, as has been already said, they are particularly rich in the felspars and epidote.

The *rutile* is the most characteristic component of the Lower Sambagawan Series. It occurs in both types of twins, the one is heart-shaped, being composed of broad crystals, the other knee-shaped. These sometimes occur in trilling according to $P\infty$ and $3P\infty$ at the same time. So far for the description of the component-minerals.

The *normal sericite-schist* has itself a special structure, being caused by an alternation of the quartz-felspar mass with fine layers of sericite together with garnet, and ores of iron-oxides ; and as the consequence of such an arrangement, the rock cleaves into thin plates. A good exposure may be observed at the quarry in Sueno near Yorii.

A peculiar structure has been observed in the course of the study,

to which the writer wishes to call special attention. It has already been pointed out on page 85, that the quartz forming the matrix has a colloid appearance, and that the felspar-grains apparently corroded at their edge are imbedded in this homogeneous quartz, as if cemented together by a plaster. It may perhaps be conveniently called the *plastered structure*. This differs, however, from Törnebohm's[1] "Mörtel-Structur," by which we understand a fine aggregate of quartz and felspar containing large allomorphic minerals of the same species.

The upper horizon[2] of the Lower division is characterized by a most peculiar rock, viz., the piedmontite-schist.

The next higher—the Middle division of the Sambagawa series—is essentially formed by an alternation of two distinct types of rocks. These are the *spotted green*, and *spotted black schists*. The *piedmontite-schist* which has just been mentioned commonly occurs in thin bands in the lower *étage* of these spotted schists. Nevertheless the writer considers it to be an integral part of the Lower Sambagawan. It will now be shortly described. So far as the writer is aware, it has not yet been found in other parts of the world, or at least up to the present not recorded in any geological literature he has seen. This fact induced the present writer lately to work it out somewhat in detail, in a separate paper in this journal Vol. I, part III, to which the reader is referred for the details.[3]

This rock is of a purple colour, hence locally called the "Murasaki" or purple rock. It is rather more compact than the normal sericite-schist; but it easily cleaves into thin plates. On a weathered surface, beautiful purple-red crystals of piedmontite or

1) Rosenbusch; 'Massige Gesteine,' 2 te Aufl. p. 42.

2) *See* Note I at the end of the paper.

3) Vide also Quart. Journ. Geol. Soc., 1887, pp. 474 *et seq*.

manganese-epidote[1] are distinctly visible in long needles with unsymmetrical wedge-shaped terminations at both ends, transversally fissured, sometimes pushed asunder into several portions ; in other cases the piedmontite is bent like crystals of tourmaline or common epidote. The best exposure is found along the banks of the Ara-kawa river near the village Minano, or near Ōtakisan in Awa. Clusters of needles may often be seen together with quartz-nests as at Miyanosawa near Obata, or at Kamakata near Ogawa. It is not easy, however, to get out a good sample, as they are so intimately imbedded in the quartz-aggregate.

The components of the piedmontite-schist are (besides the manganese-epidote) quartz, sericite, a greenish-yellow garnet, rutile, non-striped felspars, and the blood-red iron-glance. On account of the parallel position of the piedmontite-aggregate with layers of quartz-grains, transverse sections of the rock present a regular banded appearance. The clinopinacoidal section of the mineral is of rhomboidal outline, caused by the predominance of the traces of T and i, and when the face M is at the same time well developed, the section is six-sided. Parallel growth and intergrowth of two or more individuals of different sizes may be found everywhere, and these are the main causes of striation upon the crystal-faces in the direction of the axis of symmetry (ortho-axis). The colour of the piedmontite is deep violet, and often yellowish-brown. A probable explanation of the considerable difference in colours has been given in a separate paper (loc. cit.). Axial colours are:—$\mathfrak{A}$=deep-violet ;

---

1) Prof. A. Stelzner, of the 'Bergakademie,' Freiberg in Saxony, has favoured the writer with a few lines, dated 30th July, of this year, after receiving my paper on 'Some Occurrences of Piedmontite in Japan,' (this Journal, Vol. I, Part III,) which run as follows: "Die Arbeit ueber Piedmontit interessirte mich um so mehr als Sie das bestätigte, was ich an einem Gesteine vom Gebirgsrücken zwischen Tatsikawa und der Besschi-Kupfergrube, Provinz Iyo beobachtete. Wir hielten das Gestein für einen Thulitschiefer—aber ihre Bestimmung wird wohl die richtige sein."

$\mathfrak{C}$ = brownish-red ; $\mathfrak{B}$ = light-violet. The Pleochroism is distinct, the polarization-colours magnificent.

Being of a beautiful rosy-red colour, highly pleochroic and acicular in habit, it has been called by Dr. E. Naumann a tourmaline.[1)]

The piedmontite is capable of undergoing various modifications. On the one side it forms a transition into a greenish-yellow epidote, the same one described in connection with the normal sericite-schist (anté page 90) but with this difference that here the red pigment localizes itself in the centre or in irregular patches. It is a most peculiar fact that the purple piedmontite also graduates into an almost *colourless* epidote. This abnormal, colourless epidote is found in long stalky crystals with rather fibrous terminations at both extremities, and is not unlike a broad column of *Grammatite*. What the chemical nature of this epidote is, the writer cannot at present say. Anyhow he is not disposed to consider it as a variety of zoicite.

The *hematite* is represented by the blood-red hexagonal plates of iron-mica whose minute scales are found abundantly in the greenish-yellow, and also in the colourless epidote ; while another variety (iron-glance) is comparatively rare and occurs in the form of dull, opaque clumps. Thus the habitus of iron-glance deviates somewhat from that of the normal sericite-schist. The presence of iron-mica gives a considerable reddish tinge to this purple schist. In piedmontite, iron-glance is never found as interpositions.

The *felspar* occurs exclusively as grains without any sign of idiomorphic forms, and commonly larger in size than those of the quartz. It is intimately intergrown with the latter as if the felspar served as a cementing medium, thus producing the *plastered structure*. The interposition of quartz-grains is of common occurrence—a fact illustrative of the simultaneous crystallization of both minerals. Simple twins

1) Loc. cit. p. 10.

and a few feeble traces of cleavage alone serve to indicate the presence of otherwise unrecognizable felspar in the rock-mass. The precise nature of the felspar could not be made out, simply because no favourably orientated sections could be seen. Anyhow, the habitus of the mineral coincides with that in the normal sericite-schist (cf. page 88). As in the latter rock, the interposition of iron-glance and iron-mica characterizes the felspar in this schist.

Of the *sericite* the writer has nothing to say, except that it is, as usual, colourless or of a light greenish tinge, and has a fine fibrous scaly structure. Light yellowish-green, rhombic dodecahedra of garnet are tolerably abundant.

In the Island of Sikoku, the piedmontite-schist is accompanied by a blue glaucophane-schist, a brief description of which has been already given in a separate paper.[1] This rock seems to be absent or at least has not as yet been found in the collections from Chichibu.

The piedmontite-schist is characterized by its peculiar outward appearance, of which enough has been already said, and this serves as a good criterion in tracing out the lower division of the Sambagawan series.

(*c*) The Middle Division.

Spotted graphite-schist, and Spotted chlorite-schist.

The horizon of the piedmontite-schist marks the beginning of the middle division. The rocks are principally of two types, viz., (I) the spotted black schist and (II) the spotted green schist, whose manifold alternations make up a considerable thickness, constituting the

1) B. Kotō, A note on glaucophane. This Journal, Vol. I. Part I. p. 8.

Sambagawan series proper. Their geographical distribution is very wide, consequently they are the commonest rocks which may be noticed even by cursory observers during a flying visit to this district.

What the present writer calls the spotted black schist is—

I.—*The spotted graphite-sericite-schist*, which is essentially made up of felspar, sericite, graphite, both varieties of hematite, quartz, and ? chlorite together with the characteristic accessories of tourmaline, garnet, and lastly rutile. Its outward appearance is not unlike that of the 'Garbenschiefer' of Saxony. The weathered rock has the aspect of a coarse-lamellar, brown mica-schist with prominent black spots (*fig. 5, Pl. II.*). In an advanced stage of decomposition it becomes even talcose in appearance.

These spots are generally of an inflated disk-shaped form with T, l, P, y (*fig. 6, Pl. II.*), the faces being very much blurred by the compression of the rock itself. The inflated side of this deformed felspar lies parallel to the plane of schistosity of the rock; consequently the transversally fractured surface of the rock presents a cleavage-face (P in the figure) of nodules with pearly lustre.

Under the microscope these spots or nodules ($^3/_{10}-^1/_2$ centim.) were found to be of a *felspathic* nature. The crystals occur in irregular forms; lustre vitreous; simple twins are often observed by the behaviour of different polarization-colours in the two halves, but with unequal angles on the right and left. A polysynthetic, lamellar structure happens not to have been observed (excepting in a very few cases, if it is really present at all), though traces of cleavage-planes are of common occurrence. Extinction of light usually takes place at about 16°–21° with a trace of twinning-lamellæ. On the rhombic face of the basal cleavage-plane, it makes an angle of −30°, or thereabouts, in the sense of Schuster (*cf. fig. 6, Pl. II.*)

As is shown in *fig. 7, Pl. II.* the central part appears almost

black, owing to an enormous accumulation of graphite-particles, rhombohedra of garnet, tourmaline-crystals, actinolite-prisms with the combination of ∞P, $\infty P \grave{\infty}$, and other colourless grains and micro-crystals of an undeterminable nature, together with opaque iron-oxide.

On igniting a thin slice, graphite (possibly identical with the *Graphitoid* of Sauer[1]) or Inostranzeff's *shungite*)[2]) is partly removed from the felspar-substance, although a dark centre remains as before. A closer inspection with higher powers discloses, besides amorphous coaly dust, some minute inclosures of air and liquid, the latter with movable bubbles. These inclosures are arranged more or less in stripes, and appear under weak powers as black dots. It is their presence that chiefly causes the opacity of the felspar as in the minerals of the hauyne group.

As to the arrangement of the black dust, the fine particles are disposed in the most fantastic manner, thereby producing at the centre a cyclonal, at the margin a fluxional-structure, as shown in *fig. 7. Pl. II.* Dr. A. Sauer[3]) has described a similar arrangement of interposition in a felspar contained in the felsparphyllite from Plaue, Saxony ; but there the interposed mineral is rutile instead of coaly dust. This peculiar structure has evidently arisen from the squeezing accompanying the movements of the crystals during their formation. Herr Dr. Sauer is rather disposed to think that the structure as of a primary nature, arguing in support of this view that the periphery of the felspar is perfectly pure and free from inclosures of foreign substances, and moreover that the suture of twinning-lamellæ passes straightforward throughout the whole section of crystals irrespective

1) Zeitschrift d. d. geol. Gesselschaft, XXXVII Band, 1885, p. 441.

2) Neues Jahrbuch f. Min. u. Geol. etc. 1880. I Band, p. 97.

3) N. J. f. Min. etc. 1881, I Band, p. 232. 'Rutil als mikroskopischen Gemengtheil in der Gneiss—und, Glimmerschieferformation, sowie als Thonschiefernädelchen in der Phyllitformation.

of the arrangement of the interpositions. The writer is, however, inclined to view the subject from another stand-point ; namely, that the clear periphery is due to an accretion of another generation resulting from the *secondary enlargement*, the centre being the primary ; the further point in regard to the cleavage, being perhaps explicable as a result brought about by pressure induced afterwards. C. R. van Hise[1] has lately found in the slate-conglomerates of the north shore of Lake Huron what may be enlarged felspar grains, but the evidence there is not sufficiently satisfactory as to the material being of secondary origin, the line of separation between the supposed new material and the nuclei being indistinctly marked. The present case affords better evidence of secondary enlargement, owing to the circumstance that the new material on the clear periphery being free of the above-mentioned interpositions is sharply marked out from the core.

The marginal portion of the felspar nodules is fringed with, or enclosed by a green sericite,[2] which is sometimes coloured brown by the oxidation of iron contained in it ; more especially after ignition a slice of it acquires a deep brownish-red colour, becoming not unlike a common biotite, from which we may infer that our green sericite is rich in ferrous oxide. This kind of mica seems to be of a wide distribution in (metamorphic) schists outside the Japanese Archipelago ; for the writer has observed the same in a mica-schist from Killin, Perthshire, Scotland, and in all the '*Amphilogit-Schiefer*' from the Zillerthal in Tyrol. In petrographical literature it has been described under various and even self-contradictory names,

1) Bulletin U. S. Geological Survey, Vol. II, p. 44.

2) The present sericite might probably be a modified form of Phengite of Tschermak, for it contains a large amount of $Si\ O_2$, but less of $Al_2\ O_3$, as may be seen from the numbers stated in page 90.

such as green mica, hydro-mica, sericite, green biotite, chloritoid, phyllitic element, etc.

The *tourmaline* forms a very characteristic component of the spotted black schist, especially as regards its quantity and the perfection of its forms. The crystals are slender, being bounded by deuteroprism ($\infty P2$) and protoprism (∞R), and terminate in an acute rhombohedron ($-2R$) at one end, and an obtuse at the other ($+R$); the basal pinacoid not yet observed; but one pole often broken; transversal fissures rare; coaly particles often found as interpositions. The acute pole rhombohedron ($-2R$) is, as a rule, unsymmetrically developed, while the opposite obtuse end is bounded by a rhombohedral face ($+R$) well finished and very regular. And it is a very remarkable fact that the *antilogous* (+) acute pole bounded by $-2R$ is *always deeply coloured;* while the *analogous* (−) obtuse pole ($+R$) is of a *lighter shade* (*fig. 8, Pl. II*). This difference in the tinge of colour should in some way be connected with its pyroelectric properties, which, as Hauy long ago pointed out, have some close connection with the very characteristic hemimorphism of the tourmaline-crystal.

The vicinal section (*fig. 9, b, Pl. II.*) of the base of our tourmaline presents some noteworthy features which may here be briefly noticed. Some basal sections show a nucleus which is marked off clearly by a fine contour, and also by the lighter colour of the centre, while others (*fig. 9, a, Pl. II.*) show an imperfect external shell with half-covered internal parts. Examples of the former are often found in various plagioclases known as isomorphic shells (isomorphische Schichtung); while in the latter we have an example of a parallel growth of individuals of the same mineral species, the well-known vertical stripes of tourmaline-columns being due greatly to this special growth. Bearing in mind the facts stated above, we are now forced to believe that the

so-called isomorphic shell and the parallel growth do not differ from each other but really mean the same thing.

As to quantity, tourmaline is so abundant that it may be regarded as an essential component.

The quartz, sericite, garnet, the light-green epidote crystals and grains, and lastly, the calcite present no peculiarities worthy of special description.

The *structural modification* of the rock varies within a wide range from a coarse-lamellar rock to a thin-tabular graphite-slate; in the latter the nodules are scarcely visible, unless weathered surfaces are viewed. In the coarse extreme, the nodules attain more than $^1/_2$ centim. in size (*fig. 5, Pl. II.*) and at the same time the rock becomes less graphitic, while the sericite increases proportionally in quantity, and then the colour changes from black to brown. When seen with reflected light, the sericite-lamellae display a nacreous lustre.

In a fine slaty variety a transverse fracture of the rock has a banded appearance through the alternation of quartz-felspar layers with those of the coaly particles. The coarser variety is typically developed near Ōda, while the slaty one occurs at the village of Honnogami.

II.— The *chlorite-amphibolite* or spotted green schist is a thick, imperfectly schistose rock of a grass-green colour with an uneven plane of schistosity. It is full of innumerable white spots ($^1/_2$–2 mm.) on a green ground, presenting an aspect quite similar to currants in a pudding (*fig. 10. Pl. II.*) F. Becke seems to have found a similar structure in a mica-schist and gneiss rich in felspar-nodules from Selitschani in Greece, and has given to it the name *cristhic structure* [1]) from the likeness to cereal grains. Our rock has such a striking

1) Tschermak, Min. u. petro. Mitth. II Band, 1880, p. 43.

resemblance to that of the so-called 'Grünschiefer System' of Saxony, that when hand-specimens from both countries are placed side by side, it is hardly possible to distinguish them, nor are there any special distinguishing features even under the microscope.

Slides show that, under high powers, each spot is nothing but an individual grain of felspar, or composed sometimes of many grains. These felspar-dots are all, without exception, of an irregular outline, and present a characteristic cataclastic structure. They are colourless, showing quite a fresh aspect—only a few stripes; twins referable to the Carlsbad type occasionally found—polysynthetic twins exceedingly rare.

(1) The writer is of opinion that this rock is a transformed schist from a *tuff* of a felspar-pyroxene rock of an eruptive origin; and (2) it is an accepted dogma that, when minerals are subjected to a certain pressure, they usually exhibit many traces of cleavage (although all minerals have not as yet been experimentally tested); and lastly (3) a mechanical force can put in motion the molecules of minerals so as to make the particles assume a new position, the result of which is the formation of twins, gliding faces, etc.—it is, indeed, very striking to find only a few such traces of cleavage in our felspar, although the rock bears signs of having been subjected to pressure in the mass-movement.

A probable explanation of these facts may be that the felspars have once been in a condition, in which a long-sustained pressure and moderate heat, with the simultaneous presence of mineral solutions, have made the molecules in the felspars to move easily but only to a *certain extent*. The same reasoning may well be applied with equal force to the formation of schists, which have an appearance as if they were formed by the solidification of an originally gelatinous mass in the diagenetic way (*vide* page 85.)

These felspar-spots seem to have been arranged parallel in accordance with the stratification; and the spots themselves have fine, curved, approximately parallel fissures in the direction of the breadth, probably due to a strain normal to the axis of mountain-flexures, and these minute fissures have been sometimes filled up by calcium carbonate. The felspars already referred to are mainly non-striped, and occasionally a few lines parallel to the basal cleavage are to be seen; still it is not easy to prove optically the nature of the felspar. The extinction-direction with reference to the trace of cleavage is about 12°–19°, and it may probably belong to albite.[1] In saying so, the presence of orthoclase is by no means excluded.

The felspars contain also grains of the same mineral species whose presence is proved by different optical orientations. There seems, however, to be no crystallonomic relation between those grains within, and the enclosing mineral; the latter serves, as it were, as a cementing material; the former may be considered as a product of dynamo-metamorphism.

The spots of felspar are rich in other interpositions of light-yellow epidote-crystals and crystalloids, with a few actinolite-needles, and tourmaline-prisms, arranged in a most confused manner especially in the central part, as viewed in a slide made parallel to the plane of schistosity of the rock. Spots in the transverse section of the rock have a special arrangement of their interpositions, minute needles and crystalloids being disposed more or less after the fashion of streams. These diminutive bodies are not decomposition-products in the ordinary sense of the term; still they might have derived their materials in part from the felspar-substance during a dynamo-metamorphic process.

---

1) If I rightly remember, Herr Dr. Dalmer made a chemical analysis of a pure felspar from the chlorite-amphibolite of Saxony, which, as has already been pointed out, has a close resemblance to our rock, and he found it to be albite.

the smaller the felspar-grains are, the more interpositions they contain, becoming finally extremely dull so that the felspar-portion is no longer visible, thereby presenting an aspect quite similar to that of a zoicitized felspar of the so-called saussurite-gabbro.

A minuter inspection under high powers resolves the *quasi*-zoicite into a monoclinic epidote. It is found as microliths and round crystalloids. Of the various forms, some have a rhombic outline, or are needle-shaped ; while others are developed in unsymmetrical wings with a twinning-suture at the middle as is seen in *fig. 11. Pl. II.*, The optical investigation of them is made impossible by the interference of the other grains and the enclosing felspars. There are seen frequently actinolite-prisms whose basal sections often met with in slides are of an acute rhombic outline without any trace of the clinopinacoid as is usually supposed to be the case; consequently the prism shows dark longitudinal margins on account of its high refracting power— and the terminations are round and often broken. Rhombic dodecahedra of garnet together with pleochroic, yellowish-green pistacite columns ($^1/_2 - ^1/_4$ centim.) lie also imbedded in the felspars.

The general mass of the rock consists of a grass-green, lamellar-fibrous substance which encircles the porphyritic felspar-grains. This green substance is slightly pleochroic showing a deeper shade of colour when the chief section of nicol is at right angles to the axis of the fibres, and remaining dark between crossed nicols. It transmits only a faint blue light. In a hot solution of hydrochloric acid, the greater part of this green substance is removed from the slides, and the remaining portion becomes almost colourless— it is chlorite.

The minerals found as interpositions in the felspar-nodules occur here again in the *groundmass*, and the green scaly-lamellar mass is interlarded throughout by epidote-needles and grains, and especially actinolite-prisms which are like spicules in the spongy mass.

Calcite with its characteristic rhombohedral cleavage fills up the miarolitic spaces. It is, however, not a constant component. *Iron-pyrites* and *iron-glance* are tolerably plentiful, the latter with margins of blood-red iron-mica. *Quartz* and *rutile*, so abundantly present in the Lower Sambagawan series, are only of a subordinate importance.

We occasionally met with *titanite* as an enclosure in felspar in combination with the clinodome ($n = \infty P \grave{\infty}$). and prism ($l = P \infty$) as in *fig. 12. a Pl. II*; consequently its section in a slide mostly presents a rhombic outline with a deeper shade on the two adjoining sides (*fig. 12. b*) which make with each other an obtuse angle of 134°–137°. This difference in shade arises necessarily from the very form of the crystal itself. and is due to the inclination of the clinodome or prism. Thus the habitus of the microscopic crystal differs from the macroscopic type, as has already been pointed out by v. Foullon.[1]

The *structural modification* of the rock is still more astonishing, ranging from a coarse-lamellar rock to an almost compact schist. A newly fractured surface of the former variety shows felspar-spots of a tolerably large size (more than 2 mm.) with a glittering plane of cleavage. In the compact variety, it differs in no wise from a common chlorite-schist. The weathered surface, however, distinctly tells the presence of countless white spots of felspar-grains.

The compact variety is somewhat more yellowish-green in colour on account of the presence of a considerable quantity of epidote as in the rock of Ōda, 1 1/2 ri (6 km.) westward from the small town of Kodama. There also numerous clusters of black *tourmaline*-crystals, attaining the size of 1 centim. in length, are found along the plane of stratification of the rock. Slides show that the mineral has a grayish-brown colour with a slight tinge of violet; zonal structure is more

---

1) Heinrich Baron v. Foullon; Ueber die petrographische Beschaffenheit der Krystallinischen Schiefer, etc. Jahrbuch d. geol. Reichsanstalt, 1883, p. 241.

imperfectly developed than in the crystals of eruptive rocks. The mineral shows the dichroism ; $\omega =$ black, $\varepsilon =$ violet-brown. The crystals are by no means pure; sagenitic needles, knee-shaped and heart-shaped twins of rutile together with crystals of zircon occur imbedded in them. The tourmaline itself is an unexpected guest in this chlorite-amphibolite, and it is even more striking to find that rutile and zircon which usually belong to the early generation in the order of crystallization, occur exclusively confined in this tourmaline of an apparently secondary origin. The tourmaline characterizes the rocks of the Middle Sambagawan and its true home is, as has been already mentioned, the graphite-sericite schist ; but it is absent in the Upper as well as in the Lower series.

The compact variety of the chlorite-amphibolite is also extensively developed in the Dōzan-gawa near the Besshi mine in Iyo, Island of Sikoku.

(d) The Upper Division.

Epidote-sericite-gneiss.

The Upper Sambagawan is not so extensively distributed as the two preceeding *étages*, and so far as the writer is aware at present, it is mainly confined to the southern side of the Arakawa river.

The rocks belonging to this division overlie directly and conformably the *black* and *green schists* of the middle division, sometimes alternating with the non-spotted graphite-schist in its lower horizon. The characteristic feature of the epidote-sericite-gneisses with their various modifications which constitute the Upper Sambagawan, is a more or less platy structure with an uneven surface on a cleaved specimen. There seems to exist some regularity in their structural modifications ; for the lower horizon consists of a thick-

platy, more or less graphitic variety, while as we advance to the higher zone, the rock passes into a very thin, stiff, papery schist.

Beginning now with the lower étage, the thick-platy, grayish-white variety easily cleaves off into slabs of more than $1/2$ centim. in thickness. The upper and lower surfaces of the slabs are covered by a thin skin of grayish-green, soft, talcose lamellae which, when viewed by reflected light, shine with a glittering lustre. It is the presence of this layer which allows of the rock being so easily taken off.

A newly fractured surface presents layers of a snowy-white, saccharoidal mass, consisting of a fine admixture of quartz and felspar-grains, the presence of the latter being shown by their pearly lustre. The relative quantity of both minerals could not be easily ascertained on account of the approximate similarity of their optical behaviours. The felspar shows sometimes signs of cleavage; and it as well as the quartz displays an undulatory extinction which in no wise ever comes to total darkness. On the whole, the felspar seems to be the predominating constituent, and usually more porphyritic in its habitus than the quartz-grains.

Thin talcose, wrinkled lamellae were treated with a few drops of HFl, and then evaporated to a concentrated solution, out of which, on cooling, many crystals of potassium-silico-fluoride with the forms belonging to the isometric system, aggregated themselves into perfect, delicate shapes. The angle of the optical axis is as wide as in muscovite. This sericite is either colourless, or light-green, and has fibrillated texture, but is irregularly outlined; smooth lamellae are not rare, in which case it is sometimes almost impossible to discriminate the green sericite from the co-existent chlorite.

The sericite-aggregate is never found in a pure state. In the slides, sericite itself is not abundantly present, being sliced off

during the preparation of the sections, and the same holds true of the various enclosures in the sericite. It is better therefore to take off by means of a knife a thin flake of this mineral which, when seen under the microscope, shows various interpositions, especially colourless tremolite in slender prisms, resolving at both ends into tufts of fibres. The extinction-direction of tremolite is insignificant, usually less than 10°; the pleochroism not perceptible. Minute grains and needles of an almost colourless epidote occur associated with the tremolite, within the sericite-aggregate, much as in the manner of spicules in sponges.

A special feature of the minute epidote is its acute rhombic form with sinuating lines along the longer diagonal; and to a cursory view it looks not unlike small crystals of titanite (*fig. 11. b, Pl. II*), It is the common type of twinned epidote whose structure becomes manifest by different optical orientations in the two halves under polarized light. The crystals display vivid chromatic polarization-colours.

Rhombic dodecahedra of garnet occur together with the epidote. but not with the rutile. The enormous accumulation of the above-mentioned microcrystals causes a dull appearance in the otherwise clear sericite-aggregate.

In some specimens, as in that from the Kainita-pass in Chichibu, a few long stalky crystals (1–2 centim) were detected whose longitudinal terminations split into stiff fibres. They are now rusty-brown, although bluish-green where fresh, and their morphological character resembles that of the primary glaucophane from Ōtaki-san in Awa.[1] Taking into account the above-mentioned facts, the writer is rather inclined to consider the tremolite already described, as having been derived either directly or indirectly from the other minerals such as glaucophane.

1) loc. cit.

Somewhat large, pleochroic pistacite-grains are found in isolated patches, but they deserve no special mention. Some specimens are rich in calcite ; its occurrence is, however, not constant.

As we have already pointed out, the more we ascend to the higher horizon, the more papery the rock becomes, and finally this epidote-sericite-gneiss acquires such a structure as to be easily cleaved into thin laminated slabs. The microscopic habitus differs in no way from that of the foregoing variety.

A very characteristic feature of the rocks of the Upper Sambagawan is their papery structure, and also their richness in sericite-scales. Thus the weathered exposures of the rock in any road-cutting present a more or less advanced stage of disintegration, and finally become resolved into tough, slippery, silver-white splinters—a peculiar aspect of road which, if once seen, cannot be easily forgotten.

B. Architectonics of the Sambagawan Series of the Chichibu district.

(e) General Remarks.

The writer has been so far wholly engaged in describing at some length the objective representation of all the rock-varieties which constitute what is called the Sambagawan series ; he now turns to another topic— the geotectonic condition of these schists.

The Sambagawan complex forms the very lowest of the long geologic series, that makes its appearance in the Chichibu district, so far as the present state of our knowledge permits us to say. From the analogy of other occurrences, especially in the Island of Sikoku, where thanks to the munificence of our Geological Survey, the writer was placed in a position, last year, to be able to geologize the countries traveled through, and of which he hopes to give some account on another occasion as a continuation of the present study, it is highly probable (but by no means proved) that this series is superposed upon a complex of biotite-mica-schist and biotite-gneiss with great unconformabilities, which indicate a long lapse of time between the formation of the two series.[1]

1) The above supposition has been, to a certain extent, confirmed during a trip this winter (1887–8) to the provinces of Mikawa and Tōtōmi, where coarse-lameller *biotite-schist*, gray, streaked *gneiss* and then dark-gray, fine-lamellar *mica-schist* with the intercalated *quartz-schist*, in all of a considerable thickness, are developed to their full advantage. These typical Archæan rocks, which the writer saw for the first time in this country, form a long belt, beginning from the interior of Sinano, along the upper course of the Tenriū river, through the province of Mikawa, terminating at the Bay of Atsumi.

The general direction of this long, curved belt, with the convex side toward the south-east, is N.E.—S.W. ; and this belt represents the western wing of the Japan—" Schaarung " of Suess (Naumann; Die Erscheinungen des Erdmagnetismus, etc., p. 17.). Parallel to the curvature

Granitic eruptions must have taken place through the Archæan group before the Sambagawan was formed; for, along the coast to the north of the city of Imaharu, and especially in the upper course of the Nobusigegawa, east of the city of Matsuyama, both in Sikoku, the granitic bosses have severally intruded on the mica-schist and gneiss-strata; and have, in consequence thereof, produced a well-marked contact metamorphism in the latter; the deep-brown pyrope having been formed thereby in the *hornfels* of the biotite-gneiss.

Endomorphic metamorphism on the part of the intruding rock is also clearly indicated by the formation of fine-grained, aplitic granophyre-facies of granitite, this being indeed the typical biotite-hornblende variety, of which alone are formed almost all the granitic

---

of this Archæan belt, an extremely coarse-grained hornblende-granite makes its appearance on the south, convex side, where the granite occupies the greater part of what is called the Tsukude district. This rock shows many characters deviating from the common variety; first of all it is remarkable in containing a large, grayish-white plagioclase and the cross-shaped twins of hornblende together with biotite and quartz; the habitus of the granite is dioritic. In some places, as for instance, at the ascent between Hosokawa and Suyama, this Tsukude-granite becomes *gneissoid* in its aspect, due surely to shearing during the mass-movements.

Further to the south-east, this eruptive mass is overlaid by the Sambagawan series, the latter is therefore, no doubt, separated from the Archæan by a wide, geologic gap, and along the geologic boundary between the granite and the Sambagawan terranes, the Toyokawa finds its course. The Sambagawan series of this region appears younger in its whole aspect, when compared with the Chichibu type.

Still further to the south-east, this series is overlaid by the gabbro-derivatives—*tuff-amphibolite* and *tuff-pyroxenite* with intrusive masses of *gabbros* and *serpentines*, i. e. the Mikabu series of the present writer. Again, the tuff series is covered conformably by the ? pre-Carboniferous complexes of *quartzite*, *crystalline limestone*, rusty-brown *schalstein*, *graywacke*, *adinole*, *silicious slate* and, lastly, a *limestone* presumably the Upper Carboniferous—these together making up the boundary-ridge of Mikawa and Tōtōmi.

It is here out of place to give minute details; only suffice it to say that the writer's view is so far corroborated, that the so-called crystalline schists of Chichibu—the Sambagawan series—are not the oldest geologic body in Japan, and they are also not the normal schists of the accepted Archæan group. Therefore it would be a great mistake to make anyc onjecture as to the general construction of the Japanese Islands from the structural and the choric condition of the so-called Archæan group as understood at present by our geologists, since all the non-fossiliferous rocks of quite unlike origin, but having simply a crystalline, schistose habitus are indiscriminately thrown together under the category of crystalline schists in a narrow sense, and taken as the genuine Archæan. (*Note during the printing*).

mountains of Japan. According to the statements of E. Naumann,[1] the chief granitic eruption occurred at the end of the Palaeozoic, or at the beginning of the Mesozoic era in the Japanese Islands; it seems, however, not unreasonable to suppose that the granites in our country are not the product of a single eruption.

The Sambagawan series has been, perhaps, formed after the first eruption of granite. And the post-granitic schists included in the above-mentioned series are of a considerable thickness probably not less than 300 m.; and these schists in turn have been overlaid discordantly by an equally thick complex of the epidote-pyroxenite, pyroxene-amphibolite, and epidote-amphibolite which the writer is rather inclined to consider as various *modifications of the tuffs* formed in some way in connection with the eruptions of gabbro, diorite and diabase.

The rocks of eruptive origin just referred to can be seen typically developed around the conical-shaped Mikabu peaks, and accordingly the writer designates provisionally the whole complex of gabbro, diabase, the gabbro-diorite, and their derivatives under the name of the *Mikabu series.* The pyroxenites[2] and amphibolites of the above series, usually spoken of simply as chlorite-schists by our geologists, claim our special attention, as they contain an interesting mineral—the secondary glaucophane of which discriptions have been lately given by the author

1) Loc. cit. p. 40.

2) These pyroxenites and amphibolites with their numberless varieties play a not insignificant part in the geologic terrane in Japan, and they have justly full claim to special treatment and independent cartographic representation. To the writer, the above-named rocks seem to be the altered crystal-tuff (in contradistinction to the agglomerate-tuff), originally composed solely of (excepting some few cases) one mineral—the pyroxene which had been thrown up from eruptive vents, just as in the case of ejectamenta of modern volcanoes, and then deposited at the bottom of the once *Universal ocean* of the past, when dry land was comparatively rare. In order to avoid a confusion of the nomenclature, designating rocks of quite unlike origin, or conveying different meanings about the genesis of this class of green, schistose rocks, to which are attached diverse views sometimes diametrically opposite, the writer thinks it advantageous to apply the names of *clasto-pyroxenite* and *clasto-amphibolite* respectively to our rocks, thereby signifying their tufaceous origin, just as Lossen had done long ago for the kindred variety of quartzporphyries. Zeitschrift d. deutschen Geol. Gessel. XXI. 1869.

in another place.[1] To give the detailed geognostic accounts of these is, however, foreign to the scope of the present paper.[2]

(*f*) The Lower Sambagawan.

The Lower Sambagawan series crops out only in few localities along the banks of rivers and rivulets; the greater part of it is covered by the Middle division, and consequently hidden from our view.

It is, however, not very difficult to ascertain its occurrence through having among its members a characteristic purple-red piedmontite-schist which affords us the surest criterion as to the boundary of the Lower and Middle Sambagawan.

1.—A good exposure, not to mention minor occurrences, may be seen along the winding course of the Tsuki-gawa on a small, isolated island of schists amidst the younger geologic formations, to the east of the village of Ogawa, especially in Tōyama, Simo-sato, and Kamakata, all in the district of Hiki, Musasi province. Here the strata lie almost horizontal with slight dips to the north and east. It is, indeed, very striking to behold the contrast in colour between the overlying chlorite-amphibolite rich in felspar-"eyes" and the purple-red piedmontite-schist. The rosy mangan-epidote, so rare in Europe and America, and much sought after by mineralogists, occurs in rich clusters within quartz-nodules, sometimes attaining 2 centim. or more in length. Lower down we see in thick banks the whitish-gray normal sericite-schist which through weathering acquires a somewhat green colour, while a greenish-yellow pistacite becomes

---

1) This journal, Vol. I, part I; "A Note on Glaucophane."

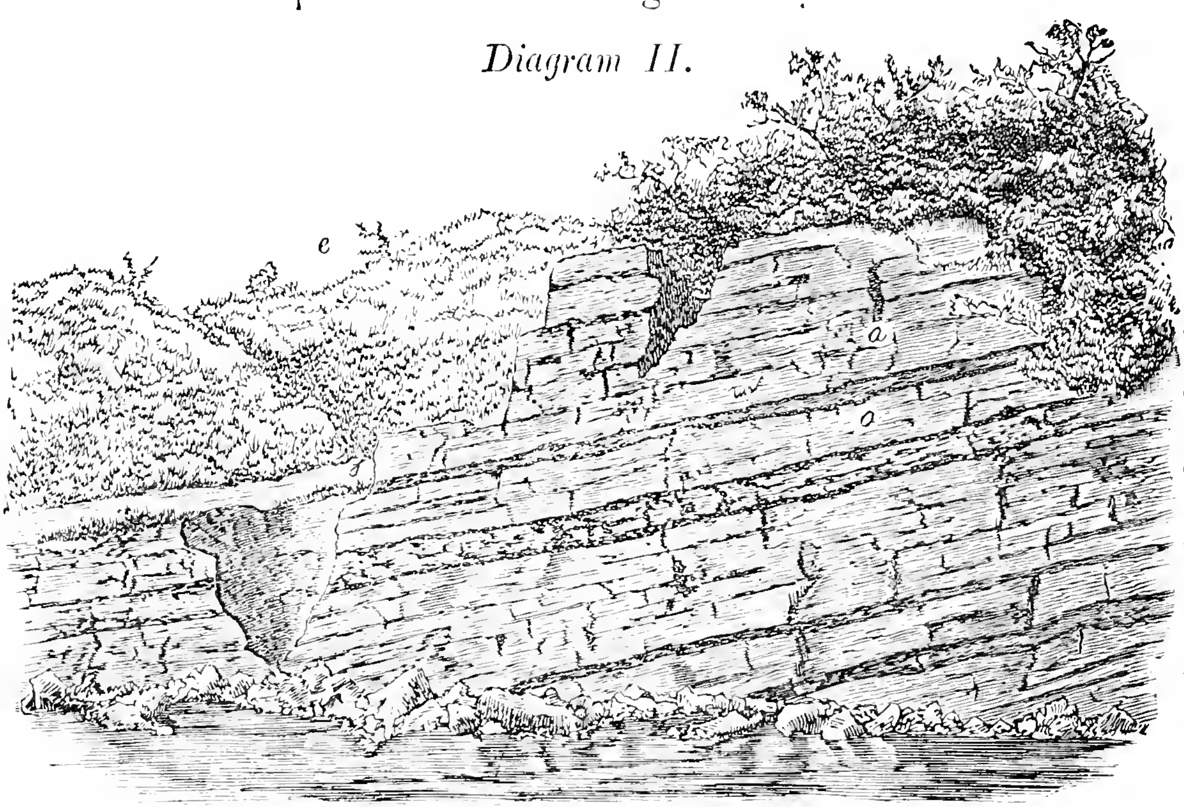
2) Prof. Geo. H. Williams, of the Johns Hopkins University, Baltimore, Md, has lately sent me a chip of a glaucophane-schist from the State of California. Of the Glaucophane of that State, we have as yet only a few records in petrographical literature. Michel-Lévy seems to have been the first to give a short description of it ("4th Annual Report of the State mineralogist of California." p. 182, 1884,—a work which is not accessible to the writer.). F. Becker has mentioned its wide distribution in the Cretaceous metamorphics of that State. (See American Journal of Science, p. 352, 1886).

On examining a slide prepared from a specimen from Berkley, California, the writer was quite surprised at finding a great similarity between the glaucophane-schist of the Kitagawa pass, Tosa (See this journal, Vol. I, p. 92.) and that of the Pacific Coast, U. S.; but our schist is of the ? pre-Carboniferous age instead of being of the Cretaceous. The writer cannot here pass over in the silence the great obligation he is under to Prof. Williams for his gift of a most valuable collection of the typical glaucophanes of Europe, for which he can scarcely find any adequate words to express his deep sense of gratitude.

conspicuously visible in small patches on the plane of schistosity of the rock and characteristically rich in microscopic rutile.

2.—About 4 km. west of Yorii, which is 12 km. south-west from the station of Fukaya on the Nakasendō railway, other good outcrops may be seen in Sueno and Kanaya along the Arakakawa; these exposures are easily accessible to observers as the two villages lie on the main road leading from the Musasi plain to the centre of the Chichibu district. In a stone-quarry in Sueno on the right bank of the river, the normal sericite-schist with abundant lamellæ of a light-greenish sericite is worked in thin slabs, into which the schist may be readily cleaved; the strike is nearly E. 20° S. with high angles of dips to the S. E., and the schist is conformably overlaid by the spotted graphite-schist. Here lies the anticlinal saddle of the Sambagawan series; for, at the Kanao bridge (Tokura bridge), only 1 km. to the north of the quarry already referred to the piedmontite-schist zone is found with its accompanying normal sericite schist, having the strike N. 30° W., but the dip being 40° N. E. Diagram II shows an illustrative exposure near the bridge already mentioned:—

Diagram II.



a.—The normal sericite-schist rich in quartz, abundant in pistacite-patches, the colour light-gray;

b, and *b′*.—the purplish-red piedmontite-schist, only one inch in thickness;

c.—the white, compact quartz-schist making up very thin zones;

d.—the green chlorite-amphibole-schist with its characteristic white "eyes" of felspar; the rock rather compact and imperfectly schistose; yellow, glittering iron-pyrites abundant;

e.—the graphite-sericite-schist with black specks of felspar, showing a thin lamellar structure; spots very promiscuous through weathering; the rock earthy; the soil produced therefrom black and slippery.

The above diagram illustrates well the junction of the Middle and Lower Sambagawan. The cliff about 7 m. in height standing in the foreground represents the latter, while the black graphite-schist of the former lies a little behind covered with grass and consequently not shown distinctly on the drawing. The overlying as well as the underlying Sambagawan rocks dip away from the sight of the observer.

3.—Again about 6 km. up the Arakawa other outcrops come to view at the village of Hon-nogami, and also close by the side of the bridge in Minano. In the latter place two thin zones of piedmontite-schist alternating with the chlorite-amphibolite have been observed in an almost horizontal position with a slight dip to the north-east, just as at Sueno already mentioned.

The occurrence of the red epidote-schist in Hon-nogami. and Minano may be ascribed to the existence of a fault running through the two villages from Kanasaki in the direction of N.20°E along the left side of the Arakawa The vertical displacement being very small, the outcrops found there are very meagre and the underlying normal sericite-schist does not show on the surface to its full advantage. The strike runs approximately N.–S. with a slight dip to the east.

4.—The next locality in which the Lower Sambagawan may be seen, is along the Sambagawa itself, which flows into the Kanna river close to the town of Onisi, in the district of Midono, Kōzuke province. This rivulet has an easterly course with a slight trend to the south, thus harmonizing well with the general strike of the whole series. Along this tectonic valley of about 5 km. in length beginning from Imogaya, just at the foot of the Eastern Mikabu, the entire complex of the Sambagawan series is typically developed ; and if any one is desirous of convincing himself of the stratigraphic order of the complex, this is just the place for him.

The general strike is N. 70° W. with variable dips to S.W.; but, where the transversal faults run across the valley (altogether 3 principal ones as in Profile IV), the geotectonic condition is considerably disturbed, and the outcrops of the piedmontite-schist and normal sericite-schist are then laid bare for inspection. As can be seen in Profile IV, two large faults occur in the lower course at Siosawa and Tsukiyosi respectively, causing a depression in the one, while the northern side has been thrown up to a considerable height, forming the fantastic cliff of the Tsubori-iwa (200 ft.). This cliff is composed of chlorite-amphibolite of the Middle division, and at its foot, along a deep ravine, the Lower Sambagawan crops out to day-light.

5.—The last outcrop which the writer has to mention lies at the north-western corner of the geologic territory of the Sambagawan series, being exposed just to the south of Obata, at the entrance of the long valley of Akihata. Here the piedmontite-schist makes a small saddle with the strike N. 50° W., the dip being very steep toward the north (Todoroki), and then vanishes again from sight on the north under the vertical cliff of the Tertiary sandstone and conglomerate of the Chōgonji-yama (str. E. 20 S.) to be seen no more in this region.

(g) The Middle Sambagawan.

This division plays an important rôle among the Sambagawan series, amounting to about 200 m. or $^2/_3$ of the whole thickness, made up of alternate layers of the chlorite-amphibolite and graphite-sericite-schist. The whole middle complex lies conformably upon the Lower division, and occupies the greater part of the area of our geological terrane. As a whole, the green schist predominates in the lower horizon, while the black schist chiefly constitutes the higher part; and both are directly overlaid by rocks of the Mikabu series, when the Upper Sambagawan is wanting.

The chlorite-amphibolite or spotted green schist is extremely rich in the white "eyes" of a *saussuritic* felspar, which become more clearly visible on a weathered surface; but sometimes, nay, indeed very frequently the fresh surface of a rather compact variety does not show any sign of the presence of the "eyes"; and this is the source of a lamentable confusion of this rock with the overlying Mikabu rocks—both being habitually indiscriminately termed *chlorite-schist*, although they are really separated by a wide stratigraphic gap. This green, spotted schist not infrequently comes together with the piedmontite-schist (i. g. in Todoroki near Obata, and Minano), and the former is rather compact and less fissile than the spotted, black schist. The graphite-schist becomes coarse-lameller (Ōda and also in the neighbourhood of Onisi) through the presence of the black "leather-bag-shaped" crystals (*fig. 6*) of felspar; and through weathering the rock acquires a brown colour, presenting an aspect quite distinct from the spotted graphite-schist.

1.—Both the chlorite-amphibolite and graphite-schist occur in an isolated district east of Ogawa, where they overlie the piedmontite-schist, and dip to the north-east, being directly covered on the north by graywacke-sandstone and in part by the Tertiary of the Ogawa basin.

2.—Along the Arakawa from Yorii to Hon-nogami, illustrative examples of the alternation of the spotted black, and green schists are exposed to view with moderate dips variable at times but generally to N.E.

3.—In the other localities, such as, on the road from Kodama to Ōda, in the environs of Onisi, in the lower course of the Sambagawa, in the Hino valley beginning from Kanai to Kami-Hino, or, lastly, in the Akihata valley, these two rocks always occur in multifarious alternations, the higher portion of which consists chiefly of the graphite-schist. The dip is usually very gradual, the prevailing strike harmonizing well with the general axis of the whole series, and it is not uncommon to find them in a perfectly horizontal position in extensive tables.

(h) The Upper Sambagawan.

The upper division is mainly confined to the southern side of the Arakawa and makes its appearance on the higher parts of the hills. This light-gray or dark coloured rock is nothing but the epidote-sericite-gneiss which in the lower horizon is thick-platy, while on the higher zone it becomes a very thin, papery schist. The latter shows on the cleaved surface a silky lustre, and through long exposure to air the rock falls into a soft, talcose, glittering, slippery mass.

The writer has always been confronted with difficulty in assigning the boundary of this and the preceding division, as the spotted graphite-schist of the Middle Sambagawan insensibly blends with that of the Upper Sambagawan forming with it a continuous series. Between the Lower and Middle divisions a well-characterized, red piedmontite-schist serves well for the purpose; while here such reliable means fail entirely. Consequently the writer in this case stands rather on weak footing in establishing the separate existence of the Upper Sambagawan. Nevertheless, the writer believes he is

in the right, basing this belief on the general dissimilarity of petrographical characters of the two divisions. Other evidences in favour of his view could be produced from observations made in the Island of Sikoku if required, but the future will decide the whole question.

The task becomes still more serious when we try to fix the boundary between this and the Mikabu series, as the appearances of the rocks of both groups are in the main similar; still the microscope affords a means of discrimination.

The typical exposure of this division may been seen on the Kainita pass between Kaiya and Misawa, where the strata lie nearly horizontal with a slight dip to the east, and the serpentine dykes variously traverse the rocks without causing any considerable disturbances in the adjoining gneiss, and the latter is partially covered by the pyroxenite and amphibolite of the Ōgiri-yama. Where the Upper division fails, the spotted graphite schist of the Middle Sambagawan is directly overlaid by the Mikabu series. These conditions are clearly shown in the following profiles.

(i) **Profiles.**

Profile I, C D—is taken from Kaiya to Misawa for a distance of about 4 km. through the well-known pass of Kainita; the rocks exposed are in the main various types of the epidote-sericite-gneiss, i. e. the Upper Sambagawan. Just at the foot of the pass near Kaiya, a red schalstein accompanied by a siliceous slate rests unconformably upon the gneiss. This gneiss has a thin-lamellar structure and may be easily cleaved into stiff papery masses. The weathered rock presents a talcose appearance. Half-way up it is covered by a diabase-amphibolite which in turn is intruded into by dykes of a bluish-green serpentine having a shelly structure. On the western

flank, the gneiss is considerably disturbed for only a short distance by two dykes of serpentine, but this serpentine by no means constitutes the normal stratified member of the Upper division. It is generally accepted by the geologists of our Geological Survey that all serpentines should form a schistose member of older schists; that is to say, they are derived from olivine-schist and the like. The writer is not able to agree wholly with them in regard to this point. The serpentine occurrences hitherto known in Chichibu always seem to point out that the rock has been derived from the diabasic, and gabbro rocks, although all serpentines in other districts may not necessarily be of like origin.

In this profile, the Upper Sambagawan is developed in its full proportions, and generally speaking, strata are horizontal with a slight dip to the east.

Profile II, EF—is taken from the region, 7 km. to the north of, and parallel to, the preceding, for an extent of about 5 km. between Sueno and Hon-nogami across the village of Fuppu. The strata all dip slightly to the east. The greater part of the section is occupied by the Middle Sambagawan, while the higher horizon is covered at two prominences by the Upper Sambagawan. The whole complex seems to have been greatly disturbed at its middle by numerous dykes of serpentine; the highest part of the pass is capped by the amphibolite of the Mikabu series. Near Sueno on the east, and Hon-nogami on the west along the Kami-Hiradani valley, the graphite-sericite-schist with large nodules is developed in its typical form.

Profile III, GH—presents exclusively the rock-groups of the Middle Sambagawan. It extends from Genda, lying to the west of the town of Kodama to Ōda, along the banks of Minaregawa, for a distance of nearly 6 km. For the study of the Middle Division, the river-banks are worthy of personal inspection in order to get an insight

into the mode of arrangement, and the relations of various rock-types which insensibly pass into one another. Near the middle of the profile at Terayama (Kōchi), a serpentine dyke is found considerably disturbing the graphite-schist; at Kōchi an anticlinal fold is clearly traceable. Before the anticlinal the strike is N. 80° W. with the dip of NE; but afterwards it changes to N. 10° E. with the south-easterly dip.

Profile IV, IK—is that of the Samba-gawa, along which all the three divisions are typically represented. In the lower course of the river alternate layers of the spotted black and green schists have a slight northerly dip, while at Siosawa, the strike changes to N. 65° W., dip N.E. The piedmontite-schist layer has strike N. 65° E., dip S.W. All the layers are in the same condition up to Imogaya. After crossing the top to the Hino valley, the strike of the Upper Sambagawan becomes N.–S. with dip to the west.

Profile V, LM—is taken along the Ogawa in Akihata for a distance of 8 km. (2 ris). The rocks exposed are mainly of the Middle division with str. N.50°W., dipping gradually to S.W. Near the Kotōge in Hino, the black schist is traversed by serpentine-dykes, and higher up we encounter the Mikabu rocks which continue far into the interior of the Sanchū valley on the south.

At the north end of the profile (Todoroki), the piedmontite-rock crops out to day-light but only for a short distance, and then again disappears under the Tertiary sandstone of the Chōgonji-yama (str. W. 10° N; dip high to NE.)

Profile VI. AB—represents the section for a distance of 2 km. from Kamagata on the south-east to Ogawa on the north-west. Many instructive exposures may be observed along the winding course of the Tsuki-gawa where the Lower Sambagawan with the normal sericite-schist and piedmontite-schist is conformably overlaid by the Middle Sambagawan complex of the spotted green and spotted

black schists. Between Kamagata and Simo-sato, the strike runs N. 15° E. with a gentle dip to S. E., being disturbed by two faults at the village of Tōyama. After passing the anticlinal of Simo-sato in the north-westerly direction, the strike changes to N. 70° W. with a dip to N. E., and then the schistose complex disappears under the greywacke-sandstone and Alluvium near the basin of Ogawa.

(j) Relations of the Orography and Geology of the Sambagawan Terrane.

The axiom, that mountains of older geologic dates are insignificant in their height when compared with those of younger origin, holds true also in the case of Chichibu. The district under question is circumscribed on all sides by mountain-ranges, built up of rocks of various ages, the very centre of which is an extensive geologic basin of the Pliocene-Tertiary of Ōmiya. The north-eastern rim of this caldron-like depression is formed by low hills of the ancient Sambagawan series.

In travelling along the Nakasendo railway, we may notice a very great contrast in the views of different parts. The back-ground is the tremendous escarpment of the Palæozoic hornstone of Ryōkami, while the borderland is a low chain of the Sambagawan series which is limited on the north by the two conical peaks of Mikabu, and on the south the diabasic Ōgiri-yama projects out from the surrounding hills. These heights are built up of the tuff-amphibolite of the Mikabu series which directly overlies the Sambagawan rocks. On account of its compactness of texture, the Mikabu series withstands the atmospheric agencies better than the other, and thus a marked contrast has resulted in the topography of this district.

The hydrographic basin of the Sanchū in the district of Kanra, discharges its waters by Onishi in the form of the Kannagawa; while

the waters of the depression of Chichibu find their exit in a narrow channel of the Arakawa by Minano and Yorii. Both rivers run across the Sambagawan series, and no sooner do the rivers enter the region, than they assume a meandering course, being directed against the strike of this series. This district is devoid of forest, and affords only poor soils.

(k) **Massive Rocks.**

Eruptive rocks within the Sambagawan terrane make their appearance as dykes, or more frequently in the form of massive exposures; the boundary between this and the adjacent rocks has become quite indistinct owing to subsequent changes in themselves and also in their mode of occurrence; still it is beyond all reasonable doubt that the eruptive rocks came out after the formation of the Sambagawan schists, as is clearly indicated by considerable tectonic disturbances in the neighboring rocks. The above-mentioned eruptives are *gabbros* and *gabbro-diorites*, both have always an intimate connection with *serpentines.*

Of late, we are acquainted from various sources with the wide distribution of gabbros and the like in our country, and ample materials are ready at hand which can only be treated on another occasion. In the present paper the writer gives the description, for the sake of completeness, of those only which occur within the district under question.

The typical rock is the *gabbro-diorite* found along the Arakawa, on the cliff of Kosaka (Kanasaki) near the (often-mentioned) Minano village. It is a whitish-gray, hipidiomorphic-granular rock of originally massive structure, but now become more or less imperfectly schistose ("grobflaserige"). The grayish mass is speckled with dark brownish, satiny patches (sometimes an inch or more in size) of a

lamellar texture. Seen under the microscope these patches consist, for the greater part, of a long, flaky, almost colourless or bluish-green variety of amphibole, a part of which has already changed into a light-green, confused mass of serpentine. The above-mentioned amphibole is ? *tremolite* (c : c = 11°–15°) of the secondary origin, produced from a diallage-like, greenish-brown pyroxene, the remains of which can frequently be met with in the still fresh cores accompanied with the *dark-brown, highly pleochroic margin of the secondary, compact, basaltic hornblende* (c : c — always 1°–2° greater than that of the green one). The last variety of amphibole is of a very special interest.

In petrographical works on gabbros, there has been much discussion about the hornblendes which occur in connection with the augite in the gabbro rocks, whether the minerals are of the primary or secondary nature. The hornblendes are principally of 3 kinds which will now be considered—(1) the *green, fibrous, uralitic*; (2) the *compact green*; and (3) lastly, the *brown, compact hornblende* All writers seem so far in accord as to the secondary nature of the first variety; but the second and the third have in common the *compact* texture, differing from each other only in colour, thus the appearance of both betrays the original freshness. On this account, J. H. Kloos[1] was led to say that, of the hornblende which encircle the core of the intact diallage, "die Annahme einer Umwandlung des Diallags in die compacte Hornblende scheint mir von vornherein ausgeschlossen zu sein......... Die randliche Verwachsung von Diallag und Hornblende würde als einfache Perimorphose zu deuten sein."

Lately, this question became one of paramount importance in petrography, especially in explaining the genesis of amphibole-schists and the "flasergabbros," and has already received much attention

1) Neues Jahrb. f. Mineralogie etc., III Beilage-Band, p. 32.

from various lithologists, such as G. H. Hawes,[2] R. D. Irving,[3] J. Lehmann,[4] Hj. Sjögren,[5] G. H. Williams,[6] and many others; and we are indebted specially to the last author for a detailed account[7] of the paramorphosis of pyroxene to amphibole.

Most American writers adhere, with good reason, to the view of the secondary nature of not only the green compact, but also the brown basaltic hornblende, which Kloos considers a matter of impossibility (op. cit). My study of Japanese gabbros confirms the view entertained by the American lithologists, and the following short account of the hornblendes may prove not to be entirely wanting in interest.

Our rock from Minano contains large porphyritic crystals of *diallage* which is undergoing paramorphosis. The large core is surrounded by a zone of perfectly compact hornblende; the interior boundary of this rim, i. e., the line of contact between it and the diallage-remains, is the most irregular possible. The most delicate tongues and shreds of hornblende extend from the outer rim into the diallage-substance in every direction in a most complicated manner, so that it is impossible either to describe or to portray it adequately. The diallage and hornblende, which are well distinguished by differences in colour and optical orientation, are not really separated by sharp lines of contact. They everywhere shade into one another by insensible gradations. The transition of substance is so gradual that it is not possible to detect the exact points where one begins and the other ends, even under the highest power of the microscope. Nowhere does

2) The Lithology of New Hampshire. Cf. R. D. Irving. (foot-note 3).

3) On the Paramorphic Origin of the Hornblende of the Crystalline Rocks of the North-Western States. Amer. Journ. Science, 1884. II, XXXVIII, p. 166.

4) Ueber die Entstehung der altkrystallinischen Schiefergesteine, etc. p. 190.

5) Geol. Fören. in Stockholm Förh. Cf. Neues Jahrb. f. Mineralogie, etc. 1884, I, p. 81.

6) Bulletins U. S. Geol. Survey, Vol. 4. The Gabbros and Associated Hornblende Rocks, p. 651.

7) On the Paramorphosis of Pyroxene to Hornblende in Rocks. Amer. Journ. Science, XXVIII, 1884, Oct. p. 259, et seq.

the secondary basaltic hornblende exhibit any signs of a fibrous structure. The diallage-core is traversed by fine lines due to its lamellar structure, and these lines sometimes directly proceed into the hornblende-substance. *Pl. III, fig. 1.* is drawn from the polished surface of the rock to half the natural size, and this shows how the marginal black hornblende is related to the diallage. The portion marked (a) between the core and the outer irregular zone is the spot where, when seen under the microscope, the hornblende sends out tongues and shreds of its own substance into the diallage, thus presenting the pseudo-*eutaxitic* structure. The green, compact hornblende of secondary origin is also found in connection with the brown variety, or sometimes developed independently; and both finally resolve themselves into a fibrous confused mass after passing through an intermediate stage of tremolite.

Now, the question is at once suggested:—Why should the brown, basaltic hornblende described before have all the appearance of a fresh mineral, while the green, compact variety as well as the diallage-core itself show more or less a faded aspect? The keenest observers have hitherto passed over this point in silence. First of all, we may ask ourselves whether the fine lamellar structure present in the diallage before us has existed since the formation of the crystals, or whether it is simply a result of pressure, by the action of which the molecules composing the mineral have been enabled to rearrange themselves in a new position for the production of the polysynthetic lamellæ. Leopold von Werveke[1] thinks it not altogether impossible that these lamellæ could be produced by the influence of mechanical power from outside; but it is not yet decided whether these should be considered as a twinning structure, or only an apparent one. Anyhow, the lamellæ may be regarded as having a pathological significance resulting from the action of external force.

1) Neues Jahrb. f. Mineralogie etc., 1883, II, p. 99.

If such be the case the compact original pyroxene, as it seems to the writer, was paramorphosed to the basaltic hornblende long before the formation of the plane of parting in the former; and by that change the ferrous oxide contained in the pyroxene became in part the ferric oxide, to which the brown colour of the new hornblende appears to be in the main attributable. When once the ferric oxide had been formed, it acted as a preventive to further chemical change in the substance of the hornblende, just as in the case of an iron-rod which, when dipped into nitric acid becomes, by virtue of the thin coating of oxide formed on its surface, passive or indifferent to the acid. The process of oxidation stated above, had advanced gradually from periphery of the pyroxene, while the inner part of it remained quite intact; and at this stage mechanical forces had acted from outside producing the lamellar, polysynthetic structure, very characteristic of diallage; and at the same time lamellæ had been formed in the marginal, brown hornblende, though fewer in number. Judd[1] says; "If lamellar twinning has been already developed in a crystal, then chemical action takes place along the gliding planes in preference to the normal solution-planes." The surface of lamellæ in our diallage therefore afforded new fields for chemical processes, and the mineral ultimately resolved itself into tremolite or serpentine respectively, while the brown, compact hornblende survived the general decomposition. The above supposition is moreover strongly supported by frequent occurrences of sharply defined patches of the basaltic hornblende within tremolite and also in a confused aggregate of serpentine.

The *felspar* of the gabbro-diorite occurs in large allomorphic crystals; its appearance is quite dull, and is of a dirty grayish-white

1) "Nature" Vol. 35, No. 18, March, 1887, p. 416. The Relation between Geology and Other Mineralogical Sciences.—Presidential Address to the Geologial Society at the Anniversary Meeting, February, 1887. See also Quart. Journ. Geol. Soc. XLIII.

colour. In spite of its dusky aspect, it displays very vivid, chromatic polarization-colours, uncommon in a partially decomposed felspar. The oblique extinctions upon the faces M and P take place at $-41°$ and $-29°$ to $-33°$ respectively. The positions of hyperbolas seen from M and P are just at the midway between those of hyperbolas of bytownite and anorthite. Along the fissures of the felspar, radiating tufts of stout needles may frequently be seen with vivid chromatic polarization-colours of green and violet tinges. The angle of extinction is oblique, but in no way do they ever come to total darkness; with hydrochloric acid they gelatinize without difficulty. G. H. Williams[1] seems to have found the same mineral in a gabbro of Baltimore, and the writer agrees with him in considering these tufts to be built up of a zeolite (Scolecite). A very similar gabbro-diorite was recently brought from Mine-oka, in the province of Awa. The felspar of that diorite presents exactly the same habitus as that of Chichibu. It may be well here to quote, for the sake of comparison, the result of the analysis of that felspar, which is as follows:—

$Si\,O_2$	43.59	
Al_2O_3	31.62	
Fe_2O_3	0.90	($Fe\,O$ wanting)
$Ca\,O$	17.25	
$Mg\,O$	0.27	
$K_2\,O$	trace	
$Na_2\,O$	1.78	
$H_2\,O$	4.51	
	100.42	

Sp. gr. 2.62

The crystals of *epidote* are plentiful in the felspar-substance, especially at the contact between the greenish hornblende and the

1) Bulletins of the U. S. Geological Survey, No. 28, p. 58.

felspar, formed by the mutual reaction of both minerals, the terminated ends of epidote-individuals projecting into the substance of felspar, as described by G. H. Williams.[1]

As has been already stated, a typical exposure of the gabbro-diorite is found at Kanasaki near the village of Minano, along the cliff of the Arakawa. There the rock passes insensibly into *serpentine* which occurs as dykes in the graphite-sericite-schist ; and the higher part of the cliff is entirely covered by a mass of the serpentine.

The serpentine is of a bluish colour : the weathered portion of it may be easily parted in shelly masses. Under the microscope in polarized light, the whole rock appears to consist of long, divergent flakes, transmitting a gray or blue light ; in short, it looks just like williamsite from Easton in Pennsylvania, although the presence of the nickel oxide in our specimen is not yet proved. Our serpentine may have probably resulted from the decomposition of secondary amphiboles, and up to the present no traces referable to the structure of the olivine-bearing variety has been discerned ; the presence of olivine is, however, not absolutely denied, as it seems to be found in other serpentines occurring in the neighbouring districts.

The serpentine is very frequently met with on the southern side of the Arakawa from Nabeyama-Fuppu to the Kainita pass in a long discontinuous chain of a dyke, running from S. 20° E. to N.W. for a distance of 10 km.; and its north-western prolongations may be traced still further at Kōchi in the Minaregawa valley (see Profile III). Its mode of occurrence can be understood clearly by referring to the profiles I and II.

The serpentine rocks lend a peculiar feature to the topography of this region by their compact texture, and they stand out in prominences from the surrounding schists and gneisses. As to its economic

1) op. cit. p. 31.

uses, a small serpentine-quarry was worked until a few years ago, at Yosino-iri in Misawa, for making small ornaments: but it is now entirely abandoned.

The foregoing is only a few, short remarks on the serpentines found within the confines of the Sambagawan series; but the great majority of its occurrences fall in the succeeding or Mikabu period, where ophitic rocks occur in large detached masses within the amphibolite and pyroxenite, accompanied by a very interesting, secondary glaucophane; the treatment of these augite and hornblende rocks lies, however, beyond the scope of the present paper.

(*l*) Conclusion.

The Sambagawan is the lowest of the long geologic series ever found in the Chichibu district, covering an area of about 270 km., and occupying the north-western rim of the central depression of Chichibu. The main anticlinal axis runs from Sueno near Yorii on the south-east to Obata on the north-west, with a trend of N. 70° W. for a distance of 27 km., touching the northern side of the Sambagawa and also the upper course of the Hino valley; and along this line we really find good exposures of the Lower Division, and consequently of the piedmontite-schist. The widest portion of this Sambagawan belt is traversed on the south by the Arakawa river and on the north by the Kanna river, the strata dipping away gradually from the main axis of elevation.

The rock-complex which comes underneath, is probably mica-schist and biotite-gneiss as developed in the Island of Sikoku; but this hypothesis cannot be established until an actual contact is really ascertained; what lies directly above is a thick series of gabbro and diabasic derivatives and serpentines.

To summarize the leading features of the three divisions, the

lower one consists of whitish-gray, normal sericite-schist, rich in quartz and sericite with the characteristic rutile and calcite. At the junction of the Lower and Middle divisions a very unique piedmontite-schist[1]) makes its appearance, alternating with the spotted black and green schists: the two latter afterwards become the predominant rocks of the Middle division. They are the chlorite-amphibolite with white specks of felspars, and the graphite-sericite-schist with black spots of felspars, these two attaining the great thickness of 200 m. or $^{2}/_{3}$ of the whole thickness. The characteristic accessory components are tourmaline and calcite. The Upper Sambagawan consists of a platy or papery epidote-sericite-gneiss rich in felspar, with the never-failing accessory of epidote.

The peculiarities of the Sambagawan rocks are the complete absence of the true muscovite or biotite, the abundance of sericite, the want of the microperthitic structure, the presence of tourmaline and piedmontite. No trace of organic remains has ever been detected. Anyhow, these rocks do not present many of the characteristics so common to the true crystalline schists.

The question now arises as to the real origin of these rocks. It is not altogether safe for an inexperienced student like the writer to advance any view concerning a probable mode of the formation of such rocks as sericite-gneiss, etc., especially in the face of high authorities in Europe. Still the practical lessons learned by personal observation in the field, have forced him utter to his views on the genesis of the Sambagawan rocks, whose nearest relatives are to

1) This is for the first time we find the piedmontite occurring in such a large quantity as to entitle it to the rank of an essential ingredient of certain rocks. The nearest relative (*thulite*) of this mineral is said to have been found in an eleolite-syenite from the Serra dos Poços de Caldas in Minas Geraes, in Brazil. Its occurrence seemed, however, at first so strange that Rosenbusch was very much astonished on hearing, from Orville A. Derby, of its presence in the syenite, as may be conjectured by his affixing the sign of interrogation to *red epidote* or *thulite*. "Massige Gesteine" 2te Aufl. p. 90.

be found in the "Granulitmittelgebirge" in Saxony, also in the Taunus, and the Ardennes.

To the present writer, it seems highly probable that the whole of the Sambagawan rocks represents a dynamo-metamorphic state of originally different rock-types. The normal sericite-schist may have resulted from a coarse graywacke, while the epidote-sericite-gneiss may have been changed from a fine variety. The chlorite-amphibole-schist might have been derived from a basic tufaceous material of an eruptive rock or rocks, whereas the graphite-sericite-schist may have its origin in a carbonaceous shale.

We have rocks analogous to the Sambagawan series in the higher strata—transition-system—but with quite a dissimilar aspect. Between the Sambagawan and the Upper Carboniferous *Fusulina* limestone in Chichibu, there lies a thick complex of rocks, being made up of alternate layers of arkose graywacke-sandstone, black slate, hornstone, adinole slate, diabase sheets, diabase-tuffs, schalstein, etc.—these are now comprised under the name of the ? pre-*Carboniferous*. To the writer, the Sambagawan rocks seem not materially to differ from these, except that the former have now been changed into apparently unlike rocks. The sericite-gneiss near Döbeln in the 'Granulitgebirge' in Saxony is, according to J. Lehmann,[1] a metamorphosed graywacke. Thus the supposition, that our sericite-gneiss and sericite-schist may have resulted from a felspar-graywacke-sandstone, appears not entirely impossible, and the same may be said of the graphite-sericite-schist which was once a black carbonaceous slate.

One fact should not here be passed unnoticed; i. e. the richness of felspar in our graywacke-sandstones; indeed, that mineral makes up the great bulk of the constituents of the rock. In ordinary sandstones formed by the deposition of washed sands along the shores and river-

1) 'Entstehung der altkryst. Schiefer.' Bonn. 1884.

beds, the felspar-grains are, as a rule, scanty, since that mineral is easily decomposable, when compared with others of the sandstone-components. From this fact, our arkose graywacke seems to differ somewhat as regards its origin from common rocks of the same name. The writer is rather inclined to consider its components to have been derived from the *assorted ejectamenta* of volcanic activity which occurred during the Sambagawan epoch in the same manner as at present, or even on a grander scale. A similar deposit is now being formed along our shores, as may be seen from the samples of sand dredged up from near the Marine Biological Station of the Imperial University at Misaki at the entrance of the Bay of Tōkyō. The younger eruptives near the Station being mostly non-quartzose pyroxene andesite, the dredged samples consist only of grains of pyroxenes, and triclinic felspars, the finer particles of the ejected materials being removed by the sorting action of waves. The normal sericite-schist and epidote-sericite-gneiss already referred to might have been changed from the arkose graywacke-sandstone, the latter having been formed in the manner just described.

Again, the chlorite-amphibole-schist of the Middle Sambagawan, alternating with the graphite-sericite-schist, seems to have been changed from the unsorted, muddy, volcanic ejectamenta deposited in a shallow sea, rather than to have been the product of a crushing *in situ* of zones of the original coarse-grained rock.[1] The result of

---

1) Association of the gneissic and other schistose rocks with igneous masses and also the similarity of the mineral components in all have often been taken, not with full justice, as proofs, that they were originally of the same rock-variety and have even been said to form the same mass of one geologic age; and this is the current view of lithologists in the modern age of the theory of *dislocation-metamorphism* in geology. This may be partially true. But it is, however, not less reasonable to suppose that the schistose rocks might be tuffs of igneous masses occurring in the vicinity, and they are, of course, respectively most nearly related in mineral composition. The absence or presence of stratification, or *foliation-stratification* can only decide the case; but to ascertain which is really developed in individual cases is a painstaking task, and errors are apt to creep in in such an observation, giving rise sometimes to differences of view, not easily reconcilable.

such a crushing process is said to bring about the formation of an apparently bedded structure called by Bonney *pseudostromatism*.[1]

Of late, Frank Rutley,[2] after studying carefully, under the microscope, the rocks of the much discussed region of the Malvern Hills, classified them into a banded and an unbanded series, the former including the different varieties of gneiss, the latter gabbro, syenite, and diorite, etc.; and he suggested that "*the gneissic rocks of the Malvern Hills may be composed of the detritus of eruptive rocks...... he looked upon them as beds of volcanic ejectamenta.*" At present, some gneiss and schists are considered in Europe as well as in America to have resulted from the deformation of a complex mass of plutonic igneous rocks, and there is an universal inclination to exaggerate the effect of what is called pressure-metamorphism, notably so since the appearance of the work of Lehmann.[3] Rutley's view may serve as a check to this tendency. His observations accord well, on the whole, with those which the writer has made in studying the rocks of the Sambagawan series.

Lastly, as to the age nothing positive can be said. The sericite-gneiss near Döbeln, in Saxony, is said to be equivalent to the Cambrian.[4] In the south-eastern flank of that Granulite-Mountain, the epidote-amphibolite of Hainichen, very similar to that of Japan, is developed as a geologic equivalent of the northern series, which is chiefly made up of phyllite, spotted schist, sericite-gneiss and an earthy graphite-schist; that is, the southern series in part represents the Cambrian rocks. In many parts of the Alps, some of the highly crystalline schists are referred to a special facies of the Palæozoic, or even said to be equivalent to the Triassic.[5] Our

1) Anniversary Address of the President, Quart. Journ. Geol. Soc. Vol. XLII. p. 65.
2) On the Rocks of the Malvern Hills. Quart. Journ. Geol. Soc. for August, 1887. p. 508.
3) op. cit. p. 107. 4) Hermann Credner: Das sächische Granulitgebirge. p. 58.
5) v. Hauer: Geologie. p. 249. Stur: Funde von Untercarbonischen Pflanzen der Schatzlarer Schichten der Centralkette in den nordöstlichen Alpen. Jahrb. Geol. Reichsanstalt, XXXIII. p. 189.

Sambagawan series may, no doubt, be correlated with some of these European types ; it is, therefore, by no means safe to give it now a final resting place in the Archæan group, as has been already done by E. Naumann.

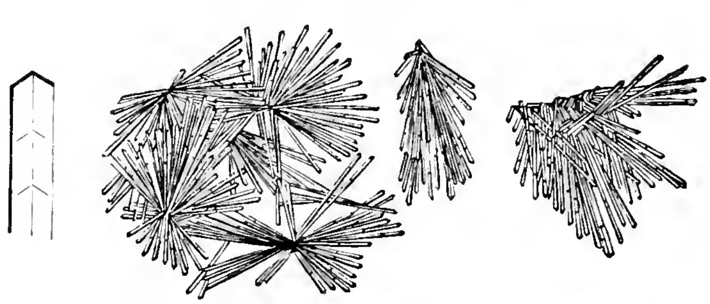
In 1885, Dr. E. Naumann, then Director of the Geological Survey of Japan, laid before the International Geological Congress in Berlin, a reconnoissance-map of the geology of Japan, in which the so-called crystalline-schist system seems to cover a considerable part of the map. But this must now be modified, as the season's works of last year in the Island of Sikoku, and in Chichibu have proved that the greater part of the so-called Archæan rocks in Japan is really represented by the Sambagawan series.

As already mentioned, the age of the Sambagawan series is still uncertain ; therefore it is advisable at present to give it a special position in the cartographical representation of the geology of Japan. With these remarks based upon microscopic and tectonic investigation, the writer concludes now the sketch of the Sambagawan series of Chichibu, by which he hopes to give some impulses to, and some bases for, further observations and studies.

I take here the opportunity of expressing my hearty thanks to Professors Dairoku Kikuchi and C. G. Knott for their kindness in undertaking a laborious and time-wasting work to see the paper through the press, and also in giving many valuable suggestions and additions. I am also largely indebted to Messrs. S. Sekino, M. Ōkawa, and H. Hirauchi for the preparation of the annexed plates. My thanks are lastly due to the Geological Survey of Japan for the free use of the topographical map.

Appendix.

Note 1.—The normal sericite-schist, accompanying the piedmontite-bearing rock, acquires now and then a coarse-lamellar structure; and in some case as that of Yanase near Yorii, a cleaved surface of such a rock shows numerous green patches of a *fuchsite*-like appearance. These patches, after being taken off with a knife-edge and mounted on the object-glass, seem, under the microscope, to consist of sericite-lamellæ, together with a large number of bundles of smaragd-green needles (0.5 mm. in length), beautifully arranged in a radial manner (see the annexed figure).



Studied more in detail, each needle is found to be provided with a median ridge, and extinction of light takes place parallel to, and at right angles with, that ridge. The pleochroism is distinct, and the deepest shade of colour is observed, when the longest extension of the needles is at right angles with the shorter diagonal of the lower nicol. Crystal-individuals are usually broken at a regular distance, and the traces of the planes of fracture meet with each other at the median ridge, making an angle of about 90°; this is probably due to the existence of a plane of cleavage or gliding upon one of the brachydomes. By the addition of a few drops of hydrochloric acid, the green crystals are seen to dissolve readily with vivid effervescence, and from this fact as well as from its form and optical properties, the mineral may be said with certainty to belong to *aragonite* (Eisenblüthe). Cf. foot-note, page 93.

Explanation of Plates II–V.

Plate II.

This plate is devoted to the illustration of some of the most characteristic structures of the rocks and also peculiar habits of the minerals composing the Sambagawan schists.

Fig. 1.—A transverse section of the *glaucophane-schist* from Ōtaki-san near the city of Tokusima, in the province of Awa, Island of Sikoku. This may serve as an admirable example of an internal flexure of schists, in which a quartz stratula of a homogeneous appearance is intercalated between thin layers built up of sericite and glaucophane. The originally horizontal and parallel bands have suffered minute foldings, simultaneous with the mass-motion of the earth's crust; and this movement has brought about the granulation of the homogeneous quartz-bands, as may be seen in the drawing, especially at the turning point of the plicature (page 85).

Fig. 2.—Brings to view, how the periphery of the felspar-nodules in the normal sericite-gneiss has grown together with the quartz grains so as to produce the *pseudo-pegmatic* structure: this figure also illustrates well the simultaneous crystallization of the quartz, and the marginal portion of the felspar (page 88).

Fig. 3.—Gives an instructive example of somewhat porphyritic felspars in a sericite-schist. Felspars are remarkably rich at their centres, in the interpositions of iron-mica, iron-glance and rutile, and are fringed with green, lamellar fibrous scales of sericite. The same interposition and trimming recur in nearly all the felspars of porphyritic habitus throughout the rocks of the Sambagawan series (page 88).

Fig. 4.—A garnet-rhombohedron found in the graphite-sericite-schist. An attempt is made to show a peculiar mode of arrangement of rutile needles within the crystal (page 91).

Fig. 5.—Is an outward appearance of a polished specimen of the typical spotted graphite-sericite-schist, drawn to natural size. A cursory glance at the figure reminds us of the "Garbenschiefer" of Saxony. The interstitial spaces are occupied by black coarsely lamellar-fibrous flakes of sericite. Through weathering, the rock acquires a brown colour, and appears just like a biotite-mica-schist of a common type. Under the microscope, the green mica reminds us of *chloritoid*, but the comparatively low grade of its hardness compels the writer rather to refer it to a green sericite-like mineral (phengite).

Fig. 6.—The flecks of the above schist are the deformed crystals of felspar, in the combination of the faces *T*, *P*, *l*, and *y*, as shown in the figure. The individuals of the felspar lie parallel with the vertical axis to the plane of the schistosity of the rocks. The plane of the basal cleavage shows a pearly lustre; and a few traces of the cleavage apparently parallel to the clinopinacoid are also discernible (page 97).

Fig. 7.—An irregularly outlined crystal of the felspar-dots already referred to in *fig. 5.* The figure brings to view a most peculiar fluidal arrangement of coaly dust; the clear external zone seems to be formed by the *secondary enlargement* of the felspar (page 97).

Fig. 8.—The most common type of the crystals of tourmalines found in the graphite-sericite-schist, showing the acute, deeply coloured pole at one end, while the obtuse termination is of a lighter shade (page 100).

Fig. 9, a.—Represents a case of the parallel growth of two tourmaline-

crystals, and *Fig. 9, b* an isomorphic layer-structure (page 100).

Fig. 10.—A general appearance of a polished specimen of the spotted green schist, showing numerous "eyes" of felspar (page 100). Drawn to natural size.

Fig. 11.—Crystalloids of epidote in the spotted green schist. Forms are various, some are heart-shaped, while others are almost elliptical. Asymmetry is the prevalent character of these twinned crystalloids. They are exceedingly minute in size and appear under weak powers as colourless dots, rich within the felspar "eyes" and in the general mass of the rock (page 104).

Fig. 12.—Crystals of titanite showing the predominating rhombic character of their crystal-faces (page 105).

Plate III & IV.

In Plates III and IV, an attempt is made to illustrate by means of six profiles the general stratigraphic arrangement of the three divisions of the Sambagawan series, together with the intruded masses of serpentines and the overlying clasto-pyroxenite and clasto-amphibolite complexes of the Mikabu series. The sections are made in various directions, and nearly at equidistant points, and they all give similar profiles, so the general result arrived at as to the stratigraphic order may be considered approximately correct. The genuine crystalline schists and gneisses have so far not yet been observed in this region. Details are given in the chapter on "Profiles" pp. 119 et seq., so that it is not necessary here to append further explanatory remarks.

A few words, however, need be said in regards to *fig. 1, Pl. IV*. This is drawn from a polished specimen of an altered gabbro from Minano. The white parts traversed by irregular fissures represent a dull, grayish-white felspar, probably *bytownite;* while the dark-shaded

portion gives an approximate idea of the appearance of the paramorphosed portion of the diallage into amphibole. The lightly shaded part (*a* in the figure) is that portion where, when seen under the microscope, the hornblende sends out tongues and shreds of its own substance into the diallage.

Plate V.

The map (as a basis for which the "Middle Section" of the reconnoissance map of topography of Japan, recently published by the Geological Survey has been used) giving the area occupied by the rocks of the Lower, Middle, and Upper Sambagawan, together with the intrusive masses of gabbros, gabbro-diorites and serpentines. The line OP indicates the main axis of the anticlinal ; while those drawn across the Sambagawan belt signify the position of the profiles given in Plates III and IV.

Abbreviations of Geographical Names.

Ki.	=	Kita	means	north	as	Ki.-Kanra.
Mi.	=	Minami	,,	south	,,	Mi.-Kanra.
Ni.	=	Nishi	,,	west	,,	Ni.-Tama.
Hi.	=	Higashi	,,	east	,,	Hi.-Ōno.
Y.	=	Yama	,,	mount or mountain	,,	Mikuni Y.
S.	=	San	,,	,,	,,	Mitsumine S.
K. or g.	=	Kawa or gawa	,,	river	,,	Tone-g.

Contents.

On the Plants of Sulphur Island

By

Samurō Ōkubo

Assistant Professor of Botany, Imperial University.

The plants of Sulphur Island in the following list were chiefly collected by Mr. Shinnosuke Matsubara, who lately submitted them to me for determination. My friends Messrs Yasushi Kikuchi, Nobutoshi Okada, and Ichirō Shishido, also made some collections of plants in the island, which they kindly showed me.

Of the flora of Sulphur Island, as far as I know, nothing has yet been published; nor have any of us ever before made an examination of the flora. It may not be without interest, then, to publish this list of plants.

The gentlemen, named above, visited Sulphur Island on November 10th, 1887. They remained on it only six hours; and, as each had other special work to do, a very short time only could be spared for collecting plants. Consequently it is very probable that further search would add many more plants to the list here given.

Sulphur Island lies between latitudes 24° 44′ 29″ N. and 24° 47′ 46″ N. and between longitudes 141° 16′ 29″ E. and 141° 21′ 10″ E. Its greatest length is about $5\frac{1}{2}$ miles, stretching from south-west to north-east, and its greatest width not more than two. According to Mr. Y. Kikuchi, it is an active volcanic island, consisting chiefly of tufa and basaltic rock. The present volcano is at the south-eastern end of the island, and is almost destitute of any vegetation. On the north-

western side of the active volcano is a low sandy plain, chiefly of volcanic ashes. The plain is covered pretty abundantly with such plants as *Vitex trifolia, var. unifoliolata*, *Cassytha filiformis*, *Ipomœa pescapræ.* Next to this plain is a plateau, where vegetation is most luxuriant, consisting chiefly of *Scævola Kœnigii*, and *Pandanus odoratissimus. Morinda citrifolia* grows also here. Some of the company found an extinct crater in this plateau. The whole island is quite destitute of fresh water.

The origin of the vegetation of Sulphur island like that of other oceanic islands, must depend upon the action of winds, currents, and birds.

As the island is uninhabited by man, there is less probability of the plants having been introduced through his agency. Its principal occupants are birds, such as *Diomedia nigripes*, *Procellaria*, *Sula*, *Cettia*, and *Hypsipetes*. There is but one species of mammalia, *Pteropus pselaphon*, which also abounds in the Bonin Islands.

The flora of the island is very like that of the Bonin Islands.

Malvaceæ.

1. Abutilon Indicum, Don.
   Coll. Matsubara and Shishido.

Tiliaceæ.

2. Triumfetta sp.?
   Coll. Matsubara.

The specimen looks very like T. rhomboidea, Jacq., but in the absence of flower or fruit it is impossible to decide as to whether it really is a species of Triumfetta or not.

Rhamneæ.

3. Zizyphus vulgaris, Lam.
   Coll. Matsubara.

Sapindaceæ.

4. Dodonæa Thunbergiana, Eck. et Zey.
   Coll. Matsubara.

Leguminosæ.

5. Phaseolus sp.
   Coll. Shishido.

Rubiaceæ.

6. Morinda citrifolia, L.
   Coll. Kikuchi, Matsubara, Okada, and Shishido.

Compositæ.

7. Bidens bipinnatus, L.?
   Coll. Matsubara.

Goodenovieæ.

8. Scævola Kœnigii, Vahl.
   Coll. Kikuchi, Matsubara, and Okada.

Primulaceæ.

9. Lysimachia leneariloba, Hook. et Arn.
   Coll. Kikuchi, Matsubara, Okada, and Shishido.

Convolvulaceæ.

10. Ipomæa pes-capræ, Roth.
    Coll. Matsubara and Shishido.
11. Ipomæa sp.
    Coll. Shishido and Matsubara.
12. Convolvulus Japonicus, Thunb.
    Coll. Matsubara.

Solanaceæ.

13. Solanum biflorum, Lour.
    Coll. Kikuchi and Okada.

Verbenaceæ.

14. Vitex trifolia, L. var. unifoliolata, Schauer.
Coll. Kikuchi, Okada, Shishido, and Matsubara.

Laurineæ.

15. Cassytha filiformis, L.
Coll. Matsubara, Kikuchi, Okada, and Chishido.

Euphorbiaceæ.

16. Euphorbia pilulifera, L.
Coll. Matsubara.

Urticaceæ.

17. Trema orientalis, Planch. var. arguta. Max.
Coll. Matsubara and Shishido.

18. Bœhmeria biloba, Wedd.
Coll. Matsubara.

Orchideæ.

19. Cirrphopetalum, sp.?
Coll. Matsubara.

20. Lusia teres, Blume.
Coll. Matsubara.

Pandaneæ.

21. Pandanus odoratissimus, L. f.
Coll. Kikuchi.

Cyperaceæ.

22. Cyperus sp.
Coll. Matsubara.

23. Cyperus sp.
Coll. Shishido.

Gramineæ.

24. Ischæmum sp.?
    Coll. Matsubara and Shishido.

Filices.

25. Davallia tenuifolia, Sw. var. Chinensis, Sm.
    Coll. Matsubara.
26. Pteris quadriaurita, Retz.
    Coll. Matsubara and Shishido.
27. Asplenium Nidus, L.
    Coll. Matsubara.
28. Nephlolepis cordifolium, Sw.
    Coll. Matsubara and Shishido.
29. Nephlolepis sp.
    Coll. Matsubara, Kikuchi, Okada, and Shishido.

Some New Cases of the Occurrence of Bothriocephalus liguloides Lt.

By

Isao Ijima, *Rigakushi, Ph.D.*

and

Kentaro Murata, *Igakushi.*

With Plate V *bis*.

As far as we are aware, that larval Cestode parasitic in man, first described by Cobbold as *Ligula Mansoni* and renamed by Leuckart as *Bothriocephalus liguloides*, has hitherto been found in only two cases (Leuckart: die menschlichen Parasiten des Menschen. II Aufl. p. 941–951). The one case was that of a Chinaman, in whose corpse Dr. Manson of Amoy found no less than 12 pieces, one free in the pleural cavity and all the rest in the sub-peritoneal connective-tissue in the region of Fossa iliaca behind the kidneys. The other case was that of a Japanese and was observed by Dr. Scheube, then of Kyōto Hospital and was communicated by him to Prof. Leuckart. In this case the worm was discharged from the urethra.

Of late we have come to know of at least six new cases of the occurrence of Bothr. liguloides. Notes on them have already appeared in some medical journals written in Japanese, but we believe, no apology is needed for the reproduction here of the accounts of these cases with such additional remarks as suggest themselves to us. The six cases are as follows:

Case I.—*Bothr. liguloides discharged from the urethra.**—We owe the knowledge of this case to the kind communication of Mr. K. Namba, a physician in the province of Echigo, who also sent us the worm for examination. He writes to the following effect: the patient was a boy, scrofulous and of weak bodily constitution. When three years old he suffered from frequent swelling of the scrotum on the right side, consequent on inguinal hernia. This complaint ceased, but after the lapse of several years, when he was nine years of age, he began one day (July 1886) to experience difficulty in urination, which had to be done often but only drop by drop. Two days passed in this way, when, while making efforts for the passage of urine, a tapeworm-like body came out of the urethra to the length of about 10 cm. On being drawn it contracted and tore off (to what length is not stated). On the following day, the patient came to Mr. Namba, who put him in warm-bath and carefully wound out the worm, that still hung out of the urethral opening and showed signs of movement. The piece thus obtained measured over 20 cm. After this, the urine passed unobstructedly and an inquiry made many days afterwards showed that the boy had since felt in his usual health.

The measurement above given must be considered as giving only a fairly approximate length of the piece extracted. The piece of the worm sent to us in spirit was only 8 cm. long (*Fig. 1*). We do not know whether this piece is the whole that was pulled out by Mr. Namba. At all events, numerous wrinkles and folds observable on the surface, show that it has greatly contracted. Both ends are not natural, so that it was not possible to determine which is the anterior and which is the posterior end. One end was greatly disfigured while the other was deeply notched in the middle-line and plainly indicated that the cut at this place must have been made when fresh. The

* This case was independently published by Murata in *Chūgai-Iji-Shimbō*. No. 181. 1887.

breadth of the piece was fairly uniform, at one place measuring 10 mm. The body was fleshy, reaching about 1.75 mm. at the thickest part; its lateral edges were rounded. The color was whitish, slightly translucent.

On the one surface there was a median depression running through the whole length. It was by no means sharply defined. The lateral halves of the body were, for the most part, more or less reflected toward the side on which this depression lay, so that at some places the cross-section would present a V-shape. Numerous wrinkles, mostly transverse, gave the lateral margin an uneven outline. The specimen appeared as if it were swollen, but sections showed that such was not the case.

Case II.—*Bothr. liguloides from the urethra.*—*Igakushi* S. Saitō of Kyōto reported, at one of the meetings of Kyōto Medical Society, of two cases of tapeworm-like parasite, which we have recognized as Bothr. ligulodes. This report was published by one of us in Nr. 185 of the *Chiugai-Iji-Shimpō*. One of the cases will be described here and the other afterwards as Case V.

According to Saito's report, the patient was a man (son of a farmer in Sayama Village, near Kyōto), 25 years of age and strong in body. Five years previously (1882) he is said to have suffered from violent gonorrhoea, at one period passing blood with urine. In half a year he recovered, but sometime afterwards, the desire for passing urine began to be frequent, sometimes as much as 15 or 16 times in a day. However it was only by great efforts that he could discharge urine. Besides, he felt now and then itching or pressing sensations at the perinaeum. This state continued until Oct. 14th 1887, when, while endeavoring for the passage of urine, a moving worm protruded itself from the urethra. Mr. Ogino, a physician of the village carefully

pulled it out until it tore off leaving a part of the body behind. The piece obtained measured then 2 feet in length, about 6 mm. at the broadest and about 1.5 mm. at the narrowest part. For two days afterwards the patient felt pain in passing urine, which moreover contained blood. The frequent but scanty discharge of urine continued longer. When Saito examined him some time after, the urine was transparent and amber-colored, without precipitate or other abnormality.

We do not know what had since become of the piece of the worm that was left in the urethra nor of the complaint in urination.

Through the kindness of *Igakushi* Saitō we were enabled to examine the piece (*fig.* 2) of the worm extracted by Ogino. It was preserved in spirit, very much twisted, rather tough and greatly shrunken, probably the effect of having been thrown into strong alcohol. It was 245 mm. long. The one end was torn and the other, with which the worm undoubtedly first protruded itself from the urethra, was natural. The latter was no doubt the head-end. At this end the body had very much contracted, forming in contrast to the long-drawn part that followed, a rounded disc about 3 mm. broad and about one-third as thick. On its surfaces fine transverse wrinkles were discernible and at the middle of the anterior margin, there was a small but distinct indentation, showing the position of withdrawn rosterum. Behind this disc-like portion, the body was very thin, almost thread-like (1 mm. in breadth) for some distance and then gradually broadened toward the hind end, where it was thin and measured about 3 mm. in breadth. Transverse wrinkles were especially abundant near the margin and numerous longitudinal ones in the median portion. At some places, three longitudinal grooves (one median and two lateral on the one surface and two (lateral) on the other, were more prominent than others. In view of the shrunken state of the specimen, no importance can be attached to these grooves.

Case III. —*Bothr. liguloides from the urethra.*—This case was observed by Mr. Toyoda, a specialist in helminthiasis in Kyōto. He has published a note of the case in *Iji-Hyōron*, Nr. 2. 1888.

The patient was a citizen of Osaka, 12 years old. On the morning of May 8th 1884, he began to discharge blood with urine, and in the afternoon a white worm appeared from the urethra while urinating. Toyoda was immediately called for. He succeeded in pulling out the worm entire. This measured about 364 mm. in length and about 12 mm. in breadth. Put in a vessel (with water?) it continued to contract and stretch and move about for nearly two hours. It was then put into glycerine for preservation. As the worm was new to Toyoda, he tried various means to identify it but in vain.

As we read his note, there was scarcely any doubt as to the identity of the worm with Bothr. liguloides. In compliance with our request, Toyoda has kindly sent us the specimen for inspection. Examination of it showed the correctness of our assumption.

The specimen (in glycerin) is fleshy and well preserved but had been colored by carmine and cut open at some places, probably in attempt at dissection. It is figured in *Fig. 3.* The length was about 105 mm. and the greatest breadth, 6.5 mm. Both ends are natural so that the diminution in size, as compared with Toyoda's measurement, is entirely due to contraction. The body was narrower near one end than the other. The narrower end was no doubt the head, the configuration of which could not however be definitely ascertained on account of an unfortunate cut at this place. The broader hind-end showed a shallow indentation at the middle-line.

The two surfaces presented the following difference, distinctly for the most part. The one surface had a distinct depression, running longitudinally at the median line. Most of the irregular transverse depressions proceeded from this median line (see the upper part of

Fig. 3). The other surface (the lower part of the same figure) was divided into three longitudinal areas (one median and two lateral) of about equal breadth, by two lines of shallow depression. The distinctness of these areas was brought forth more by the fact that the lateral areas greatly bulged out in comparison with the median, than by the presence of those grooves. Moreover transverse constrictions were to be found mostly on lateral areas.

Case IV.—*Bothr. liguloides from the eye.*—This case has been kindly communicated to us by *Igakushi* R. Satō of Utsunomiya. He also placed the worm at our disposal. It was preserved in spirit, 25 mm. long, narrow (1.5 – 4 mm.) and flat. It was very much twisted and shrunken up, but microscopical investigation left no doubt of its being Bothr. liguloides. The one end was cut and the other broken into shreds.

As to its origin Sato informs us that it was at the end of 1883 when he came across a patient with blepharitic symptom. He was a young man, 17 years of age, living at Kanazawa in Province Kaga, where Satō then resided. (In *Chingai-Iji-Shimpō*, No. 181, in which one of us published a note of this case, it was stated by mistake that the locality was Utsunomiya). The affected place was the region of the inner angle of the left eye. At this place, not only the eye-lids but also a part of the conjunctiva around the Plica semilunaris was in a state of severe inflammation. At a spot just over the Caruncula lachrymalis, Satō observed a whitish spot which seemed to protrude itself. This was taken hold of by a pincette and pulled out, when it proved to be the worm in question.

Case V.—*Bothr. liguloides from the eye.*—We know one more case in which the parasite was located in the orbit. For information and

the specimen, we are indebted to *Igakushi* Saitō (see Case II).

The patient was a girl, 15 years old, living at or near Kyōto. On March 10th 1875, a vesicle-like protuberance formed itself, without any assignable cause, on the white of the left eye, mid-way between the cornea and the outer angle. Three days after, a physician, Mr. Shingū, examined and found it to be about of the size of the tip of little finger, soft and white, somewhat resembling cod-ovary in appearance. In two hours, he observed an elongated macaroni-like body, which on being slowly pulled out was found to be a worm.

The parasite (in spirit) has been tolerably well preserved. Length, 120 mm; breadth 3–6 mm. It is represented in *fig. 4*, in natural size. The hind end (the lower end in the figure) is not natural, having been apparently torn in the fresh state. Near the head-end, the body is broadened and terminates round, but with an invagination at the apex. A number of transversal and longitudinal rugae were found as usual. On the greater part of the one surface, we noticed the division into three regions by two longitudinal grooves as described in the worm of Case III. For some distance there was also a distinct median groove in the middle region. The specimen was cut into sections (*Figs. 6 & 8*), of which we shall have to speak later on.

Case VI.—*Bothr. liguloides from the subcutaneous tissue of the thigh.*—This case relates to a soldier belonging to the Nagoya Garrison. The parasite was extracted by Mr. S. Nagao, Army Medical Officer, who not only supplied us with the following information but also kindly permitted us to prepare sections from his specimen.

The patient was a native of Toyama, in the Province of Etchiu. In the summer of his fifteenth year of age, that part of the right leg just above the knee-joint on the inner side swelled, without any apparent reason for it. In the interior of the swelling a hard mass

was to be felt. There was no pain. It was somehow treated by a local physician and disappeared in about 10 days. A year after, the swelling reappeared at the same place but again subsided in about the same length of time. From this time until his enlistment in the Nagoya Garrison, the same swelling often recurred, invariably during summer. The patient did not definitely remember if it took place *every* year or if there were years in which it did not occur. The enlistment was in May, 1885. In July of the same year, the usual swelling appeared on the inner side of the lower one-third of the right thigh. It was observed that the swelling shifted its position up and down by itself to a small extent. It caused no trouble and soon disappeared. The next year passed without the appearance of the swelling. But in 1887, at the beginning of July, the swelling manifested itself this time at Scarpa's triangle. It did not at all interfere with the patient's general health and dispersed in a few days. In September of the same year the swelling reappeared on the inner side of the middle of the thigh. As it gave him pinching pain, Mr. Nagao was consulted. The latter found a hard mass of the size of a fist, situated in the subdermal tissue at the above-mentioned spot. It could be shifted to a certain extent. The surrounding tissue was inflamed and swollen. Attempts were made to test if it contained anything obtainable by means of inserted syringe, but in vain. Iodine-tincture was administered for about 40 days. This had no desired effect; on the contrary, the swelling enlarged and the pain increased to such a degree as to make the patient incapable of performing his duties. He was then taken into the hospital. Carbolic-acid water was injected into the swollen tissue and cold wrapper applied. In 5 days there was indication of suppuration and so a warm wrapper was substituted for the cold. In 4 days more the swelling suppurated and was cut open. Together with thin pus the worm described below came out of the pus-cavity.

The latter, situated in the subcutaneous tissue, was traversed by trabeculæ of connective tissue in various directions. The wall of the cavity was at some places smooth, as if lined by serosa.

The worm is undoubtedly Bothr. liguloides. The specimen (in spirit) was about 88 mm. long and 3.5 – 6.5 mm. broad. The head-end is represented in *fig.* 5, twice magnified. The other end was torn. The involution at the front apex was distinct. Numerous irregular furrows, both transversal and longitudinal, were present as usual. Of longitudinal furrows, the two that divide one of the surfaces into three longitudinal areas, were unmistakeably recognizable (see *fig.*). When fresh, the body was soft and translucent.

The removal of the worm, which was undoubtedly the cause of the almost annual swelling, took place just nine years after this occurred for the first time.

To the above six cases we might add one more which however could not be tested by us. Once during the dissection of a subject in the Anatomical Institute of the University, Dr. Disse found a worm imbedded in the subcutaneous connective-tissue of the left inguinal region. According to our informant, Mr. Takesaki of the pathological Institute, who was the eye-witness of the discovery, the worm was about one foot and a half long and tapeworm-like but unsegmented. It was new to Disse. Takesaki, whom we have shown specimens of Bothr. liguloides, believes that it was the same worm. It is said that the worm was preserved, but unfortunately it could nowhere be found.

We have then before us *at least* 7 sure cases (the case of Scheube inclusive) of the occurrence of Bothr. liguloides in Japan. The

patient of Scheube was infected presumably in Kiushiu (see Lt.). Of the six cases mentioned by us, two occured at Kyōto and the rest at Osaka, Kanazawa, Toyama and Province Echigo respectively. This justifies us in believing that the parasite has a very wide distribution throughout the whole country. We may further assume that special research in this direction would show that the parasite is by no means so rare as it seems to be.

According to Prof. Leuckart the real seat of the worm is the connective-tissue as was found in Manson's case. This is fully borne out by the case in which the worm was found in the subcutaneous tissue (Case VI) and also by the two cases in which it was located evidently in the connective-tissue space around the eye-bulb (Cases IV & V.)

Leuckart made it highly probable that the worm has the power of changing its position—of moving through tissues to a certain extent. Three of the cases just mentioned put this beyond doubt. In two cases (IV & V) namely, the worm was found to have pierced the conjunctiva and to protrude itself and in one case (VI), the periodical swelling of the thigh, evidently caused by the presence of one and the same worm, was found to vary in position almost everytime it appeared, between the part just above the knee on the inner side and the Scarpa's triangle. It was moreover observed that the swelling changed its position of its own account during its existence.

Under such circumstances, Leuckart's explanation of the exit of the worm from urethra, that it had secondarily bored its way into the urinary apparatus, requires no additional evidence to prove its correctness.

All the known cases, except the two in which the parasite was found in the orbit, tend to show that it is mostly located in the lumbar or the pelvic region. The entrance into the urinary organ

is effected probably after the worm has acquired a considerable size. The wandering into the orbit must have taken place when it was yet of small size, but whether as hooked embryo or as small larva it is difficult to say.

As to how long the parasite may exist in the human body, we call attention to case VI, in which a worm caused the periodical swelling of tissues for nine years. In this connection it is to be noted that Scheube's patient suffered hæmaturia more than five years before the discharge of the worm from the urethra. In cases I and II either the swelling of the scrotum (said to be the consequence of inguinal hernia) or gonorrhœa (?) with hæmaturia occurred five or six years previous to the discharge. It is of course impossible to say what relation, if any, existed between these symptoms and the worm in these three cases.

The preserved and much contracted specimens that came to our view, do not allow anything definite to be stated of the configuration of the larval cestode in question. The head-portion of the worm represented in *fig. 4* has been cut into horizontal sections, one of which is shown in *fig. 6*. The involution of the apex appeared in such sections as a narrow branched indentation, which was not very deep. Neither the papilla-like apical projection seen by Leuckart (the partly evaginated head) nor the two grooves characteristic of Bothriocephalus, were observable in any of the specimens. The broadening near the anterior end is apparently the result of contraction. In general the body seems, to judge from what we have seen, to broaden gradually toward the posterior end (*figs 2–5*).

The numerous wrinkles and folds on the surface are undoubtedly the effects of contraction and of the preserving fluid. Whether the one or two more or less prominent longitudinal grooves or depressions, which we have taken notice of in our description of specimens,

are to be looked upon in the same light (as Leuckart does) or not, we are unable to say.

We have studied the finer structure of the worm by sections cut from five specimens, but we cannot add anything of importance to what is already known from the investigations of Prof. Leuckart.

The cuticula covers the entire surface of the worm. It is homogeneous, thin but sharply defined, and does not stain with carmine.

Like other Vermes there is immediately beneath the cuticula a system of fine circular and longitudinal fibers (*fig. 7*, on the right side). Circular fibers run externally to, but in close apposition with, longitudinal fibers. In both layers, the fibers run isolated and parallel with one another. Longitudinal fibers could distinctly be seen in cross-sections.

With respect to strongly developed bundles of longitudinal muscle-fibers as well as those isolated fibers that run in all directions through the mesenchyma, we have found just the condition as described by Prof. Leuckart, except that we have failed to recognize the especially thick grouping of muscle-bundles along the course of lateral nerve-trunks. Anteriorly the muscle-bundles concentrate themselves toward the circumference of the involuted head (*fig. 6*), just in the same way as we see it in the larva of Bothr. latus.

As was pointed out by Leuckart, the number of excretory vessels to be seen on cross-sections is very great. Those running in the neighborhood of lateral nerve-trunks are of much larger caliber than those in the peripheral or the median part (see *fig. 8*). In sections of the head-part (*fig. 6*), we have met with but a very few number of small vessels.

Larval Cestode (Sparganum) resembling the human Bothr. liguloides was found by us in Inuus speciosus as well as in Mustelus itatsi, but we reserve its description for a future opportunity.

P. S.—Since the above notes were in type, we were favored by Mr. K. Takahashi of the Medical College with following informations, which we will here add as :—

Case VII.—*Bothr. liguloides from the eye.*—The patient was a girl, eleven years old, native of Ko-aiki Village in the province of Kōzuke. In Spring of last year she suffered from conjunctivitis. From January of this year, the upper eye-lid of the left eye began to swell and redden with intervals of comparative repose. Even during such an interval, the eye-lid seemed to be somewhat thicker than usual. On March 16th, during a school-exercise she felt pain in the eye, so that she was compelled to return home. However the pain soon subsided and the next morning she was able to attend the school. On 19th a swelling was noticed on the eye, which was investigated by Mr. Hagiwara, a physician in the town of Mayebashi. According to him, the swelling was of the size of a small bean, was situated on the eye-bulb beneath the conjunctiva, showed no signs of inflammation and could be shifted for a certain extent. On cutting the conjunctiva open, a worm protruded itself. It was then drawn out by means of a pincette, during which process the patient felt a slight pain. It seems that the worm was originally situated in the region of the Fornix of the upper eye-lid but had changed its position so as to come beneath the conjunctiva bulbi.

The extracted worm, shown us by Takahashi, was about 25 mm. long (in spirit). Without doubt it was only a part of the entire worm. The greater part of the specimen was split lengthwise into two parts. The head-end was present. It had just the configuration as represented in our *fig. 4* or *5*, but measuring only 2 mm. in breadth.

Explanation of Figures (Pl. V *bis.*)

Fig. 1.—A piece of Bothriocephalus liguloides in alcohol (Case I). Nat. size. *a* represents the shape of one of the cross-sections of the piece.

Fig. 2.—Bothr. liguloides in alcohol (Case II). Nat. size. Very much shrunken. Above is the head-end.

Fig. 3.—Bothr. liguloides in glycerine (Case III). Nat. size. The head-end is above. *a*, supposed shape of its cross-section. Here and there longitudinal cuts on the body.

Fig. 4.—Bothr. liguloides in alcohol (Case V). Nat. size. *a*, outline of a cross-section near the hind end.

Fig. 5.—Anterior part of a Bothr. liguloides (Case VI). Twice magn. Preserved in alcohol. *a*, its supposed cross-section.

Fig. 6.—A horizontal section of the head-part of the worm represented in *fig. 4*. Colored by borax-carmine. About 10 times magn. Outlined by means of Camera lucida.

Fig. 7.—Peripheral part of the body of Bothr. lig. (*fig. 4*) stripped off, seen from inside. Drawn without the use of Cam. luc. *mes* = mesenchyma, *mus* = bundles of muscle-fibrils.

Fig. 8.—Part of a cross-section of the same worm. About 37 times magn.; stained with borax-carmine. *m*, *m* = the median plane. *n*, *n* = the two nerve-trunks in section. *x*, *y* = spaces produced by the tearing of mesenchyma. Cross-sections of numerous excretory vessels (ex) and of strongly colored muscle-bundles (mus) are to be seen.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

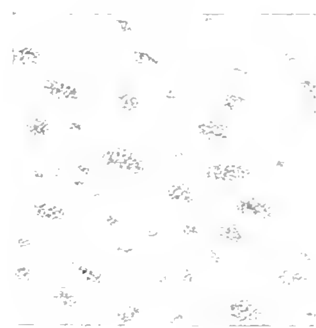


Fig. 6.

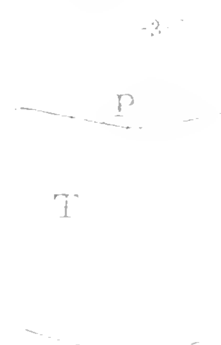


Fig. 7.

Fig. 8.

Fig. 9.

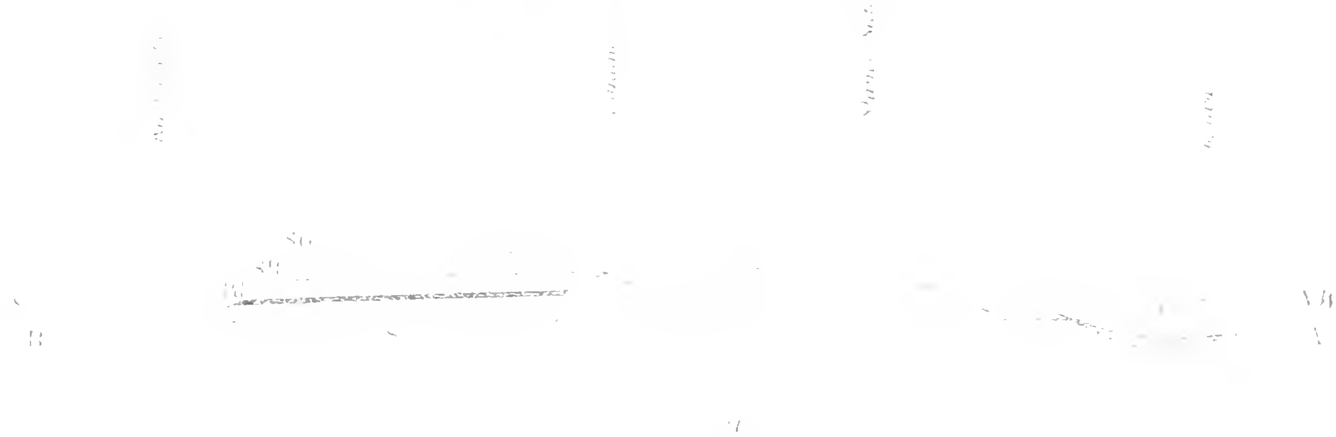
Fig. 4.

Fig 12.

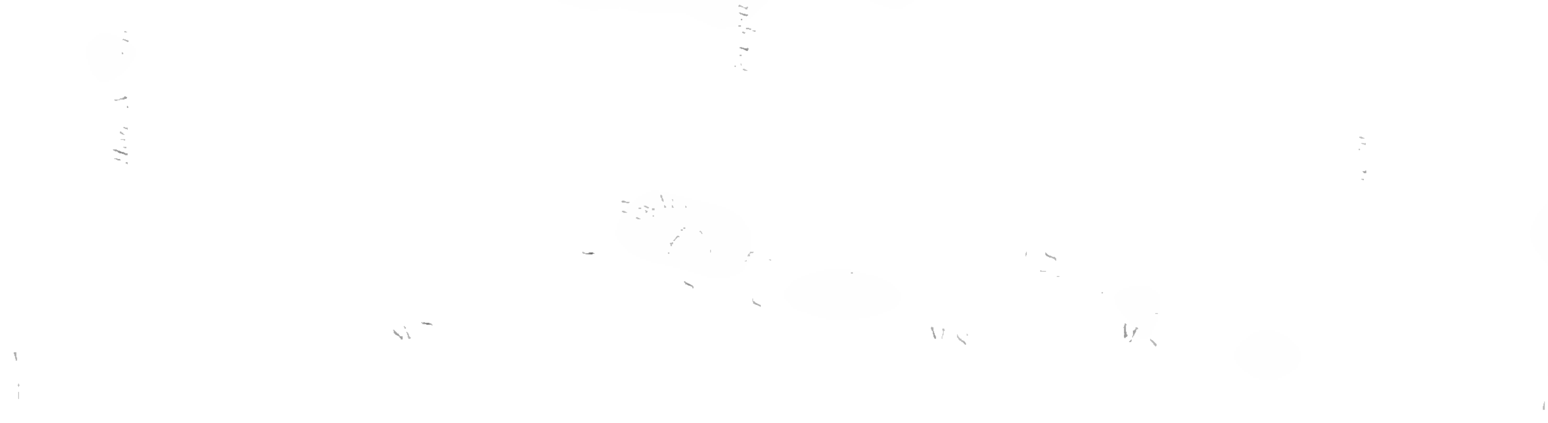
Fig. 10.

Fig. 11.

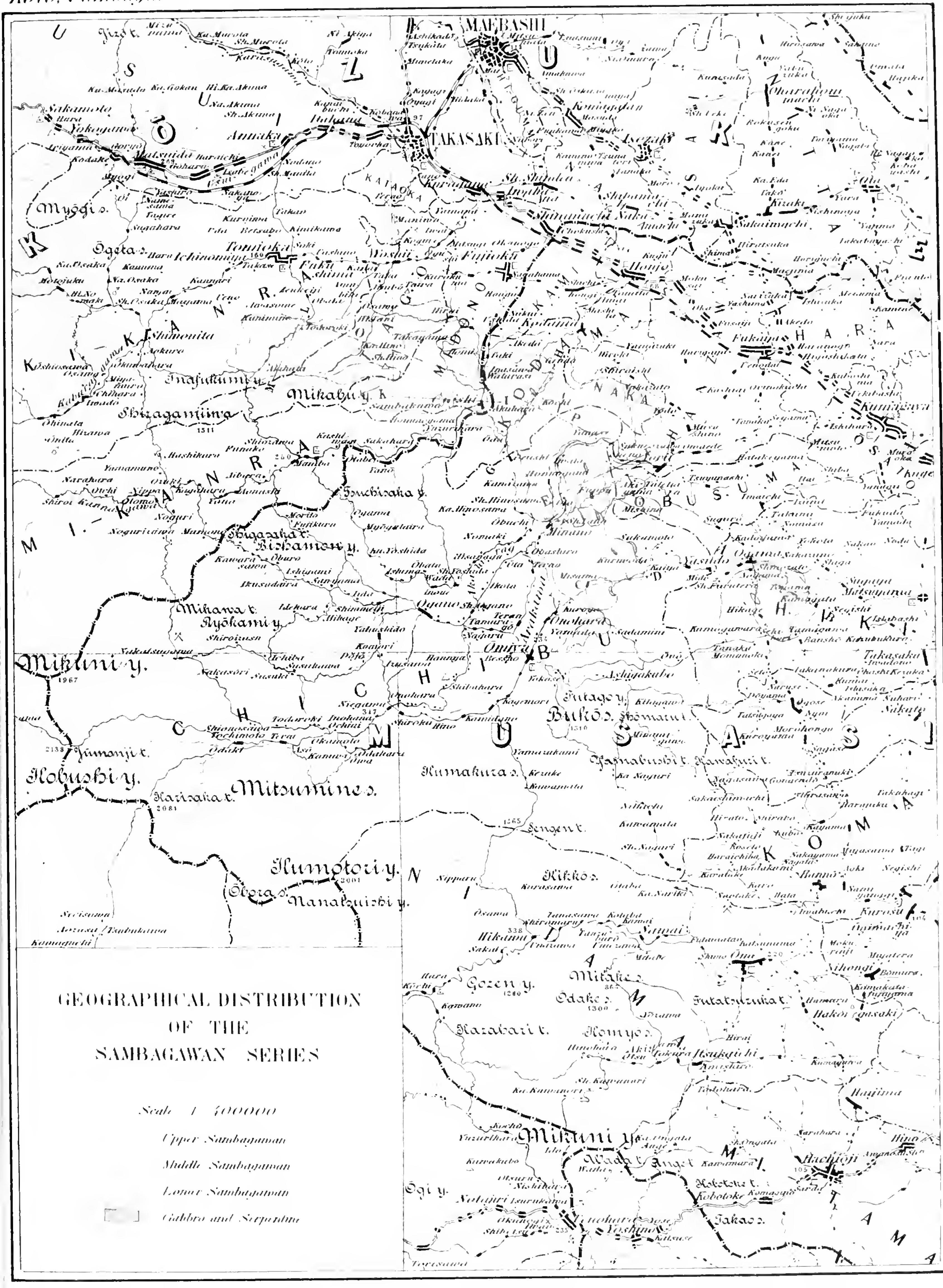
Fig 12

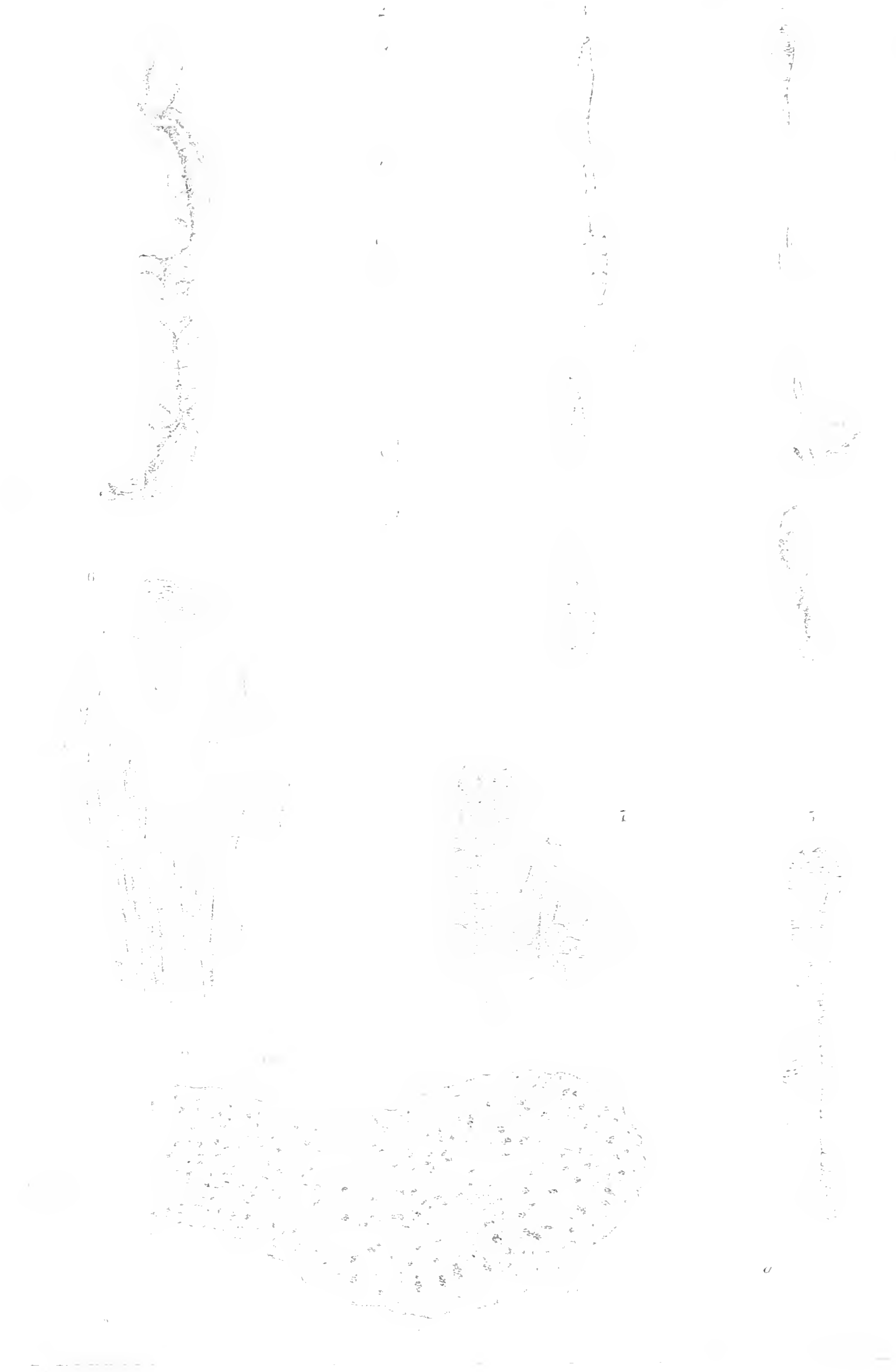


Profile VI. ... SG ... green ... schist ... SB spotted ... black ... schist ... Al. Alluvium.



Profile III. SB ... black schist ... SG spotted ... Al. diluvium.


GEOGRAPHICAL DISTRIBUTION
OF THE
SAMBAGAWAN SERIES
Scale 1 : 400000
Upper Sambagawan
Middle Sambagawan
Lower Sambagawan
Gabbro and Serpentine
MAEBASHI
TAKASAKI
Annaka
Myōgi s.
Tomioka
Fujioka
Honjo
Fukaya
Kumagaya
Shimonita
Mikabu y.
Shiraganiiira
Dōuchizaka t.
Bishamon y.
Mihawa t.
Ryōkami y.
Mikuni y.
Jumonji t.
Hobushi y.
Harizaka t.
Mitsumine s.
Kumotori y.
Ōbora s.
Nanatsuishi y.
Futago y.
Bukō s.
Yasnabushi t.
Hanafuri t.
Kumakura s.
Sengen t.
Kikkō s.
Hikami
Mitake s.
Gozen y.
Odake s.
Kazabari t.
Komyo s.
Futatsuzuka t.
Itsukaichi
Mikuni y.
Wada t.
Angel t.
Kobotoke t.
Hachiōji
Takao s.
Ōgi y.
Hanno
Ōmiya
Ogano
Ome
Kobotoke



Errata.

(Vol. II.)

Page	60,	6th	line, for 0·01 and 0·04 read 0·1 and 0·4.
,,		14th	,, ,, 90 read 0·9.
,,		24th	,, ,, 0·05 ,, 0·5.
,,		26th	,, ,, 0·06 ,, 0·6.
,,	71,	27th	,, ,, 0·76 ,, 0·96.
,,	144,	4th	,, ,, pescapræ read Pes-capræ.
,,	145,	No. 10	,, pes-capræ ,, Pes-capræ.
,,	146,	No. 15	,, Chishido ,, Shishido.
,,	147,	No. 28	and No. 29 for Nephlolepis read Nephrolepis.

a full account of his instruments and modes of operation. This account is reproduced, very much in his own words, in Section III of the present paper. That Section, therefore, is peculiarly Mr. Tanakadate's contribution. For the rest—the combining of the observations by the method of Least Squares, the preparing of the Charts, etc.—

A Magnetic Survey of all Japan

Errata.

Page 170, 11th line from bottom, for "20th" read "22nd"

,, 186, 8th ,, ,, bottom, ,, "in" ,, "is."

Specimen Page opposite page 190, line marked (12), for "Log. $\sqrt{M\ H}$" read "Log. $\sqrt{M\ H}$."

Page 192, 10th line from top, for "Yahrbuch" read "Jahrbuch."

,, 246, 13th ,, ,, ,, for "Spica" read "Arcturus."

that gentleman left his own observations all but reduced, along with a full account of his instruments and modes of operation. This account is reproduced, very much in his own words, in Section III of the present paper. That Section, therefore, is peculiarly Mr. Tanakadate's contribution. For the rest—the combining of the observations by the method of Least Squares, the preparing of the Charts, etc.—

A Magnetic Survey of all Japan

carried out,

by Order of the President of the Imperial University,

by

Cargill G. Knott, D. Sc. (Edin.), **F. R. S. E.,** Professor, and
Aikitsu Tanakadate, Assistant Professor of Physics,
Imperial University, Tōkyō, Japan.

With Plates VI – XV.

Prefatory Note:—In the Summer of 1887, the Imperial University of Japan at last saw its way to carry into effect a proposal which had been made by me some years previously. Along with Mr. Tanakadate and Messrs. Nagaoka and Imagawa, I was accordingly instructed by the President of the University to make a Magnetic Survey of all Japan within, if possible, an interval of two and a half or three months.

I regret exceedingly that in the preparation of the account which follows I have not had the continued assistance of Mr. Tanakadate.

Before setting out for Scotland in January of this year, however, that gentleman left his own observations all but reduced, along with a full account of his instruments and modes of operation. This account is reproduced, very much in his own words, in Section III of the present paper. That Section, therefore, is peculiarly Mr. Tanakadate's contribution. For the rest—the combining of the observations by the method of Least Squares, the preparing of the Charts, etc.—

I am alone responsible. As a check upon my own calculations, Messrs Ashino, Hirayama, Kano, and Kimura, graduating students in Astronomy, Mathematics, and Physics, kindly went through the labour involved in combining the observations by the method of least squares. Messrs Nagaoka and Imagawa have, of course, always been at hand and have rendered most efficient aid throughout the whole process of reduction. To Mr. Nagaoka especially are my best personal thanks due for the many ways in which he has lightened my labours.—C. G. Knott.

Arrangement of Matter.

The Paper is divided into Five principal Sections, as follows:

Section I.—Historic Retrospect and General Description of the Aim and Methods of the Present Survey.

Section II.—Particular Account of the Equipment and Modes of Operation of the Northern Party.

Section III.—Particular Account of the Equipment and Modes of Operation of the Southern Party.

Section IV.—Final Reduction of the Observations and General Conclusions.

Section V.—Comparison of Results with those of previous Observers.

Section I.

The earliest determination of any of the magnetic elements in Japan was probably made by Inō* about the beginning of the present century. In all the charts which were at that time made by

---

* This Inō was certainly a remarkable man. As an Appendix to the present Memoir I give a short biography.

him, he represents the magnetic north as being identical with the true geographical north. Dr. Naumann, in a paper on secular changes of magnetic declination published in the Transactions of the Seismological Society of Japan (Vol V, 1883), has given an interesting discussion of some of Inō's bearings, which seem to differ from the true bearings by amounts that can be explained only on the supposition of considerable local magnetic deviations. That the average value of the Declination over Japan in Inō's days was zero is no doubt true; but, by assuming his magnet to point always true north and south, Inō appears to have fallen into appreciable error in laying down the positions of certain of the mountains in north Japan. Inō began his geographical survey of Japan in 1800 A. D. and finished it in 1818.

In 1860 Mr. Arai, the present superintendent of the Meteorological Office, measured the magnetic declination at a locality in Yedo (now Tōkyō) and found it to be 3° 11′ West. In 1882 a second determination by the same gentleman gave the value 4° 24′ W. From these two determinations we find by a simple calculation that the mean secular variation of magnetic declination during the interval was 3′. 3 per annum. If we assume this rate to hold throughout the century we find, on reckoning either from 1882 or 1860, that the year of zero declination was 1802. This year falls within the period during which Inō made his survey; and the concordance between the various observations is, to say the least, striking.

In 1880, Mr. Otto Schütt of the Geological Survey Department made a series of observations of all the elements at certain stations in the south easterly part of Japan.* His furthest north-west point

* "*Ein Beitrag zur Kenntniss der Magnetischen Erd Kraft*" published in the *Mittheilungen der Deutschen Gesellschaft für Natur und Völkerkunde Ostasiens* (22tes Hest. 1880).

was Oiwake at the base of Asama Yama; while fully half his stations lay on a route encompassing Fuji Yama. At these latter, however, only the Declination seems to have been measured. The observations bring out very markedly the disturbing effect of volcanoes upon the magnetic characteristics of any district. The horizontal forces measured by Mr. Schütt appear to be on an average about 2 per cent. greater than the values indicated by the observations of the present survey,—a difference which, surely, must be due to instrumental error.

From March 16th to August 20th, 1883, Mr. Wada of the Meteorological Observatory, Tōkyō, took complete hourly observations of the Declination. It may be well to give here the mean hourly values during the whole interval of 5 $^1/_6$ months.

Diurnal Variation.					
Hour.	Declination.	Hour.	Declination.	Hour.	Declination.
0 a.m.	4° 16′.71	8 a.m.	4° 13′.46	4 p.m.	4° 18′.72
1 „	16.54	9 „	13.89	5 „	17.57
2 „	16.24	10 „	15.21	6 „	16.68
3 „	15.97	11 „	17.25	7 „	16.79
4 „	15.82	12 „	18.99	8 „	16.93
5 „	15.69	1 p.m.	20.16	9 „	16.85
6 „	14.84	2 „	20.39	10 „	16.76
7 „	13.76	3 „	19.96	11 „	16.70

The mean value is 4° 16′.75 and the mean diurnal range is 6′.93, These hourly observations of Mr. Wada are the most complete and valuable of the kind that have been made in Japan. It may be remarked that within the last few months the same gentleman has got into fair working order a complete set of Mascart's self-registering magnetographs. Unfortunately they were not ready for use during the months of our survey, so that we have no means of applying even

approximate corrections so as to reduce our results to one epoch.

We had hoped before starting to have had the cooperation of the observers at the Naval Observatory, where for some years fairly systematic measurements of the magnetic elements have been made. Since 1883, the declination has been taken regularly every day at 7 a.m.—not a very good hour if probability of steady values or of good means is aimed at. The Horizontal Force and Dip have been taken each twice a month, the former always in the afternoon between 2 p.m. and 5 p.m., and the latter usually immediately thereafter, excepting on a few occasions when it was observed in the morning. The Horizontal Force was measured with the Deflecting Magnet at one distance only. The mean values for the years 1885–6–7 are given here, being calculated from the tables published in the Annual Reports of the Naval Observatory. The Horizontal Force was measured in foot-grain-second units; but the values are here reduced for convenience to the centimetre-gramme-second units.

Mean Annual Values of the Magnetic Elements at the Naval Observatory, Tōkyō.

Year.	Dip.	Horizontal Force.	Declination W.
1885	49° 36′.2	.29098	4° 4′.2
1886	49° 36′.7	.29151	4° 6′.2
1887	49° 39′.4	.29180	4° 7′.1

All show evidence of an annual rate of increase. It will be noticed more especially that the annual rate of change of Declination is much smaller (about 1′.5) than the value deduced from Inō's and Arai's observations.

Another fact deducible from the Naval Observatory Reports is that, so far as can be judged from the limited number of observations made, the magnetic conditions during the summer months of 1887

seem to have kept fairly constant. The several measurements of the Horizontal Force do not differ amongst themselves by so much as 1 in 1000; the Dips agree to within 7′; and the mean monthly declinations to within 3′.

To regard all the observations of the present survey as made at the same epoch is, therefore, in the circumstances, the safest course. Indeed it would be impossible with the data at our command to apply monthly corrections having any claim to probability.

During the years 1882–3 a magnetic survey of Japan was carried out by Messrs. Sekino and Kodari, members of the staff of the Geological Survey Department. Their chart was presented at the International Geological Congress at Berlin in 1885, and is reproduced, along with the observations, in a pamphlet of Dr. Naumann's, entitled "Die Erscheinungen der Erdmagnetismus in ihrer Abhängigkeit vom Bau der Erdrinde" (Stuttgart, 1887). In this pamphlet, which contains a full account of previous magnetic work in Japan, Dr. Naumann calls special attention to the apparent trend of the lines of equal magnetic declination, and suggests a possible relation between their form and the geological structure of the country.

Some three or four year ago, however, an inspection of the results of this first survey and a consideration of the routes chosen convinced me that it would be unsafe to deduce from them any definite conclusions as to the general magnetic characteristics of Japan. For example, with the exception of a few in the neighbourhood of Niigata and Sado Island and a few between longitudes 136° and 138° E., there were no stations along the western coast of Japan. North of latitude 38° N. there was but a single line of stations, confined to the Ōshiū-kai-dō (the Ōshiū High-way). Then the observations were made in two sets, the one in 1882 between August 18th and December 7th, and the other in 1883 between September 10th and

December 24th. The observations were made usually about 9 a.m. or 3 p.m., but not with absolute regularity. Thus to make the results comparable amongst themselves, corrections due to the diurnal, annual, and secular variations ought to be applied. Of these the diurnal variation is the one which will tell most in the circumstances: but I am not aware if any attempt has been made to reduce all the observations (especially of the declination) to one hour.*

Considering then the manner in which Sekino's survey was carried out, we are I think justified in regarding the observations as an insufficient basis for any safe generalization.

It thus appeared that the thing to be desired was a new survey—what might be called a preliminary survey of all Japan, special attention to be paid to the distribution of stations, and the whole to be completed within as short a time as possible. The scheme finally decided upon was briefly as follows:—To form two parties, the one or Northern Party to survey the northern and eastern parts of Japan, including some stations in the Island of Yezo, the other or Southern Party to survey the south-western parts including one or more stations in Korea. This required two sets of instruments. Already the University possessed the ordinary Kew Portable Magnetometer; and with this the Northern Party conducted their survey. The Dipping Needle used by the Northern Party was loaned by the Royal Society Committee; and I take this opportunity of cordially thanking them for their kindness to me personally as well as to the Imperial University of Japan. In the work of the northern party I was ably assisted by Mr. H. Nagaoka, one of the graduates of the year.

* There seems to be a slight inaccuracy in Dr. Naumann's remarks (See page 8 of the pamphlet) on the reduction of these observations to one hour. I understand from Mr. Sekino that no diurnal correction was applied to the observations made at the northern stations (Aomori and Iwanuma being excepted). At the southern stations corrections were applied in accordance with the three-hourly observations that were being made in Tokyo at the time, the observations being reduced to the mean of the 9 a.m. and 3 p.m. observations.

The southern Party was under the charge of Mr. Tanakadate, who was assisted by Mr. K. Imagawa, an elective student of Physics. This party used a Dipping Needle of the usual Kew pattern (Dover, Charlton Kent, Circle No. 24); but measured the declination and horizontal force by means of an instrument constructed by Mr. Tanakadate himself. This will be fully described further on.

Each Party was completed by the addition of a University servant.

In selecting the stations we aimed at two things; 1st, a fairly good distribution, 2nd, a shunning of local disturbances due to volcanic rocks. As the observations will show, the second condition was extremely difficult to fulfil—indeed practically impossible except by leaving out large tracts of country. This was especially true in the northern part of Japan, where magnetic rocks abound. We always tried however to give volcanoes a wide berth, as these had been shown by previous observers to be great sources of disturbance, especially as regards the declination.

As the tabulated values of the various elements sufficiently indicate by the dates attached the chronological order of the stations, it is enough to give a very general indication of the routes.

The northern Party left Tōkyō on June 20th, 1887, and proceeded by rail and jinrikisha northwards to Shiogama. From here they took boat to Ishinomaki, thence up the river (the Kita-kami-gawa) to Ichinoseki, and on by road to Morioka. Here they turned due east and made over the hills to Miyako, a coast town of considerable importance. A Japanese junk then took them northwards to Kuji, whence by packhorse and jinrikisha they proceeded by Hachinohe to Aomori. From Aomori they crossed to Hakodate, from which as a centre they made expeditions by sea to Sapporo and Nemuro. The return journey was made on the western side of the great central ridge, coast towns such as Akita, Sakata, Niigata, etc.,

being included as well as the more easily accessible inland stations. The west coast was left at Takata in latitude 37°, and the route continued over the high central mountains to Kōfu down the river (Fuji-kawa) to Hara, and then round the south-eastern coast to Chōshi and so back to Tōkyō.

The southern Party left Tōkyō on June 22nd and proceeded, partly by steamer partly by land, westwards along the South Coast to Osaka. From here an excursion was made to Shikoku; and, after their return to Osaka, they pursued their westerly course by steamer along the northern shores of the Inland Sea. By August 1st they reached Nagasaki, whence they made a trip to Korea and back again, which occupied nearly three weeks. The subsequent route lay round Kyūshū and finally, chiefly by steamer and boat, along the North Coast of the western portion of the main Island. Their last station was Nanao lying on the east side of the peninsula of Noto which projects northward into the Sea of Japan.

The Northern Party completed the prescribed survey by September 3rd. Two other stations, however, at which observations were made on September 25th and 26th, were subsequently included. The distances travelled by road, railway, and water were roughly as follows:

By road ...	... ...	1350 miles.
,, railway	... ...	400 ,,
,, water	... ...	1700 ,,

The Southern Party completed their work on October 14th. The distances travelled by road, railway, and water were as follows:

By road ...	... ...	1100 miles.
,, railway	... ...	300 ,,
,, water	... ...	2450 ,,

Almost all the travelling by water accomplished by the Northern Party occurred during six days of their stay in Yezo—otherwise, with a few unimportant exceptions, their route lay wholly overland. On the contrary, the Southern Party accomplished over all nearly a month's travelling by water.

Section II.

The Northern Party, as already mentioned, were provided with the Portable Magnetometer (Elliott Brothers, No. 64) and Dipping Needle of the well-known Kew Pattern (Barrow & Co. London, No. 24). In addition to these was the indispensable Chronometer (Negus Sidereal, No. 1669).

The chronometer was systematically checked by Sextant Observations, usually of the sun, sometimes of a planet or star. When circumstances were favourable, equal altitude observations were taken. Generally, however, this method was impracticable, since the plan of survey decided upon prevented the Party sojourning more than 16 or 17 hours at most of the stations. The chief source of error in determining the time by single altitude observations lay in the somewhat uncertain values of the latitudes and longitudes of many of the stations. Frequently the only choice was to estimate these coordinates from the authoritative maps prepared by the Meteorological and Military departments. For a few important stations (Hakodate, Niigata, etc.) recent determinations of latitude and longitude were available: in other cases Inō's observations were utilised. The clock-error was necessary for the determination of the declination, and the rate for the determination of the horizontal intensity. Any possible error in the latter can affect the value of the horizontal intensity by, at the most, *one* in the 5th significant figure. As the declination was, in the great majority of cases, determined by

observation of the azimuth of Polaris, a few seconds' error in the determination of the clock-error was insignificant. Occasionally, however, cloudy weather prevented the observation of Polaris, and in that case the Sun's azimuth was taken instead. It was customary indeed, if the station were reached early enough in the afternoon, to take a Sun's azimuth, in case the night should not prove propitious; and in not a few cases the declination was obtained by both methods. Once or twice, on broken nights, Jupiter, Arcturus, or Spica was utilised for obtaining the true astronomical meridian.

The azimuth was taken in the usual way by means of the small mirror fitted to the Kew instrument. If Polaris or other star were being observed, four transits were taken,—first, two with the star in front of the observer, the mirror being reversed on its Y's between the observations, and then two similarly with the star behind. If the sun were being observed, two transits were taken, one fore and one back, the mirror being in each case reversed between the contacts. In this way the small errors of adjustment were taken proper account of.

Immediately before or immediately after the transit observations, the magnetic declination was taken, the magnet being always viewed in the inverted as well as in the proper position. During the whole survey, the point on the scale corresponding to the magnetic axis never varied by as much as the tenth of a small division.

On two occasions only did continued wet weather prevent the azimuth observation being made—namely at Ichinoseki (No. 7) and at Ueda (No. 37)

In the final working up of the results only those stations are chosen at which the azimuth was obtained by means of Polaris observations. In all except one case, these Polaris observations were made between the hours of 8 p.m. and 12 p.m. local time: in that

one case (at Sekiyama No 36) they were made at 3 a.m. Now it is just at these hours that the magnetic declination is subject to the slowest change and has besides a value which can hardly differ more than 1′ of arc from the mean value for the day. Where a complete series of hourly observations throughout a whole day is out of the question, observations of declination made a little before midnight will give a fairly good approximation to the true mean value. When the sun was used for finding the azimuth, it was usually before 8 a.m. or after 4 p.m. On one occasion, however, (at Ebisu, No. 33) circumstances compelled a transit of the sun to be taken at about noon.

The dip and horizontal force observations were usually made at early morning before eight o'clock or in the afternoon after four. As there was but one tripod, it was necessary to remove the one instrument so as to make way for the other. It was found more convenient generally to observe the dip first, and then make the deflection and vibration experiments. If necessary a declination was taken and the azimuth obtained as usual from two transits of the sun. If it was a morning observation and if the declination had been obtained by means of Polaris the night before, this final declination was dispensed with. If it was an afternoon observation, however, and the sun visible, a solar azimuth and magnetic declination were always taken, to guard against the mischance of a cloudy night. The dip and magnetometer observations were make by both observers alternating. Thus the one who observed the dip during this set of observations would operate with the magnetometer during the next set, and *vice versa*. The one who was not observing recorded and kept time.

The sextant observations were all undertaken by Mr. Nagaoka, who had carefully trained himself to the work before the Expedition started. The sextant used was one of Browning's (London), No. 6295.

The travelling was usually effected by means of jinrikisha; and there cannot be the least doubt that the rattling and jolting did not increase the steadiness of the chronometer. Frequent sextant observations were therefore absolutely necessary, so that a fairly accurate value of the mean daily rate could be obtained. There always must be, however, a certain doubt as to whether the mean rate so obtained is really the true rate when the chronometer is resting. It is highly probable, in fact, that irregularities must result from such a jolting as *jinrikisha* give; we can only hope that these irregularities balance each other in the long run.

The results obtained will be discussed along with the observations of the Southern Party. Their mode of operating was in many details quite different from the mode adopted by the Northern Party, and calls for a full account both of instruments and methods.

The following account is drawn up, almost in his very words, from Mr. Tanakadate's own descriptive notes.

Section III.

With the Exception of the theodolite, which was fitted up both for transit and for magnetometric observations, the South Party resembled the North Party in its equipment. A chronometer, (Negus Sidereal, 1629), a dip circle, a box of tools and necessary books, and a tent, completed the out-fit.

The dip and vibration Experiments were always carried out by Mr. Imagawa; while Mr. Tanakadate undertook the chronometer rating, and the declination and deflection experiments. Usually the one acted clerk to the other, noting down what the latter read off. In the vibration experiments, the observer signalled the vibrations, and the instants of signal were timed by Mr. Tanakadate from the chronometer and noted down. In the transit-observations, however,

Mr. Tanakadate worked alone by the usual eye and ear method.

The object aimed at being to obtain a general magnetic survey of Japan, it was advisable as far as possible to eliminate local and diurnal disturbances. The stations had all originally been chosen after a careful study of the distribution of volcanoes throughout the country. If further we assume that all mountains are a possible source of disturbance—of underground sources of disturbance it is impossible of course to take any preliminary account—the following rough rule, which guided us in most cases, will probably be found useful:—A station should be so placed that no mountain as seen from it shall subtend a vertical visual angle greater than 5°. Thus, let there be a mass of magnetic substances of volume v and susceptibility k at a distance r from the station; and suppose that this mass is a cube* of height h. Then, if I be the intensity of the magnetic field in which the mass is placed, the disturbance at the station will be $k\,I\,v/r = k\,I\,(h/r)^3 = k\,I\tan^3\theta$, where θ is the visual angle. Take $I = \cdot 4$, $k = 10$ (bad iron), $\theta = 5°$, and the value of the disturbance comes out ·0027. In the most favoured circumstances, however, it is highly improbable that more than 1 or 2 p. c. of iron is present in the substance of the mountain. Hence we may safely assume that a mountain at such a distance will only affect the 5th decimal place. In a few cases, such as Wakwan (No. 62) Hagi (No. 72) Hamada (No. 73), we were compelled by circumstances to break the above rule; but in no case did the visual angle amount to 10°.

The station was usually chosen in an open field within 1 or 2 kilometres of some village or town. Occasionally a cotton field was cleared sufficiently to permit the tent to be pitched.

On a few occasions dip observations were made at two or three spots in the vicinity of the chosen site; and if the values came out

---

* Such an assumption gives of course a higher value than is likely to be. [C. G. K.]

within the limits of errors of observation, the place was assumed to be free from local disturbance. At two stations, Hamada (No. 73) and Maizuru (No. 78), this dip test was made after the regular series of magnetometer observations had been completed. At the former station, the values differed by as much as 20′. at the latter by 3′. From the nature of the environment such discrepancies were just what might have been expected.

At Minabe (No. 59) we first made a series of hourly observations of the declination; and this proved such an easy matter with the form of declinometer used that at all subsequent stations declination observations were made in sufficient number to obtain a diurnal curve. Excepting when the number of distinct observations was less than five, these diurnal variations are shown in diagram (Plates XII to XV). The general smoothness of the curves is a sufficient proof of the efficiency of the electromagnetic declinometer, to be described below.

The diurnal variation of the Horizontal Intensity was observed twice, namely, at Hiroshima (No 61) on July 29th, and at Miyazaki (No. 69) in September 1st. The values are all given in the tables at the end of the memoir. A comparison of the two sets of observations shows how very dissimilar are the measured variations of the horizontal force in the two cases, although the relation between the temperature and magnetic moment of the bar magnet comes out very similar in the two cases. From these results it is possible to obtain in the usual form an expression for the Moment of the magnet in terms of the temperature.

On two occasions, at Minabe (No. 59) on July 22nd, and at Shioya (No 80) on October 5th, a series of observations of the dip was made throughout the day. These gave nothing definite (see complete list in Table) although the variation of declination on these days was as usual.

At each place, three observations at least of all the elements were made, one in the morning, one near noon, and one in the evening. In the final tabulation, the arithmetic means of the dips and horizontal intensities are given. The mean declination is however obtained by a different method. From the complete observations made at Hagi (No. 72) and Hamada (No. 73), means were taken by summing the 24 ordinates corresponding to each hour of the civil time. In both cases this mean came out lower than half the sum of the principal maximum and principal minimum by one-tenth their difference. Thus if d, d' be the maximum and minimum respectively, the true mean was found to be

$$\frac{d+d'}{2} - \frac{d-d'}{10}$$

This rule is applied to all the observations. In several cases the maximum and minimum are only inferred from the neighbouring points.*

The magnetometer used by the South Party presents many points of novelty, both in construction and in mode of using. It combines the ordinary apparatus for the measurement of the horizontal force with a special form of declinometer invented by Mr. Tanakadate and described in a paper published in the *Proceedings of the Royal Society of Edinburgh* (1884–6).† The whole apparatus is built up upon a theodolite, which in its ordinary form as an alt-azimuth instrument serves for all the astronomical observations necessary for finding the latitude, longitude, and meridian of the station, and the true sidereal time. The instrument is shown in Plate VI, mounted for its

* A glance at the curves will show that this method gives a mean in neither case differing by as much as half a minute from the value of the declination as observed between the hours of 9 and 12 p.m., so that (if we disregard the possibility of the existence of a magnetic storm at these hours) the methods adopted by the North and South Parties to obtain a good mean declination give quite concordant results. [C. G. K.]

† Also in the *Rigakukyōkwa Zasshi* (Vol. II).

several purposes. It consists essentially of a theodolite, a mirror magnetometer, a declination coil, a small galvanic cell, a resistance box, a vibration case, a deflection bar, and a bar magnet. At any given station, the base of the theodolite is fixed and adjusted once for all, and, if possible, never changed throughout the whole series of experiments. Only in this way can a really satisfactory series of diurnal observations of the declination be made. That this might be done, a second tripod was necessary for mounting the dipping circle and vibration apparatus.*

The instruments will be described (1) as a Declinometer, (2) as an apparatus for measuring the horizontal force, (3) as an alt-azimuth or transit instrument for determining the astronomical meridian, the clock error, and latitude.

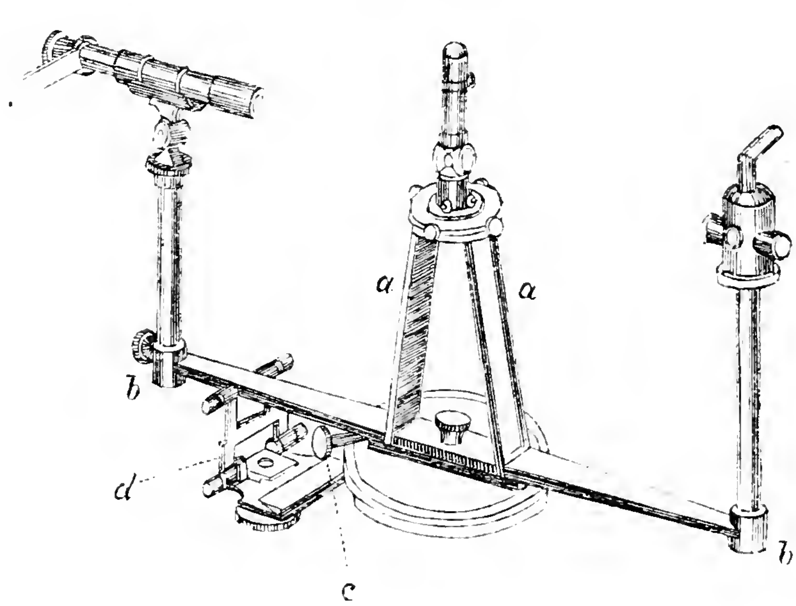
The theodolite, which formed the basis of the whole, was one of Negretti and Zambra's construction. To fit this up as a magnetometer, the compass needle had first to be removed, and into the hollow space left in the centre of the theodolite base, was fitted what may be called a magnetometer stage. In figures 1 and 5, Plate VI., this stage may be seen, and its form is specially will shown in figure 5. In the subjoined cut, it is shown in clearer detail. The chief points aimed at in its construction were ease of adjustment and facilities for clamping it either to the base of the theodolite or to the Y's. The disk-shaped base of the stage with its circumferential ring-clamp (*c*) nearly fitted the circular cavity left after removal of the theodolite compass needle. This ring-clamp, when tightened, abutted against the sides of the receptacle, and fixed the whole stage to the Y's of the theodolite. When loosened, its two halves collapsed upon the disk-shaped base of the stage, which then simply rested, unclamped, upon the plat-

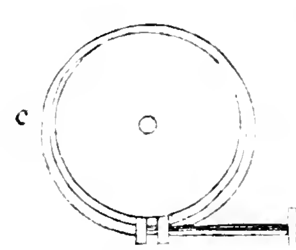
* A second tripod might will be added to the Kew set of instruments, and a distinct saving of time be effected in making a series of observations throughout the day. [C. G. K.]

form of the theodolite. The central part of the stage (a a) rose up to a height which fell somewhat short of the level of the Y's, and ended in a socket intended for the insertion of the base of the magnetometer proper. The last essential part of the stage was a stout brass bar (b b) running through between the Y pillars. From its ends rose uprights, to one of which was fixed the telescope and scale

Fig. 1.

($\frac{1}{2}$ natural size).





($\frac{1}{6}$ natural size).

used in the readings, and to the other a lamp for night work, which also played the rôle of counterpoise. The fixing of the stage to the base of the theodolite instead of to the Y's was effected by means of a clamp (d) attached to the brass bar and attachable to the rim of the theodolite. In this way the stage could be clamped to either base or Y's, singly or together, as occasion might require.

The magnetometer case (Plate VI, fig. 2, and Plate VII, fig. 3), when set in position on the central ring socket of the stage was centred by means of four adjusting screws. The magnetometer is shown dissected on Plate VII. Figures 1 and 2 show the magnet and mirror suspension. The magnet (Fig. 2. m) is a small hollow cylinder piercing the mirror centrally perpendicular to its plane. Mirror and magnet are fastened to an aluminium stem ($a\,a'$), whose lower end is broadened, so that it may when necessary be securely gripped by the vice $s\,s'$ shown (magnified) in figures 5 and 6 (Plate VII).

The suspension was by means of a spider line. Several experiments were made to test the torsional effect of such a suspension. A full account of these was given in the Rigaku Kyō Kwai Zasshi (Vol. II, p. 108); but it will suffice to say here that the torsion due to a twist of 180° on a spider line of the length indicated cannot cause an error of 1″ of arc in the orientation of a suspended magnet. The spider-line was drawn out directly from the living animal, and a suitable length attached to the mirror and the disk d (Figs. 1 and 2), from which the magnet and mirror were to be suspended.* The disk d is a fan-shaped horn damper, whose weight is nearly the same as that of the mirror and magnet, and whose purpose, as such, is to remove all torsion out of the spider line, after the suspension has been carefully mounted in the case. The case consists of four parts, as follows (See Plate VII, fig. 3). (1) The top cover—a glass tube capped by a bell glass, below which is a shelf (b) with a triangular hole and a slit wide enough to let the horn damper pass through easily. (2) The glass tube, on which the top cover is slipped, and to which it is fixed by a clamp (c) at any height within certain limits.

* There cannot be the least doubt that the spider-line is *the* perfect mode of suspension of a small light mirror, such as is used in delicate galvanometers and magnetometers. The zero point *never* changes, so far at least as the suspension is concerned. [C. G. K.]

the height being adjusted to suit exactly the length of the suspension. (3) The magnet chamber—a brass tube with two circular glass windows, a little larger than the magnet-mirror, and facing usually north and south. Two small windows are also made facing east and west, so that, in the process of centering, the middle of the magnet can be seen from these directions. (4) The vice, which grips the stem (a') when the magnetometer is being carried about. This vice occupies the lowest part of the magnetometer casing, lying inside the part $n\,n$ shown in Figure 3. The details of its construction are indicated in Figures 4, 5, 6 (Plate VII). Two brass springs ss' ss' are fixed below at $s's'$. A tube tt, with two side openings through which the springs bulge out when the vice is to be released, can be made to slide up and down inside the magnetometer case. When this perforated tube is down (Fig. 6) the vice closes; when it is up (Fig. 5) the vice opens. The vice-springs grip the stem a' of the suspended magnet. When they are opening so as to leave the magnet free, there is considerable risk that the stem may stick to one of them; and to free it by a jerk or tap would be liable to break the delicate suspension. To obviate this, two thin strips of brass (pp' pp') are fixed to the sliding tube at $p'p'$, and are so adjusted that their upper sharpened edges press always against the inside surface of the vice-springs. Thus, as the tube tt slips up, the ends pp glide along the inner faces of the springs and gently detach the stem, should it chance to be sticking to either arm of the vice. The up and down motion of the sliding tube is effected by means of a special combination of rack and screw. Into a rectangular socket cut in the sliding tube, a small piece of brass (r, Fig. 4) fits. This piece of brass, which must be inserted after the sliding tube has been slipped into position, is toothed on its outward facing surface so as to fit into a screw cut internally on the ring nn (Figs. 3 and 4.). A vertical slit, cut in the

tube of the magnetometer case, serves as a guiding slot in which the small toothed piece moves. The ring *nn* is milled on the outer surface. When it is turned by the finger, the toothed piece in gearing with it rises or falls according to the direction of turning; and with it the sliding tube also moves, and opens or closes the vice.

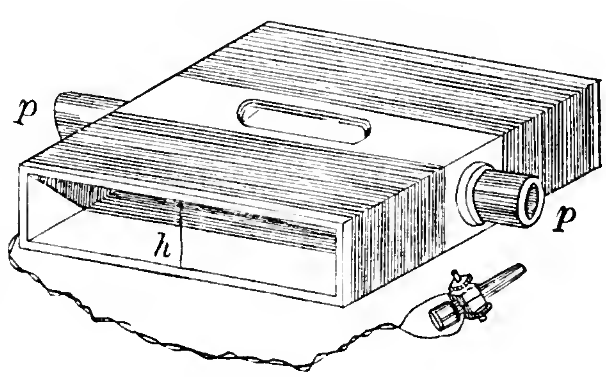
The magnetometer case expands below into a disk (*ll*), with a hemispherical knob on its lower surface. This knob rests in a tetrahedral hole cut in the base-plate. By screws passing through the disk and bearing on the base plate, the magnetometer case can easily be adjusted to a true verticality. (See Fig. 3, Plate VII, also Fig. 2, Plate VI).

To mount the magnet and damper in the case, after they have been connected by the spider-line, the top-cover must first be unclamped and removed. The small screw, *b*, is uncrewed a little so as to leave the triangular space or notch in the shelf quite open. The top is then held inverted in the one hand; and with the other the stem *a'* is carefully lifted until the damper hangs free. The damper is then dropped through the slit, the magnet and mirror carefully deposited on the table near by, and the screw *b* tightened so as to jam the "handle" of the fan-shaped damper up against the sides of the notch. The top piece is now held up in its proper position, so that the magnet and mirror hang freely from it. These are next gently lowered into the tube-case of the magnetometer, and the top is clamped in the position which makes the mirror hang level with the circular windows. The stem *a'* will now hang in its proper position between the arms of the vice, which should be opened before these operations are begun. If the hanging is done carefully by an experienced hand there should be little or no twist on the spider line. Any twist can however be easily removed by gripping the stem below the magnet with the vice, inverting the magnetometer case, unscrewing the small

screw b, and so leaving the damper to hang freely. It is for this reason indeed that the fan-shaped body, d, is called by that name. For, in virtue of its shape, it very speedily comes to approximate rest as it swings in the small space between the bell-glass and the shelf, and of course comes to final rest only when all twist has been taken out of the spider line. If the suspension is left for a few hours in this position, say overnight, we are safe in assuming that the twist has been practically eliminated. A slight tip will now bring the handle of the damper into the notch, where it is securely clamped by screwing the screw b home. The magnetometer, when placed erect, is now ready for use. This magnetometer is an essential part of the apparatus required both for the declination and deflection experiments. The Declinometer will now be described.

The peculiar feature of the electromagnetic declinometer is a coil of wire of a convenient form, whose axis can by a simple experimental method be accurately made to coincide with the magnetic meridian. The coil is shown in Plate VI. Fig. 1, and also in the annexed cut. It is wound on a flat rectangular frame of brass in two separate parts, a certain portion in the middle being left vacant. Looked at from above, it is square. Two pivots $p\,p$ project from the middle of the sides in a direction perpendicular to the axis of the coil.

Fig. 2.
($\frac{1}{4}$ natural size).



These pivots are hollow, and are made of the same external diameter as those of the telescope belonging to the theodolite. The upper and lower surfaces are pierced so as to allow the magnetometer to project above the coil. The middle part of the frame where no wire is coiled is equal to the height of the frame and

one-fifth its breadth. These dimensions were carefully calculated * so as to make the error due to an excentric position of the magnetometer a minimum.

To bring the coil into adjustment, it is necessary to operate as follows. Place the stage and magnetometer on the theodolite, and mount the coil with its pivots resting on the Y's (Plate VI, fig. 1). Adjust the Y's into an approximate east and west direction by sighting the freely hanging mirror edge-on through the pivot cores. Lay the coil horizontal, so that the ends of the coil now face north and south. On one of these ends, which is partly closed, a small pin-hole is made accurately at the centre. On the other end, exactly opposite it, a wire (*h* in the cut) is stretched. When this wire is sighted through the pin-hole and *through the small hollow magnet*, then we know that the magnet is truly centred with regard to the coil. If the wire cannot so be seen, the magnetometer must be adjusted in position by means of the screws provided in the ring socket in which the base of the magnetometer rests. If every thing is in accurate adjustment, a reversal of the coil, so that what was at first the east pivot becomes west and *vice versa*, will still leave the pin-hole, magnet, and wire in one and the same line. If it is not so, then the pin-hole and wire are at fault. It is evident, however, that once the adjustment is made, it is made once for all as far as the coil is concerned—that is until the wire should give way. The magnetometer being thus centred east and west, it must next be centred north and south. This is done with sufficient accuracy by sighting the magnet through the hollow pivot of the coil, and working the north-and-south adjusting screws until the magnet hangs central.

If the small telescope with attached scale and lamp-counterpoise

* The details of the calculation are given in the Paper in the *Proceedings of the Royal Society of Edinburgh* (1884–6) already referred to.

are now mounted in position (see Plate VI, Fig. 1), everything is ready for making a determination of the magnetic meridian. The stage and all its belongings are clamped to the base of the theodolite and rendered quite free of the Y's; and, consequently, the coil can be turned round independent of everything else. The magnetometer stage is adjusted until some convenient division on the scale, as reflected from the magnet mirror, is brought to coincidence with the cross-wire of the observing telescope.

The coil is now put in circuit with a small cell, which is shown in Plate VI, Fig. 1, hanging from the centre of the theodolite. This is done by putting the terminal double-plug (see small cut on page 184) into one of the holes of the resistance box, whose terminals (see Plate VI figure 1) are joined to the poles of the battery. At the first trial the direction of the current should be such as to make the magnetic field due to the current in the coil have the same (general) direction as that of the earth. This is readily judged of by the quickened movement of the magnet. The reflected image of the scale will in general be seen to move. With the current always on, let the azimuth of the coil be shifted until the originally observed reading of the scale is brought back again to the cross-wires. Since the magnetometer and telescope have been absolutely fixed in position during the whole operation, this gives to the first approximation the direction of the declination. The current in now reversed by simply turning the double-plug half-way round in its hole. In general, the result of this will be that the image of the zero scale reading will slowly move to one or other side of the cross-wire. The resistance in circuit is then adjusted until the current is such as to cause the time of oscillation of the magnet to be some three or four times as long as that under the earth's force alone. The original division of the scale is again brought back to the cross wire by careful adjust-

ment of the azimuth of the coil. If the current is now broken and the scale image does not shift, it is certain that the magnetic axis of the coil lies in the magnetic meridian. The reading on the theodolite gives its azimuth.

During the series of operations, however, the magnetic meridian might have changed—during the forenoon, indeed, the change is quite perceptible in five minutes. Or, the magnetometer stage might itself have got shifted somewhat in the process of adjustment. In either of these cases there will be a shifting in the scale image at the instant of making the final break in the circuit. By adjusting, however, until at the final break the scale image remains absolutely unaffected, we know that at that instant, which should be noted on the chronometer, the magnetic meridian is determined. An exactly similar observation must be made with the coil reversed as regards east and west; and then the mean of the two readings so obtained may be assumed to give the declination at that instant corresponding to the mean of the two instants of adjustment, quite independent of any error resulting from any (slight) deviation from perpendicularity between the axis of the pivots and the line of magnetic force at the centre of the coil due to the current in it. An experienced hand can make the interval between the two final adjustments as small as $2\frac{1}{2}$ minutes.

For the determination of the horizontal force, the declinometer coil must be removed and in its place a deflection bar substituted as show in figure 5, Plate VI. This bar is made of brass and has a V-groove on its upper surface—or rather two V-grooves extending the one to the east and the other to the west, when the instrument is mounted ready for use. Where the bar rests on the Y's, it is made in the form of a semi-cylinder—the upper surface being flat, the lower having the same curvature as the pivots of the theodolite tele-

scope and the declinometer coil. Between the Y's, the bar swells out into an oblate ring, through which the magnetometer projects. A semi-circular groove is cut in front of the ring, so that the magnet-mirror can be sighted by the small telescope. Over the brass ring and surrounding the magnetometer a ring of wood is placed, with three semi-circular grooves cut in it at suitable places. It facilitates observation by cutting away all extraneous light, while at the same time the central groove in combination with the groove on the brass ring gives a clear front to the mirror. The side grooves are needed during night observations to allow the lamp-light to illuminate the scale. On the V-groove of the bar there are four stops, two on each side of the centre. The deflection magnet rests in the groove, and the stops are so placed that the two distances of the magnet from the centre are obtained simply by slipping the magnet along the groove from one stop to the other, without having to lift it out. The stops are placed so as to make the ratio of the two distances the best possible, according to the usual rule.

The instrument is obviously available for use either according to the method of sines or the method of tangents. The former method is the preferable one; and, in using it, it is necessary to clamp the stage to the Y's, and free it from the base of the theodolite. The operations are then conducted exactly as with the Kew Instrument. The temperature of the bar is measured by means of a thermometer placed on the V-groove outside the further stop. It is advisable to dust with a small brush the surfaces of the magnet and stop just before they are brought together. The chronometer time is taken as the final deflection is adjusted. The beginning of the experiment is given by the first time record in the vibration experiment, which it was found most convenient to make first. The mean of these times is taken as the time corresponding to the value of the horizontal force as finally deduced.

The vibration experiment is made in a vibration box somewhat similar in construction to the one used in the Kew Instrument (See Plate VI, Fig. 3). It is mounted on a second tripod, so that the magnetometer stage need never be removed until the theodolite has to be used for the astronomical observations. The magnet employed during the survey was a solid steel bar of square section and polished on all sides. Its length was 7·0024 cms. and its breadth ·803 cms. at 0° C. Its mass was 35·061 grammes, and its moment of inertia 145·15 (cm^2. gr.) at 0° C. It was suspended by two loops of silk from the end of a silk fibre freed from twist in the usual way by means of a brass weight as heavy as the magnet. To get the magnet horizontal, the base of the vibration case was first levelled by means of a spirit level. The magnet was then lowered down till it just rested on the floor. Then, by raising it gently, the observer could readily tell whether or not it was horizontal by the way in which it left the floor. A few slight taps with the brush on the dipping end, if dipping end there was, and a repetition of the same operation until the magnet left the floor as a whole simultaneously, were sufficient to make it practically horizontal. It was then raised to the height of the side window in the box and steadied by means of a fine brush. The telescope was adjusted and focussed till a clear image of the scale was seen reflected from the polished side of the magnet. A horizontal swing of about half a degree was given to the magnet by the approach and removal of a screw-driver; and the experiment conducted in the usual way. The observer signalled the instants of transit of the middle point of the swing, and these were noted down by the recorder, who was posted at the other extremity of the tent with the chronometer. The eye and ear method is no doubt more accurate, the observer himself recording the transits; but to this there is the objection that the chronometer must be brought close up to where the

observer is, thereby producing a possible magnetic disturbance. A specimen sheet is here reproduced of a single complete experiment for determining the horizontal force. The torsion and arc corrections are applied in the usual way; and the time correction is applied so as to reduce the time-unit at once to the mean solar second.

As already mentioned, it was found most convenient to take the vibration experiment before the deflection experiment, thereby saving the time necessary for the adjustment of the swinging magnet. From the first recorded swing to the last deflection adjustment the whole experiment took 23 minutes.*

The observations of Dip were made in the usual way. The magnetization of the needle was reversed by means of the large magnets that form an essential part of the apparatus. It might be remarked, however, that, if the electromagnetic declinometer is used, it would be much more convenient for the observer to provide himself with a suitable coil for reversing the magnetization of the dipping needle. The large magnets would not then be required, and the necessity done away with of constantly removing them 15 or 20 yards from the tent whenever a declination experiment was to be made. For, closed though these magnets are, they cause an appreciable disturbance within a distance of a few yards from the magnetometer.

On the occasions on which a series of observations of the dip

* It is impossible, I believe, to go through these operations with the Kew Instrument so rapidly as this. The gain of time with Mr. Tanakadate's instrument seems to be in the deflection experiment, in which the small deflected magnet is much more quickly damped than is possible with the comparatively large magnet used in the Kew Instrument. In my opinion there is quite an unnecessary amount of manipulation required in using the Kew Magnetometer as at present constructed. A small magnet mounted in a manner similar to Mr. Tanakadate's hardly ever requires a renewal of the suspension line, delicate though that is. It requires but to be placed in position, and the deflection experiments begun. A slight modification in this direction would enable us to dispense with the deflection box altogether, and considerably shorten the time required to make a deflection experiment. [C. G. K.]

Specimen Page of Observations for Finding the Horizontal Force.

Observer,

Vibration Experiment.

	Temp.	Arc	Torsion.
Initial	30.03 C	81 to 134	
Final	30.00 C	85 to 129	
Mean	30.15 C	24.25	4.75
		59′.8	11.70

Torsion after Exp^{nt}.

82	91	
123	133	
102·5 −	112 =	8·5
		4.75

after 20 Vib^{ns}. 42^m 14^s·0
„ 30 „ 43 18.4
„ 40 „ 44 23.0

For arc and torsion 1 div. corresponds to the arc 2′·466

Time.

Final.		Initial.		Difference.
45^m	27^s.5	16^h 40^m	5^s.2	322^s.3
	34.0		11.6	.4
	40.6		18.0	.6
	47.0		24.5	.5
	53.5		30.9	.6
	59.9		37.5	.4
46	6.3		44.0	.3
	12.7		50.3	.4
	19.1		56.5	.6
	25.6	41	3.1	.5
	32.1		9.6	.5

No. of Vib^{ns}. 50 322.46

Time of single Vibration, T_o 3.2246

Log. Tab.	Log. T_o	(1)	0.50848
Tab. I.	Log. $\left(1 \pm \frac{s}{86400}\right)$	(2)	$\bar{1}$.99879
Tab. II.	Log. $\left(1 - \frac{a^2}{16}\right)$	(3)	$\bar{1}$.99999
Tab. III.	Log. $\sqrt{1 + \theta\ (2\pi - \theta)}$	(4)	0.00012
Tab. IV.	Log. $\sqrt{1 + \mu\ H/M}$	(5)	0.00056
(1) + ... + (5)	Log. T.	(6)	0.50794
	Log. $\pi\sqrt{I}$		1.57806
	Log. $\sqrt{MH}$		1.07012
Tab. V.	Log. $(1 + \alpha\ t)$		0.00014
	Log. $\sqrt{MH}$		1.07026

Place, Miyazaki 1887. Sep. 12

Ended 17^h 0^m.9
Mean of times 16^h 50^m.5

Deflection Experiment.

Temperature.	Initial	Final	Mean
	29°.0 C	29°.0 C	29°.0 C

Mean Temp^r. 29°·6

	$r_1 =$		$-r_1 =$		φ_1
	a	b	a	b	
Dir.	53° 49′ 20	233° 49′ 10	53° 49′ 10	233° 49′ 20	
Rev.	41° 46′ 50	221° 47′ 10	41° 54′ 40	211° 54′ 50	
Dif.	12° 02′ 30	12° 02′ 00	11° 54′ 30	11° 54′ 30	11° 58′ 22.5
	$r_2 =$		$-r_2 =$		5° 59′ 11.3
	a	b	a	b	φ_2
Dir.	61° 21′ 30	241° 21′ 30	61° 20′ 20	241° 20′ 30	
Rev.	34° 08′ 20	214° 08′ 40	34° 30′ 20	214° 30′ 30	
Dif.	27° 13′ 10	27° 12′ 50	26° 50′ 00	26° 50′ 00	27° 01′ 30
					13° 30′ 45

Tab. VI	Log. r_1^5	(1)	7.38749
Log. Tab.	Log. sin φ_1	(2)	$\bar{1}$.01825
(1) + (2)	Log. r_1^5 sin φ_1	(3)	6.40874
Tab. VI	Log. r_2^5	(4)	6.81279
Log. Tab.	Log. sin φ_2	(5)	$\bar{1}$.36858
(4) + (5)	Log. r_2^5 sin φ_2	(6)	6.18137
Log. Tab.	Log. $\left(\frac{r_1^5 \sin \varphi_1}{r_2^5 \sin \varphi_2} - 1\right)$	(7)	$\bar{1}$.83019
Tab. VI	Log. $1/2\ (r_1^2 - r_2^2)$	(8)	$\bar{3}$.13014
Tab. VI	Log. $\{1 - 2\mu\ r_1 r_2 (r_1 + r_2)\}$	(9)	$\bar{1}$.99991
Tab. VII	Log. $(1 + 3\alpha\ t)$	(10)	0.00071
(6) + ... + (10)	Log. M/H	(11)	3.14232
	Log. $\sqrt{M/H}$	(12)	1.07026
	Log. $\sqrt{M/H}$	(13)	1.57116
(12) − (13)	Log. H.		$\bar{1}$.49910
(12) + (13)	Log. M.		2.64142

(3) − (6)
0.22475

$H =$ .31557

$M =$ 437.95

was taken, the azimuth was adjusted every third observation. This is certainly sufficient; since, as is well known, a small error in the azimuth causes an error in the dip of a higher order of small quantity. *

To make the observations that are necessary for finding the astronomical meridian, all the magnetometric pieces of apparatus must be dismantled and the theodolite telescope mounted in position. (See Plate VI, Fig 4.) The observations were made as much as possible on stars, since it is always objectionable to expose such an instrument to direct solar light and heat. Then, again, there is more chance of instrumental error when the theodolite is turned into the prime vertical, inasmuch as all the declination observations in this country happen to be made near the meridian.

In general the perfection of observation aimed at was to obtain transits of eight conveniently destributed stars across an approximate meridian, and, by combining these observations, deduce the values of the four unknown quantities,—the clock error, the two azimuth errors, and the collimation error.

The approximate meridian was found by taking two transits of Polaris, the telescope being reversed by turning the Y's through 180° between the two transits. The mean of the two azimuths read was taken as corresponding to the instant midway between the noted

* The following proof, from its simplicity, may be of interest. Let V be the vertical, H the horizontal component, and θ the dip; then $\tan\theta = V/H$. If the plane of the needle makes a small angle a with the magnetic meridian, the apparent dip θ' is given by the equation

$$\tan\theta = \frac{V}{H\cos a} = \frac{V}{H}\left(1 + \frac{a^2}{2} + \ldots\ldots\right) = \tan\theta + \tan\theta . \frac{a^2}{2}$$

$$\text{or, } d\,(\tan\theta) = -\tan\theta . \frac{a^2}{2}$$

$$\text{which gives } d\,\theta = -\sin 2\,\theta . \frac{a^2}{4}$$

If we put $a = 10'$, which is roughly the range of the diurnal variation, we find for the greatest possible value of $d\,\theta$, the utterly insignificant quantity $0''\cdot4$.

times of transit. By applying a clock error reckoned from the preceding set of observations, we obtained an approximate hour angle; and from this with assumed latitude it was easy calculating the astronomical azimuth of Polaris. The finding of this azimuth was much facilitated by the use of a table of Polaris azimuths, previously prepared, and tabulated with latitudes and hour angles as arguments. On applying the level correction, we have the azimuth to a first approximation. This value was used to set the instrument in the approximate meridian.

From the list of stars given in the Berlin *Yahrbuch*, four stars were now selected—if possible, two near the zenith and two near the horizon, and lying in pairs on each side of the zenith. Each star's transit was observed across three of the micrometer wires. Immediately after the last transit was taken the star was bisected by the horizontal wire and the zenith distance read off on the vertical circle. The striding level was read twice, once with the telescope pointing north, and once with it pointing south. The level in connection with the vertical circle was also observed, the telescope being then set to zero reading.

The azimuth was now read, and the Y's turned through 180°. Another set of four stars was chosen and their transits observed in the same way.

On a fine night, from $1\frac{1}{2}$ to 2 hours' observations sufficed for carrying out the complete transits of the eight stars; and from these the collimation, azimuth, and clock errors could be calculated. A similar set at the same spot the next night furnished the means of calculating the clock rate.

On one occasion, at Miyazaki (No. 69) the clock rate was obtained by making two sets of observations not quite 8 hours apart—the one set in the evening, the other set in the morning.

From the Zenith distances of the observed stars, the latitude of the place could be calculated by applying the usual corrections—reduction to the meridian and refraction.

When a sufficient number of stars could not be observed, the first observation of Polaris was utilized for the purpose of obtaining the azimuth to a closer approximation, the newly evaluated clock error and latitude being employed, and the calculation made without the use of the Polaris azimuth table.

At four stations (Nos. 52, 56, 67, 71) observations of the sun alone were possible. In these cases, the local time was determined from altitude observations, and the azimuth from prime vertical transits.

On two occasions, at stations (62) and (80), Polaris was observed through clouds and the clock error was assumed.

Section IV.

General Results of the Survey.

I now pass to the consideration of the principal results obtained.

There were 81 stations in all, of which 50 belonged to the Northern Party, and 32 to the Southern, the recreation ground of the Imperial University being a common station to both Parties. The Parties might with equal accuracy be termed the Eastern and Western Parties, inasmuch as all the stations of the former lie to the east of the longitude line 138° E., and (with the exception of three, including Tokyo) the stations of the latter lie to the west of that line. Roughly speaking, this line separates the main island of Japan into two regions markedly different as to their geological characters. Compared to the Western portion, the Eastern and Northern portion is highly volcanic. Here, consequently, considerable magnetic disturbances are to

be looked for. It is the more important, then, to have as many different stations as possible, so that in the final combining of the results a good selection may be made. This conviction grew upon us more and more as the work progressed, as may be inferred from the less scattered distribution of the later stations as compared with that of the earlier ones. Indeed, only by a much closer distribution of stations can we hope to make out anything very definite regarding the magnetic characteristics of the north-east parts of Japan. In the subjoined table are given the stations with their latitudes and longitudes. The dates on which the observations were made will be found in the detailed tables forming Appendix B to the paper. The stations are numbered from 1 up to 81; Nos. 1 to 50 being the Northern Party's stations arranged chronologically, and Nos. 50 to 81 the Southern Party's similarly arranged. In the other tables to be given here, the stations will usually be represented by their numbers.

After the individual observations were finally reduced, rough charts were prepared showing the broad features of the various isomagnetic lines. From these it was easy to tell where the principal centres of disturbance lay. Guided by this and other considerations, a selection of 50 points was made. These are indicated in the four succeeding tables by being represented in thicker type. The coordinates of the Mean Station of these fifty chosen stations are:—

Latitude............ 36° 30′ N.
Longitude......... 137° 9′ E.

Referred to this Mean Station as origin, the four sets of fifty observations—representing respectively the Dip, the Horizontal Force, the Total Force, and the Declination—were combined by the method of Least Squares. The results will be discussed in the order just named. For the sake of showing how far the formula deduced from the 50 chosen stations is applicable to the remaining 31, the various tables

Table I.

The Co-ordinates of the Stations.

	Station.	Latitude.	Longitude.
		° ′ ″	° ′ ″
1.	Ishibashi	36 25 54	139 52 15
2.	Yabuki	37 10 48	140 18 15
3.	Matsukawa	37 47 42	140 23 45
4.	Shiraishi	37 59 18	140 33 15
5.	Shiogama	38 19 38	141 2 15
6.	Ishinomaki	38 26 8	141 19 30
7.	Ichinoseki	38 55 30	141 2 24
8.	Hanamaki	39 25 0	141 1 45
9.	Morioka	39 43 0	141 8 0
10.	Miyako	39 38 48	141 59 15
11.	Kuji	40 11 17	141 48 0
12.	Hachinohe	40 30 12	141 29 30
13.	Gonohe	40 33 0	121 15 15
14.	Nobechi	40 52 19	141 8 15
15.	Aomori	40 51 5	140 44 45
16.	Hakodate	41 46 30	140 44 30
17.	Sapporo	43 4 0	141 22 45
18.	Kiitup	43 4 33	145 7 15
19.	Nemuro	43 20 5	145 35 15
20.	Hirosaki	40 35 30	140 24 0
21.	Ōdate	40 16 36	140 28 15
22.	Nōshiro	40 12 30	140 0 0
23.	Akita	39 42 12	140 4 15
24.	Kariwano	39 33 48	140 18 0
25.	Yokote	39 18 12	140 25 15
26.	Shimo-innai	39 3 0	140 21 15
27.	Shinjo	38 45 30	140 14 45
28.	Sakata	38 57 31	139 51 15
29.	Yamagata	38 15 0	140 16 45
30.	Yonezawa	37 54 30	140 7 45
31.	Oguni	38 4 12	139 42 15
32.	Nakajō	38 1 12	139 20 30
33.	Ebisu	38 5 0	138 26 15
34.	Niigata	37 51 1	139 2 30
35.	Kashiwazaki	37 23 36	138 32 0
36.	Sekiyama	36 56 0	138 11 30
37.	Ueda	36 24 0	138 14 15
38.	Takanomachi	36 8 54	138 26 30
39.	Kōfu	35 39 0	138 35 45
40.	Hara	35 7 44	138 48 30
41.	Hakone	35 12 30	139 1 30
42.	Ōtsu	35 15 48	139 42 30
43.	Hōjō	35 0 22	139 53 0
44.	Katsuura	35 8 0	140 19 15
45.	Tōgane	35 33 30	140 21 45
46.	Chōshi	35 43 0	140 50 30
47.	Kioroshi	35 50 48	140 9 30
48.	Shimmachi	36 17 12	139 7 0
49.	Ōmiya	35 54 18	139 36 30
50.	Tōkyō	35 42 35	139 46 0
51.	Shimoda	34 40 16	138 57 15
52.	Shimizu	35 0 10	138 30 15
53.	Nagoya	35 10 40	136 55 0
54.	Kamiyashiro	34 30 25	136 45 15
55.	Nagahama	35 23 48	136 46 0
56.	Hyōgo	34 39 20	136 10 30
57.	Tokushima	34 4 0	134 35 0
58.	Kōchi	33 32 50	133 33 15
59.	Minabe	33 46 0	135 20 0
60.	Okayama	34 38 0	133 55 30
61.	Hiroshima	34 23 10	132 27 0
62.	Wakwan	35 5 0	129 1 30
63.	Meiho	35 4 6	128 53 45
64.	Pusan	35 7 36	129 3 0
65.	Fukuoka	33 36 8	130 21 45
66.	Nakatsu	33 36 8	131 11 30
67.	Saganoseki	33 14 30	131 53 15
68.	Hichiyamura	32 25 15	131 39 30
69.	Miyazaki	31 55 11	131 26 30
70.	Yatsushiro	32 30 43	130 37 0
71.	Nagasaki	32 45 0	129 52 30
72.	Hagi	34 25 3	131 22 30
73.	Hamada	34 53 41	132 5 45
74.	Matsue	35 28 24	133 4 0
75.	Imaichi	35 21 35	132 50 0
76.	Kanōmura	35 28 33	134 12 30
77.	Koyamamura	35 31 6	134 10 30
78.	Maizuru	35 26 52	135 20 45
79.	Obama	35 30 20	135 44 45
80.	Shioya	36 18 2	136 14 0
81.	Nanao	37 3 19	137 0 15

comparing the observed and calculated values of the elements contain also these 31.

Each of the charts representing the various systems of lines (see Plates VIII—XI) shows two sets of lines. The full lines are the graphic representation of the Mean Formula for the whole country worked out in the usual way by the method of Least Squares. The dotted lines give a better idea of things *as they are*, being drawn by free-hand interpolation amongst the neighbouring stations. The eye can thus tell almost at a glance, not only to what degree of approximation the full lines agree with the true state of things, but also in what parts of the country magnetic disturbances are most pronounced.

In combining the combinations, I assumed linear expressions in longitude and latitude for the Dip, the Horizontal Force, and the Total Force. It is quite clear, however, that any attempt to represent the Declination by a simple linear function would end in utter confusion. On the Declination Chart (Plate XI) it will be seen that, roughly speaking, the dotted lines run parallel to the general trend of the country itself and have a somewhat parabolic or hyperbolic form. To add other two terms involving the squares of both the longitude and latitude would have increased the labours of calculation enormously. I therefore contented myself with adding a term involving the square of the longitude, assuming, so to speak, a parabolic curve with its axis parallel to a meridian line. The result was much more satisfactory than I had expected it to be.

In the working out of the calculations, the stations were all of course referred to the mean station (36° 30′ N. Lat., 137° 9′ E. Long.), and the co-ordinates were expressed in angular units. From the Formulæ so obtained, the lines shown on the charts were constructed. The kilometre was then taken as the unit in which the co-ordinates were to be expressed, the particular change-ratios being those which

held for the Mean Station. The lengths in kilometres of one minute in arc of latitude and longitude at that station are :—

1 minute of Latitude.........1.85 kilometres.

1 ,, ,, Longitude......1.49 ,,

This transformation of units being effected, it was then easy to find the angle at which any given iso-magnetic line at the Mean Station cuts the meridian line, and the greatest rate of change per kilometre of the particular element at that Station.

I.—The Dip.

The fifty selected observations, when combined by the method of least squares, gave the following formula expressing the Dip (θ) in terms of the co-ordinates :—

$$\theta = 50° 28'.6 + (1.141\,\varphi - .1556\,\lambda)'$$

φ and λ are the latitude and longitude co-ordinates referred to the mean station (36° 30′ N. Lat., 137° 9′ E. Long.) and measured in minutes of arc.

In the annexed table, Table II., the values of the Dip as observed at all the stations are given, and along side of them the values as calculated from the Formula.

The selected stations, on which the calculation depended, are indicated by having their numbers printed in heavier type. Including the other stations serves to indicate to what degree of approximation the formula applies to them also. This will be best shown by giving the probable error of a single observation in the different cases.

If we take all the 81 stations, the probable error comes out ± 11′.33.

If we neglect Nos. 39, 41, and 73, whose differences are very large indeed, the probable error becomes ± 8′.54. Nos. 39 and 41

are respectively Kōfu and Hakone, stations which lie in what is one of the most disturbed regions in Japan. Hakone indeed is quite peculiar magnetically, and is included amongst the 81 stations more as a curiosity than otherwise. It is situated in a hilly and highly volcanic region on the shores of a deep tarn-like lake. A small piece of stone picked up from the ground and brought near the declination magnet had a large effect upon it. Neither Kōfu nor Hakone belong to the selected stations. No. 73, Hamada, near the extreme westerly end of the country, does however belong to the selected stations. It will be noticed that the contiguous station, No. 72, Hagi by name, shows also a large difference between the observed and calculated values of the Dip, but that the difference is positive whereas the difference in the case of No. 73 is negative. The same approximate balancing of differences exists also in the case of Kōfu and Hakone. We shall consider this point more in detail subsequently.

If we take into account only the Fifty selected stations, the probable error is $\pm$ 9′.56.

Four of the selected stations stand out prominently by virtue of their large differences. These are Kashiwazaki (No. 35). Hichiyamura (No. 68), Hagi (No. 72) and Hamada (No. 73). If we neglect these, the probable error becomes $\pm$ 6′.96. Hichiyamura is one of the Kyūshū group of stations (Nos. 65-71), all of which lie in a very volcanic region and all of which have their observed Dips less than the corresponding calculated Dips.

It is worthy of note that the extreme Yezo stations, Kiitup and Nemuro (Nos. 18 and 19), fit in remarkably well with the general formula—better indeed than do the less extreme stations Hakodate and Sapporo (Nos. 16 and 17). At Hakodate (which is one of the Fifty) the proximity of Hakodate Peak, a hill of volcanic rock 1,100 feet high, might well be a source of disturbance. At Sapporo again,

Table II.

The Mean Dips for all the Stations.

	Dip.							Dip.					
Station.	Observed.		Calculated.		Difference.		Station.	Observed.		Calculated.		Difference.	
	°	′	°	′	°	′		°	′	°	′	°	′
1	50	7·3	49	57·6	+	9·7	42	48	33·8	48	40.2	−	6·4
2	50	57·3	50	45·6	+	11·7	43	48	32·6	48	20·4	+	12·2
3	51	13·8	51	15·0	−	1·2	44	48	30·6	48	26·4	+	4·2
4	51	20·9	51	38·4	−	17·5	45	49	2·5	48	54·6	+	7·9
5	51	48·7	51	57·0	−	8·3	46	48	43·1	49	0·6	−	17·5
6	51	56·2	52	1·8	−	5·6	47	49	32·5	49	15·6	+	16·9
7	52	32·0	52	37·8	−	5·8	48	49	56·0	49	56·4	−	0·4
8	53	17·7	53	11·4	+	6·3	49	49	39·0	49	25·2	+	13·8
9	53	10·9	53	31·8	−	20·9	50	49	12·0	49	10.2	+	1·8
10	53	36·0	53	18·6	+	17·4	51	47	59·4	48	9·6	−	10·2
11	54	7·4	53	57·6	+	9·8	52	48	41·3	48	36·6	+	4·7
12	54	23·3	54	21·6	+	1·7	53	48	58·2	49	3·0	−	4·8
13	54	39·4	54	27·6	+	11·8	54	48	16·8	48	25·2	−	8·4
14	54	44·1	54	52·2	−	8·1	55	49	17·6	49	24·6	−	7·0
15	54	47·1	54	54·0	−	6·9	56	48	42·3	48	44·4	−	2.1
16	55	35·5	55	55·2	−	19·7	57	48	0·4	48	4·2	−	3·8
17	56	41·5	57	18·6	−	37·1	58	47	32·0	47	42·6	−	10·6
18	56	48·4	56	44·4	+	4·0	59	47	46·3	47	40·8	+	5·5
19	57	7·1	56	58·2	+	8.9	60	48	47·9	48	54·0	−	6·1
20	54	24·6	54	37·8	−	13·2	61	48	39·6	48	50·4	−	10·8
21	54	12·5	54	16·2	−	3·6	62	50	9·3	50	10·2	−	0·9
22	54	17·8	54	16·2	+	1.6	63	50	27·0	50	10·8	+	16·2
23	53	37·6	53	19·8	+	17·8	64	50	1·2	50	13·8	−	12·6
24	53	30·5	53	28·8	+	1·7	65	48	6·5	48	15·0	−	8·5
25	53	5·0	53	11·4	−	6.4	66	48	2·0	48	7·8	−	5·8
26	52	51·1	52	52·8	−	1·7	67	47	20·9	47	31·8	−	13·9
27	52	37·2	52	33·6	+	3.6	68	46	27·7	46	55·2	−	27·5
28	52	51·9	52	51·6	+	·3	69	45	51·3	46	10·2	−	18·9
29	52	1·6	51	58·8	+	2·8	70	46	40·0	46	58·2	−	18·2
30	51	31·6	51	37·2	−	5·6	71	46	57·1	47	21·6	−	24·5
31	51	47·2	51	52·8	−	5·6	72	48	44·1	49	16·8	−	32·7
32	51	49·2	51	52·2	−	3·0	73	50	12·2	49	28·8	+	43·4
33	52	3·7	52	4·8	−	1·1	74	50	1·8	49	58·8	+	3·0
34	51	56·9	51	47.4	+	9·5	75	50	13·1	49	54·0	+	19·1
35	51	55·4	51	16·2	+	39·2	76	49	43·4	49	49·8	−	6·4
36	50	55·7	50	48·0	+	7.7	77	49	53·8	49	52·8	+	1·0
37	50	3·3	50	11·4	−	8·1	78	49	21·8	49	37·2	−	12·4
38	49	40·6	49	52·8	−	12·2	79	49	30·4	49	37·8	−	7·4
39	50	5·7	49	16·2	+	49·5	80	50	30·8	50	27·6	+	3·2
40	48	33·3	48	39·0	−	5·7	81	51	15·1	51	12·6	+	2.5
41	47	25·7	48	42·6	−	76·9							

we were compelled by very wet weather to make our observations on the stone pillars in the meteorological observatory,* and this may have been a source of disturbance.

If we now express the co-ordinates in kilometres, the Formula for the Dip becomes

$$\theta = 50° \, 28'.6 + (.6168 \, \varphi - .1044 \, \lambda)'$$

Let u be the angle between the Line of Equal Dip drawn eastward and the longitude line drawn northward; and let r be the rate of change of Dip per kilometre of distance measured in a direction perpendicular to the Line of Equal Dip; then we find in the usual way for the mean station

$$u = 80° \, 23'.6$$
$$r = 0'.626$$

II.—The Horizontal Force.

The fifty selected observations, when combined by the method of least squares, gave the following formula expressing the Horizontal Force (H) in terms of the co-ordinates:—

$$H = .29482 - .0000617 \, \varphi - .0000117 \, \lambda$$

H is measured in absolute C. G. S. electro-magnetic units; and φ and λ, the latitude and longitude co-ordinates referred to the mean station (36° 30′ N. Lat., 137° 9′ E. Long.), are measured in minutes of arc.

In the annexed table, Table III., the values of the Horizontal Force as observed at all the stations are given, and alongside of them the values as calculated from the Formula. As before the selected stations are indicated by having their numbers printed in heavier type.

* The observatory is built wholly of wood; but the tile-roof or the stone pillars themselves may have been appreciably magnetic.

Table III.

The Mean Horizontal Forces for all the Stations

Station.	Horizontal Force Observed.	Calculated.	Difference.	Station.	Horizontal Force Observed.	Calculated.	Difference.
1	.29143	.29317	— .00174	42	.29655	.29762	— .00107
2	.28952	.29009	— 57	43	.29630	.29854	— 224
3	.28785	.28836	— 51	44	.29607	.29760	— 153
4	.28830	.28692	+ 138	45	.29533	.29604	— 71
5	.28755	.28531	+ 224	46	.29696	.29511	+ 185
6	.28753	.28470	+ 283	47	.29377	.29516	— 139
7	.28426	.28309	+ 117	48	.29367	.29420	— 53
8	.28032	.28132	— 100	49	.29428	.29528	— 100
9	.28180	.28010	+ 170	50	.29642	.29590	+ 52
10	.27876	.27975	— 99	51	.29991	.30033	— 42
11	.27983	.27789	+ 194	52	.30046	.29942	+ 104
12	.27687	.27695	— 8	53	.29904	.29986	— 82
13	.27615	.27695	— 80	54	.30199	.30247	— 48
14	.27576	.27576	± 0	55	.30026	.29950	+ 76
15	.27727	.27615	+ 112	56	.30295	.30301	— 6
16	.27318	.27274	+ 44	57	.30603	.30562	+ 41
17	.26766	.26762	+ 4	58	.31006	.30826	+ 180
18	.26667	.26485	+ 182	59	.30494	.30621	— 127
19	.26025	.26356	— 331	60	.30455	.30400	+ 55
20	.27819	.27739	+ 80	61	.30714	.30592	+ 122
21	.27743	.27848	— 105	62	.30469	.30576	— 107
22	.27854	.27908	— 54	63	.30663	.30589	+ 74
23	.28275	.28091	+ 184	64	.30908	.30556	+ 352
24	.28224	.28127	+ 97	65	.31019	.31031	— 12
25	.28309	.28203	+ 106	66	.31000	.30972	+ 27
26	.28226	.28313	— 87	67	.31137	.31057	+ 80
27	.28524	.28428	+ 96	68	.31367	.31378	— 11
28	.28465	.28381	+ 84	69	.31562	.31577	— 15
29	.28651	.28614	+ 37	70	.31490	.31416	+ 74
30	.28836	.28750	+ 86	71	.31392	.31380	+ 12
31	.28748	.28721	+ 27	72	.30886	.30657	+ 229
32	.28762	.28765	— 03	73	.30045	.30391	— 346
33	.28950	.28807	+ 143	74	.30006	.30148	— 142
34	.28699	.28823	— 124	75	.30022	.30207	— 185
35	.28878	.29055	— 177	76	.30144	.30066	+ 78
36	.29152	.29245	— 93	77	.29984	.30053	— 69
37	.29629	.29442	+ 187	78	.30097	.29996	+ 101
38	.29502	.29528	— 26	79	.30018	.29946	+ 72
39	.29261	.29694	— 433	80	.29555	.29621	— 66
40	.30150	.29873	+ 277	81	.29399	.29284	+ 115
41	.30490	.29827	+ 663				

If we take all the 81 stations, the mean probable error of a single observation comes out ± .00108.

If we throw out Nos. 39, 41 and 73, which are again conspicuous by the magnitude of their Differences, No. 41 (Hakone) being especially so, the probable error becomes ± .00091. It will be noticed that No. 64, one of the Korean stations, is more conspicuous even than No. 73 in respect to the magnitude of the Difference ; and that No. 19 (Nemuro) does not fall far short of it. If these also be neglected, the probable error comes out ± .00084.

If we take into account only the fifty selected stations, the probable error is ± .00080.

Here four Stations stand out prominently by virtue of their high differences. These are Shiogama (No. 5), Hōjō (No. 43) Hagi (No. 72) and Hamada (No. 73). If we neglect these the probable error becomes ± .00064.

Two of these four neglected Stations, namely Hagi and Hamada (Nos. 72 and 73), were also amongst the Stations that were similarly treated in the discussion of the Dip. Kashiwazaki (No. 35), which was one of the four neglected in the discussion of the Dip, is fairly prominent also by reason of the magnitude of the difference between the observed and calculated Horizontal Forces. Hichiyamura (No. 68), however, the remaining one of the four neglected Dip observations, is characterised by an exceptionally small difference between the observed and calculated values of the Horizontal Force. A similar remark applies to Shiogama (No. 5) and Hōjō (No. 43), whose Horizontal Force differences are large, but whose Dip differences are comparatively small.

And now, expressing the co-ordinates in kilometres, we find

$$H = .29482 - .00003335\ \varphi - .00000785\ \lambda$$

From this we obtain the following values for the inclination of the Line of Equal Horizontal Force to the direction of geographical north, and for the greatest rate of change of Horizontal Force per kilometre :

$$u = 103° \; 14'.9$$
$$r = .000034$$

III.—The Total Force.

The Total Forces were calculated from the Horizontal Forces and Dips ; and the values at the fifty selected stations, when combined by the method of least squares, gave the following expression for the Total Force (F) in terms of the co-ordinates :—

$$F = .46407 + .000094 \varphi - .000045 \lambda$$

F is measured in absolute C. G. S. electromagnetic units : and φ and λ, the usual geographical co-ordinates referred to the Mean Station (36° 30′ N. Lat., 137° 9′ E. Long.), are measured in minutes of arc.

Table IV. is constructed after the same fashion as Tables II. and III.

If we take all the 81 Stations, the mean probable error of a single observation comes out ± .00128. If we neglect Nos. 11, 18, and 38, which are conspicuous by the hugeness of their differences the mean probable error is at once reduced to ± .00099.

If we take into account only the fifty selected Stations, the mean probable error is ± .00089.

It will be noticed that, out of the six Stations (Nos. 5, 35, 43, 68, 72, 73) which were conspicuous in Tables II. and III. by the magnitudes of the Differences, only one is so distinguished in Table IV. With the exception of Kashiwazaki (No. 35), these Stations just enumerated are rather to be distinguished by the smallness of the differ-

ences between the observed and calculated values of the Total Force.

Running our eye down the difference columns in the Dip and Horizontal Force Tables, we are struck by a general tendency for the differences at any one locality to have opposite signs. That is to say, where the observed value of the dip is greater than the calculated value, the observed value of the horizontal force is, in the majority of cases, less than the calculated value. More particularly, of the 81 Stations only 26 are characterised by having their dip and horizontal force differences of the same sign; or, if the selected stations are alone considered, of these 50 only 17 are similarly characterised.

It would thus appear that the magnetic disturbances in Japan are of a nature to affect the *direction* rather than the *amount* of the total force—a result quite in accordance with the usual laws of magnetic action. In this connection there are two very interesting cases that seem to call for special remark. To bring out their peculiarities the more distinctly, it is advisable to draw up in tabular form the differences only for the stations that are to be discussed. They are as follows:—

Station		Differences		
No.	Name	Dip	Hor. Force	Total Force
39	Kōfu.........	+ 49′.5	− ·00433	+ ·00075
41	Hakone......	− 76′.9	+ ·00663	− ·00104
72	Hagi.........	− 32′.7	+ ·00229	+ ·00035
73	Hamada.....	+ 43′.4	− ·00346	+ ·00071

These form pairs of contiguous points. In all four the Dip and Horizontal Force differences are exceptionally large; whereas the Total Force Differences are all distinctly smaller than the mean probable error for the whole. Here we have evidence in both cases of a

Table IV.

The Mean Total Forces for all the Stations

Station.	Total Force Observed.	Total Force Calculated.	Total Force Difference.	Station.	Total Force Observed.	Total Force Calculated.	Total Force Difference.
1	.45454	.45632	— .00178	42	.44809	.45017	— .00208
2	.45961	.45940	+ 21	43	.44754	.44823	— 69
3	.45967	.46263	— 296	44	.44690	.44790	— 100
4	.46163	.46329	— 166	45	.45053	.45002	+ 51
5	.46479	.46390	+ 89	46	.45009	.44971	+ 38
6	.46635	.46374	+ 261	47	.45272	.45222	+ 50
7	.46729	.46727	+ 2	48	.45624	.45757	— 133
8	.46901	.47001	— 100	49	.45452	.45401	+ 41
9	.47023	.47149	— 126	50	.45365	.45255	+ 110
10	.46975	.46879	+ 96	51	.44813	.44887	— 74
11	.47746	.47233	+ 513	52	.45515	.45196	+ 319
12	.47548	.47491	+ 57	53	.45554	.45725	— 171
13	.47737	.47583	+ 154	54	.45379	.45391	— 12
14	.47763	.47807	— 44	55	.46039	.46025	+ 14
15	.48083	.47895	+ 188	56	.45905	.45903	+ 2
16	.48343	.48414	— 71	57	.45741	.45730	+ 11
17	.48743	.48971	— 228	58	.45924	.45714	+ 210
18	.48710	.47967	+ 743	59	.45371	.45358	+ 13
19	.47937	.47986	— 49	60	.46234	.46223	+ 11
20	.47800	.47837	— 37	61	.46499	.46485	+ 14
21	.47438	.47642	— 204	62	.47555	.47800	— 245
22	.47728	.47730	— 2	63	.48155	.47829	+ 326
23	.47677	.47423	+ 254	64	.48104	.47821	+ 283
24	.47458	.47283	+ 175	65	.46455	.46604	— 149
25	.47130	.47120	+ 10	66	.46359	.46380	— 21
26	.46741	.46980	— 239	67	.45956	.45990	— 34
27	.46984	.46845	+ 139	68	.45536	.45588	— 52
28	.47151	.47064	+ 87	69	.45316	.45365	— 51
29	.46564	.46550	+ 14	70	.45887	.45920	— 33
30	.46350	.46398	— 48	71	.45988	.46256	— 268
31	.46473	.46603	— 130	72	.46828	.46793	+ 35
32	.46530	.46673	— 143	73	.46941	.46870	+ 71
33	.47088	.46952	+ 136	74	.46710	.46929	— 219
34	.46564	.46698	— 154	75	.46920	.46930	— 10
35	.46825	.46536	+ 289	76	.46628	.46621	+ 7
36	.46252	.46368	— 116	77	.46546	.46658	— 92
37	.46147	.46057	+ 90	78	.46260	.46302	— 42
38	.45591	.46250	— 659	79	.46227	.46228	— 1
39	.45612	.45537	+ 75	80	.46478	.46542	— 64
40	.45550	.45187	+ 363	81	.46971	.47039	— 68
41	.45069	.45173	— 104				

very obvious kind of disturbance. Take, for example, the first pair. Kōfu, the more northerly station, has an increased dip and a diminished horizontal force; whereas Hakone, the more southerly station has a diminished dip and an increased horizontal force. Now this is exactly what would happen if between the two stations there existed material of magnetic permeability greater than the average for the earth's crust. But we know that these stations are separated by rocks, which presumably belong to the volcanic system of Fuji-yama, and are almost certainly highly magnetic. It is not at all surprising to find such huge disturbances; but it is peculiarly interesting to find that the disturbance in the Dip largely accounts for the disturbance in the horizontal force, the total force being comparatively slightly affected. This Kōfu-Hakone disturbance falls in remarkably well with our preconceived ideas as to the perturbing effect of volcanoes upon magnetic distribution.

The other pair of stations, Hagi and Hamada, present exactly the same magnetic features. Thus it is Hamada, the more northerly station, which has the increased dip and the diminished horizontal force. It corresponds therefore to Kōfu. Magnetically considered, Hamada and Hagi are just a repetition of Kōfu and Hakone. But here the likeness apparently ends; for between Hamada and Hagi no important mass of volcanic rocks is known to exist. Of course there are magnetic rocks other than volcanic; and the apparent absence of the latter does not necessarily imply the absence of the former. All we can say is that between Hagi and Hamada a more than usually magnetic mass lies.

A careful scrutiny of the various tables of differences does not reveal any distinct tendency for the stations to occur in groups, excepting in one case only. That exceptional group embraces the Kyūshū stations. For them both the dips and total forces are smaller

than the mean formula requires. This may have some significance. From the pretty scattered distribution of the stations we should hardly expect to find them adjusting themselves into groups characterised by similar errors. The disturbances are generally of such a local character that they must almost certainly affect in different ways the magnetic elements at stations which are never very close together.

And now expressing the co-ordinates in kilometres we find, for the Total Force,

$$F = .46407 + .0000508\ \varphi - .0000302\ \lambda$$

from which we obtain, as in the other cases,

$$u = 59°\ 16'.5$$
$$r = .000059$$

IV.—The Declination.

As already stated, it is impossible to regard the Declination as at all expressible by means of an equation involving only first powers of the co-ordinates. Fortunately for the labours of calculation, the general form of the isogonic lines for Japan may be taken as fairly parabolic. On this assumption, then, the fifty selected observations were combined by the method of least squares, and the result is embodied in the following formula expressing the Declination (δ) in terms of the co-ordinates:—

$$\delta = 4°\ 53'.3 + (.241\ \varphi - .109\ \lambda - .000231\ \lambda^2)'$$

The latitude and longitude co-ordinates (φ, λ) are referred to the Mean Station (36° 30′ N, Lat., 137° 9′ E. Long.) and measured in minutes of arc.

Table V. is constructed in the same fashion as Table II., III. and IV.

If we take into account all the 79 observations—at two of the Stations (Nos. 7 and 37) the Declination could not be observed—the

mean probable error comes out $\pm$ 13′ .4. There are three Stations, however, which have such large differences that, when they are neglected, the mean probable error is reduced to $\pm$ 8′. These Stations are Miyako (No. 10), Hakone (No. 41) and Pusan in Korea (No. (64). The manner in which the declinations at these Stations diverge from the values given by the formula suggests local disturbances of quite a limited area of action. This is peculiarly true of Hakone (No. 41). That the disturbance at Pusan (No. 64) is of quite a local character is proved by the fact that at the contiguous stations (Nos. 62 and 63) the differences are comparatively small. Miyako (No. 10) is another striking instance, for which, however, no certain conclusions can be drawn as, unfortunately, observations at neighbouring points were not taken.

This whole region indeed, lying between the thirty-eighth and forty-first parallels of latitude—and more especially the central and eastern portions of it—seems to be one of marked magnetic disturbance. It is highly volcanic and contains nearly a score of distinct volcanoes, of which four are especially prominent. These are* Iwaki-san (5,260 ft.) in the northwest near Hirosaki (No. 20), Ganju-san (7,000 ft.) in the centre near Morioka (No. 9), Moriyoshi-san (5,800 ft.) to the south of Ōdate (No. 21), and Chōkai-san (7,100 ft.) to the north of Sakata (No. 28). This great spread of volcanic rock, however, is confined entirely to the Central and Western portions. The Kita-kami River, which flows from its sources a little to the north of the fortieth parallel due south to Ishinomaki (No. 6), is virtually the dividing line between the volcanic and non-volcanic deposits. In the hilly country to the east of this river the rocks are largely schists and granites. Nevertheless in this part also the dis-

* See Professor Milne's "The Volcanoes of Japan," Transactions of the Seismological Society of Japan Vol. XI. (1886).

Table V.

The Mean Declinations for all the Stations.

Station	Declination West. Observed. ° ′	Calculated. ° ′	Difference. ′
1	4 1·6	4 28·3	— 26·7
2	4 31·5	4 34·6	— 3·1
3	4 29·6	4 39·6	— 10·0
4	4 33·7	4 43·4	— 9·7
5	5 5·5	4 41·7	+ 23·8
6	5 4·1	4 39·9	+ 24·2
7	—	4 56·6	—
8	5 25·2	4 57·9	+ 27·3
9	5 2·5	5 1·1	+ 1·4
10	5 47·0	4 47·9	+ 59·1
11	4 37·7	4 58·6	— 20·9
12	4 32·6	5 11·4	— 38·8
13	4 57·7	5 10·9	— 13·2
14	5 31·7	5 17·4	+ 14·3
15	5 22·2	5 22·0	+ ·2
16	5 31·3	5 35·3	— 4·0
17	6 0·0	5 46·2	+ 13·8
18	4 46·0	4 43·3	+ 2·2
19	4 21·3	4 37·8	— 16·5
20	5 23·3	5 22·4	+ ·9
21	5 15·7	5 16·9	— 1·2
22	5 32·3	5 21·6	+ 10·7
23	5 9·3	5 13·4	— 4·1
24	5 9·2	5 9·0	+ ·2
25	5 23·1	5 4·0	+ 19·1
26	5 11·7	5 1·1	+ 10·6
27	4 58·9	4 51·6	+ 7·3
28	5 11·0	5 5·2	+ 5·8
29	4 32·6	4 50·1	— 17·5
30	4 32·6	4 46·5	— 13·9
31	4 51.4	4 54·0	— 2·6
32	5 0·3	4 56·9	+ 3·1
33	5 0·0	5 6·5	— 6·5
34	5 9·4	4 58·4	+ 11·0
35	5 3·4	4 55·6	+ 7·8
36	4 36·2	4 51·7	— 15·5
37	—	4 43·9	—
38	4 2.8	4 38·6	— 35·8
39	4 32·4	4 30·1	+ 2·3
40	4 29·4	4 20·7	+ 8·7
41	2 24·0	4 19·8	— 115·8
42	4 19·2	4 13·3	+ 5·9
43	4 23·4	4 7·6	+ 15·8
44	4 1·5	4 5·0	— 3·5
45	4 13·6	4 10·0	+ 3·6
46	4 19·2	4 6·5	+ 12·9
47	4 25·9	4 16·7	+ 9·2
48	4 45·7	4 34·2	+ 11·7
49	4 35·9	4 24·0	+ 11·9
50	4 21·0	4 19·1	+ 1·9
51	4 13·0	4 12·4	+ ·6
52	4 2·1	4 20·7	— 18·6
53	4 31·9	4 35·7	— 3·8
54	4 21·8	4 27·0	— 5·2
55	4 45·1	4 42·5	+ 2·6
56	4 34·1	4 36·4	— 2·3
57	4 26·0	4 29·5	— 3·5
58	4 20·8	4 23·8	— 3·0
59	4 30·1	4 23·4	+ 6·7
60	4 38·1	4 38·8	— ·7
61	4 27·1	4 35·5	— 8·4
62	4 25·0	4 31·4	— 6·4
63	4 42·7	4 30·1	+ 12·6
64	3 36·6	4 32·7	— 56·1
65	4 17·8	4 17·6	+ ·2
66	4 22·6	4 20·9	+ 1·7
67	4 12·0	4 17.6	— 5·6
68	3 59·4	4 5·2	— 5·8
69	3 58·4	3 58·7	— ·3
70	3 59·2	4 2·9	— 3·7
71	3 33·7	4 3·0	— 29·3
72	4 30·1	4 33·4	— 3·3
73	4 38·8	4 42·1	— 3·3
74	4 51·6	4 51·3	+ ·3
75	4 48·9	4 49·6	— ·7
76	5 9·0	4 50·9	+ 18·1
77	5 5·0	4 51·5	+ 13·5
78	4 59·4	4 47·3	+ 2·1
79	4 54·1	4 46·5	+ 7·6
80	4 59·2	4 55·2	+ 4·0
81	5 5·7	5 2·4	+ 3·3

turbances in the Declination are very marked. There is no evidence whatever of a Declination less than 5° W. until we reach the stations on the North East Coast. That is to say, the isogonic line of 5°, which general considerations would lead us to expect to run right across the region from S. W. to N. E., seems to form an extra distinct contour round the schistose granitic portion on the East. The declinations at Hanamaki (No. 7) and Miyako (No. 10) are especially large. So great in fact was the value of the declination at Miyako—just about a whole degree greater than was expected—that at first sight it looked like a misreading of a whole degree on the azimuth circle. Three distinct observations, however, were taken at this station—two by means of the sun and one (three transits in all) by means of Polaris. All the values agree to within 7′, so that the large value of the declination is undoubted. This peculiarity may be purely local, or it may be of wider extent. Our stations are not sufficiently numerous to enable us to come to any sure conclusion on this point. It should be mentioned that a little to the south of Miyako considerable quantities of iron ore are known to exist.

If we take account only of the 50 selected Stations, the mean probable error of a single observation is ± 7′. Three of the Stations are conspicuous by reason of the largeness of the differences between the observed and calculated values. These are Ishibashi (No. 1), Shiogama (No. 5) and Hachinohe (No. 12). It will be noticed that the declination at Shiogama is greater than is given by the formula, and that the declination at Hachinohe is less. These Stations lie respectively at the extreme South and at the extreme North of the region of schists and granites that has just been discussed. Only a number of observations at neighbouring localities could, however, determine whether the peculiarities presented by Shiogama and Hachinohe belong to a general system or are to be regarded as purely local.

Regarding Ishibashi it seems difficult at first to suggest a cause of such a large discrepancy. The Declination observed here was (neglecting the altogether peculiar Hakone) the second smallest observed by the Northern Party. Being the very first station of the whole survey, it was feared for a time that some mistake had been made by the observer. To make sure of this Mr. Nagaoka was despatched towards the end of September to Ishibashi to make a redetermination of the Declination. The observation was made at a slightly different locality; and the declination was measured at 9.45 a. m. by means of a Sun's Azimuth. The result did not differ by one minute from the value as found in June by means of Polaris. Here again, then, there can be no question as to the very peculiar smallness of the Declination at Ishibashi. Geologically considered Ishibashi lies just within the Eastern wing of the "Schaarung" (so called by Süss),* within which lie the volcanic regions of Japan. The boundary between this volcanic region and the non-volcanic region to the East is the continuation to the south of the similar boundary already described as coinciding in position with the Kita-kami-gawa. Now, although to all appearance Ishibashi is situated on alluvial soil, yet, according to the geologists, it just lies within this Schaarung. Hence it is quite probable that volcanic rocks exist hidden from view but so distributed as to cause a distinct magnetic disturbance. The values of the declination obtained by Sekino throughout this region show considerable irregularities, Ishibashi itself being characterised by a declination somewhat smaller than the average. His value for this station (namely, 4° 16′.6) is, however, much larger than ours. We must therefore either assume a change of 15′ or so in five years, or, what is much more probable, a very local source of disturbance.

If we neglect these three stations (Nos. 1, 5, 12) the mean

* See Dr. Naumann's Pamphlet already mentioned, page 17; also Professor Koto's Paper already published in this Volume.

probable error becomes ± 5′ .9—a value by no means considerable in the circumstances.

This seems the natural place to refer to Dr. Naumann's discussion of the iso-magnetic lines of Japan in relation to its geology. Basing on the broad features of Sekino's chart, Naumann finds in the form of the isogonic line of 5° W. a close relation to the so-called *Fossa Magna*. Just where this great break in the geological continuity of the country occurs, there a large sinnosity seemed to show itself on the isogonic line. This *Fossa Magna* almost stretches right across the central part of Japan in a nearly north and south direction. Fuji-yama is included in it and so, it is generally supposed, is the line of volcanic islands stretching south-easterly. The *Fossa Magna* hardly reaches the Northern coast of Japan; but, if continued northwards, it would be found to run between the Peninsula of Noto on the west and the Island of Sado on the east. Now it is just at this region that Sekino's 5° Isogonic line makes a great bend to the north, doubling back just over the island of Sado and then, after an easterly sweep, continuing north-easterly across the country. It is extremely doubtful whether the observations warrant such a delineation of the line of 5° Declination. A careful scrutiny of Sekino's numbers brings out certain discrepancies which should not altogether be neglected. Further, there is a complete lack of observations along the coast to the south and south-west of Sado—just where observations seem most called for. The stations chosen are all inland, and show striking irregularities in the values of the declinations. True, the declinations at the three Stations on Sado are all considerably less than the values at mainland stations immediately to the east; whereas we should expect to find them greater. But that seems hardly a sufficient reason for making the isogone of the form represented. For it is well known that the isogonic lines at and near islands often present

irregularities of quite a local description. Hence, in default of evidence which could only be obtained by a series of observations along the coast of the main island, it seems to me more prudent to draw the isogonic line of 5° fairly normal and represent the disturbance due to Sado by a small isolated contour round that island. In this way it is shown on our charts. In short, knowing as we do that an island is enough by itself to cause considerable disturbances, we are justified in explaining the peculiarities that are as being due to the existence of the Island of Sado rather than to the existence of the *Fossa Magna*. Indeed there does not seem to be any very extraordinary disturbance after all in this region. There are as marked disturbances in the north of Japan, where no *Fossa Magna* exists; and these disturbances find a sufficient explanation in the presence of volcanic rocks of all kinds. A mountainous volcanic region is certain to present magnetic irregularities; and in Japan there are two regions specially to be noted as such. A glance at the four charts will at once pick them out. The one is the great central mountainous region, just where the *Fossa Magna* is. The other is the part between the 38th and 40th parallels; but here there is nothing geologically comparable to the *Fossa Magna*. If peculiarly abnormal irregularities had been found to exist in the central region only, there might have been reason in connecting them with a very peculiar geological structure. But, since the northern region is quite as abnormal magnetically as the central one, the most logical course is to look upon these abnormalities as due to something common to them both. And this common element is simply a prodigious developement of volcanic rocks.

There are six stations which may be said to belong to the central hilly region. These are Sekiyama, Ueda, Takanomachi, Kōfu, Hara, and Hakone—Nos. 36 to 41 inclusive. Excepting Hara, which is on the coast, these stations all lie at considerable elevations. Thus Seki-

yama and Kōfu are nearly 1000 feet above the sea-level; Ueda is about 1500; Takanomachi and Hakone about 2500.* It is not surprising then that at the two last-named places the values of the various magnetic elements should be so distinctly abnormal. Takanomachi lies just on the northern side of the water-shed, which here divides the basins of the Fuji-kawa and Shinano-gawa. Immediately to the south the high mountains are crossed by a pass 4,500 feet high; and on the other side of this pass lies Nagasawa (3,100 feet), where an observation of Dip was made. This Dip (see Appendix B) is remarkable for its smallness, being half a degree smaller than the Dip at Takanomachi, and a whole degree smaller than the Dip at Kōfu. In fact, just where it is most mountainous, there are to be found the greatest magnetic irregularities. It is impossible of course in the circumstances of the case to decide as to whether height itself has a direct influence on the values of the magnetic constants. The volcanic nature of the rocks is more than enough to account for all irregularities.

Another very interesting group of stations is the Korean group. Here there are three principal stations, Wakwan, Mēho, and Pusan (Nos. 62, 63, 64); but quite a number of observations were made at points in the near neighbourhood. These are given in the complete lists in Appendix B. Excepting Mēho, all these stations lie on the shores of Pusan Harbour. This harbour is formed by a bay opening to the south with a large island filling up fully half the entrance. On a sharply projecting cape pointing towards the northwest end of the island lies Wakwan, the Foreign Settlement. At the head of the bay, some 3 miles to the north is Pusan itself; while due east from Wakwan about 3 miles across the bay is Kurosaki with Kurosaki Cape

* These heights are estimated from aneroid observations made during the survey, taken in combination with the daily charts of the meteorological office, which furnish the sea-level readings of pressure for the whole of Japan.

half a mile further south. On the eastern shore of Chearegdow—as the large island already mentioned is called—lies Omizutare, looking over against Kurosaki Cape. A little to the south is Shiinokijima, that is Shiinoki Island, helping to fill in the space between Chearegdow and Kurosaki. Shiinokijima is about half a mile long; and the observations were taken at the extreme southwestern and northeastern ends. Thus Wakwan, Pusan, Kurosaki, Kurosaki Cape, Shiinokijima Cape, Shiinokijima and Ōmizutare form a circle of points surrounding Pusan Harbour; and no two of them are further distant from one another than four or five miles. Within this very limited area very striking irregularities exist. The declination varies by nearly a whole degree; the dip by fully the same amount; the horizontal force by 1.5 per cent. of the whole value. Perhaps the most striking irregularity of all is that displayed by the value of the dip at Shiinokijima Cape as compared with the values at Shiinokijima proper (half a mile to the south-west) and at Kurosaki Cape (a mile and quarter to the north-north-east). As Mr. Tanakadate has pointed out, the difference of fully a degree in the dips at the two Capes suggests a kind of horseshoe magnet with its poles at these points. The whole series of observations is a very good example of the powerful effect of local disturbances. It should be said that the whole district is quite hilly and craggy.

We shall now follow a very usual custom and group our stations according to the geological character of the rocks in their neighbourhood. This can only be done in a very general way, since the geology of Japan is still known but in the broad. There is, of course, no doubt as to the geological characteristics of some stations; but there are many regarding which we can, by study of the geological map, draw no very sure inferences. The following grouping of stations is, therefore, only a tentative one. Three groups are distinguished. In

the first group stations with undoubted volcanic surroundings are included. In the third we put stations which are as characteristically non-volcanic. The rocks, in this case, may be schist or granite, or stratified rocks of some determinable geologic age, including alluvium. The second group includes what may be termed the doubtful stations. They may be resting on non-volcanic rocks or soil, but so close to rocks of truly volcanic origin as to warrant us in regarding them almost as belonging to the first group. It is highly probable, indeed, that a station lying near the edge of a stretch of volcanic rocks should be characterised by considerable irregularities in its magnetic elements. Hence the doubtful or intermediate second group may easily compare in the magnitude of its mean errors with the first group. The groups are as follows :—

Group I.—Volcanic : Shiogama, Ichinoseki, Hanamaki, Morioka, Gonohe, Nobechi, Aomori, Sapporo, Nemuro, Ōdate, Kariwano, Yokote, Shimo-innai, Shinjō, Yamagata, Ebisu, Sekiyama, Takano-machi, Kōfu, Hakone, Ōtsu, Hōjō, Shimoda, Wakwan, Mēho, Pusan, Nakatsu, Hichiyamura, Nagasaki, Nanao.

Group II.—Intermediate : Yabuki, Matsukawa, Shiraishi, Hachinohe, Hakodate, Hirosaki, Nōshiro, Akita, Kashiwazaki, Ueda, Hara, Hagi, Kanōmura, Koyamamura, Shioya.

Group III.—Non-volcanic : Ishibashi, Ishinomaki, Miyako, Kuji, Kiitup, Sakata, Yonezawa, Oguni, Nakajō, Niigata, Katsuura, Tōgane, Chōshi, Kioroshi, Shimmachi, Ōmiya, Tōkyō, Shimizu, Nagoya, Kamiyashiro, Nagahama, Hyōgo, Tokushima, Kōchi, Minabe, Okayama, Hiroshima, Fukuoka, Saganoseki, Miyazaki, Yatsushiro, Hamada, Matsue, Imaichi, Maizuru, Obama.

The mean probable errors for the different groups are given in the following tables—the first being based on the results for all the stations, the second taking account of the fifty selected stations only.

For all the Stations.

	Group I.	Group II.	Group III.
Dip	14′.3	11′.5	8′.4
	8′.9		6′.8
Horizontal Force	.00135	.00095	.00089
	.00095		.00082
Total Force	.00142	.00117	.00130
	.00117		.00082
Declination	19′.2	10′.3	9′.5
	10′.3		6′.7

Where two different values for the mean probable error are given, the second is calculated after those stations, which are characterised by *very large* differences between the observed and calculated values, have been thrown out. Thus in group I, Nos. 39 and 41 are neglected in the calculation of the second values for the Dip and Horizontal Force ; No. 38 for the Total Force ; and Nos. 41 and 64 for the Declination. In Group II. there were no stations prominent by the relative hugeness of their differences. In Group III, No. 73 is neglected for the Dip and Horizontal Force ; Nos. 11 and 18 for the Total Force ; and No. 10 for the Declination.

For the Fifty Stations only.

	Group I.	Group II.	Group III.
Dip	7′.5	13′.4	8′.9
Horizontal Force	.00075	.00084	.00082
Total Force	.00111	.00114	.00074
Declination	11′.0	6′.3	5′.7

The general conclusion to be deduced from both these tables is that the mean probable error is greater the more volcanic the region.

We shall now bring together for ease of reference the values of the elements for the mean station, the directions in which the iso-magnetic lines pass through this station, and the maximum rate of change of each element per kilometre of distance. The tabulation of the declination constants offers some difficulty, as the isogonic lines are not straight but parabolic. If, however, we express the latitude and longitude co-ordinates in kilometres, we obtain the following transformed expression for the declination :—

$$\theta = 4° \; 53'.3 + [.1303 \; \varphi - .0732 \; \lambda - .000104 \; \lambda^2]'$$

From this we may estimate the constants u and r, not only for the mean station, but also for other stations. It is easy to see that the values of u and r will be the same for all points having one and the same longitude. In this way the following table has been constructed; u being, as formerly defined, the angle made by the direction of the iso-magnetic line drawn eastward and the meridian line drawn northward, and r representing the rate of change per kilometre in a direction perpendicular to this line.

The Constants for the Mean Declination at different longitudes.

Longitude of Point.	u	r
130°	114° 37′	.143
134°	83° 39′	.131
138°	55° 39′.9	.158
142°	38° 33′.2	.209
146°	28° 10′.6	.277

And, finally, collecting the results for the mean station (36° 30′ N. lat. and 137° 9′ E. long.), we have the following condensed table of the magnetic constants of the mean station as calculated from Fifty selected stations conveniently distributed over all Japan.

The Magnetic Constants for the Mean Station.			
	Mean Value.	u	r
Dip	50° 28′.6	80° 23′.6	0′.626
Horizontal Force	.29482	103° 14′.9	.0000343
Total Force	.46407	59° 16′.5	.0000591
Declination	4° 53′.3	60° 40′.6	0′.119

V.—The Diurnal Variation.

Plates XII to XV show graphically the daily march of the Declination at various Stations, as observed by Mr. Tanakadate. For the earlier curves comparatively few points were taken ; but as the survey progressed the observation of successive declinations was found to be such a simple matter with the electromagnetic declinometer that Mr. Tanakadate made it a special feature of his work. The chief value of such observations in the present case is that from them a thoroughly good mean for the day may be obtained. As a whole, the c rves are of a character which speaks well for the accuracy and sensitiveness of Mr. Tanakadate's instrument. The sensitiveness indeed depends simply and solely upon the fineness of graduation on the theodolite circle. In judging of the merits of the curves, we must bear in mind the conditions of the experiment. For not only will any possible magnetic storms disturb the general smoothness of the observations, but also windy and wet weather will almost certainly cause disturbances in measurements made by an instrument mounted on an ordinary tripod under a tent.

Taking the 17 best and most complete sets of observations (see Appendix B). we find for the mean daily range for the three months beginning July 4th the value 8′. This is about 1′ greater than the mean diurnal range obtained by Mr. Wada during his five months of

observation in 1883. In the circumstances we may regard the conditions of the diurnal variation as essentially the same in the years 1883 and 1887.

It will be noticed that the curves obtained at Wakwan (No. 62) and Meho (No. 63), two of the Korean Stations, have peculiarly large ranges—nearly 11′ in both cases. The two curves are so similar and so smooth that it is difficult to believe their exceptional character to be other than a real thing. It will be further noticed, however, that during the month of August generally the diurnal range is distinctly greater than the mean just given. We may, indeed, by grouping the different sets of observations according to months, obtain an indication of the annual change of range. These monthly means of diurnal range are as follows :—

Month.	Number of Sets of Observations.	Mean of Diurnal Range.
July....................	4	8′
August	6	9′.5
September	6	6′.9
October	1	6′.5

Even if we were to throw out the two excessive Korean values, the August mean would still be distinctly higher than the July or September mean, being in fact 8′.8. All this is quite in accordance with well established facts.*

There is one other feature presented by Mr. Tanakadate's curves, which seems worthy of remark. It is that the hour of maximum deviation appears to come distinctly earlier in the later curves. This is especially well marked in the curves on Plate XIV. At Hagi, Hamada, Matsue, indeed, the maximum seems to fall at noon instead

* Compare Table VI. in Art. METEOROLOGY, (Section TERRESTRIAL MAGNETISM), Encyclopaedia Britannica (Ninth Edition.)

of between 1 and 2 p. m. as is usually the case. Whether this is a local peculiarity or not cannot of course be decided from the material to hand; but the otherwise unexceptional character of the observations would lead us to regard this feature also as something that has a real existence. In any case, however, it is of interest to find that satisfactory self-consistent observations of diurnal variation can be made during a rapid magnetic survey, and that, notwithstanding the continual change of locality, the observations are sufficiently precise to indicate the monthly march of the diurnal range. This is, of course, quite to be expected if there is truth in the view so strongly advanced by Balfour Stewart, and supported by Schuster's interesting application of the Gaussian theory—the view that the diurnal variation is chiefly due to electrical movements in the upper regions of the atmosphere. In such a case, local conditions should have a comparatively small influence.

On two occasions the Southern Party took hourly observations of both the Horizontal Force and Dip. The observations are given in detail in the complete tables in Appendix B. As they do not seem to bring out at all distinctly the well known features of the diurnal changes, they can only be regarded as materials for obtaining a specially good mean for the day. They have, consequently, not been shown graphically. The two sets of hourly measurements of the Horizontal Force may be however useful as an indirect means of finding the temperature co-efficients of the moment of the magnet employed. The determination at Hiroshima gave the following expression for the magnetic moment (M) of the magnet in terms of the temperature (t) in degrees centigrade :—

$$M = 438.25 - .286\,(t-28) + .003\,(t-28)^2$$

Section V.

Comparison of Present with Former Results.

This Section will necessarily be a short one, inasmuch as magnetic measurements made in Japan have been somewhat limited in number. Excepting the observations made by Messrs. Sekino and Kōdari, and the few made by Mr. Schütt already referred to, no complete satisfactory observations have been made at any stations out of Tōkyō.

During the various expeditions sent out from year to year by the Tōkyō University for the measurement of the force of gravity by pendulum swinging, attempts were made at the same time to obtain measurements of certain of the magnetic constants. The first expedition of this kind was to the top of Fuji-yama under the direction of Professor Mendenhall. This was in the year 1880. In 1881, Messrs. Tanakadate, Fujisawa, and Tanaka, then students of physics, who had rendered efficient service in the first expedition, proceeded to Sapporo accompanied by Professor Chaplin, and there made like observations on gravity. In 1882, the expedition, consisting of Mr. Tanakadate, and two students of Physics, Messrs. Sakai and Yamaguchi, proceeded to Kagoshima and Naha, in the extreme South-west of Japan. And finally in 1884, Mr. Tanakadate accompanied by Messrs. Sawai, Hayasaki, and Saneyoshi, three students of Physics, proceeded to the Bonin Islands and there swung their pendulums. In all these expeditions magnetic measurements of a kind were made. In the third expedition only was any attempt made to measure the dip. This was done by balancing the inductive action of the earth's field upon an iron wire by means of a current circulating in a helix surrounding the wire. The wire and helix were placed alternately vertical and horizontal ; and the ratio of the current strengths required to effect the balance in the two cases gave the tangent of the

angle of dip. In the other expeditions the dip was not measured. Excepting at the Bonin Islands the horizontal force was measured by swinging various magnets which had been already swung at Tōkyō, and which were again swung at Tōkyō on the return of the Party. The declination was observed at Kagoshima and Naha by means of an ordinary theodolite needle, the compass card being graduated only to half-degrees. At the Bonin Islands, however, the measurements of the horizontal force and of the declination were of a much higher order of merit. Here, indeed, Mr. Tanakadate used his first form of electro-magnetic declinometer; and carried out complete series of deflection and vibration experiments for the determination of the horizontal force. The accounts of these various determinations are given in the Memoirs of the Science Department of the Tōkyō University (Nos. 5 and 7) and the several Appendices to No. 5. For the sake of comparison the results are here reproduced. The Horizontal Forces as given for all except Bonin are calculated on the assumption that the Horizontal Force at Tōkyō was .2964. This was the value obtained by the Bonin Island Party before they started on the expedition, and agrees perfectly with our own value.

Station.	Year.	Dip.	Horizontal Force.	Declination.
Fuji-yama Top	1880.		.2810	
Sapporo	1881.		.2659	
Kagoshima......	1882.	44° 56′	.3131	3° 18′.5 W.
Naha	1882.	38° 19′	.3354	2° 25′.5 ,,
Bonin	1884.		.3166	2° 3′.13 ,,

The latitudes and longitudes of the last three stations are as follows :—

	Latitude N.	Longitude E.
Kagoshima............	31° 35′ 30″	130° 30′ 10″
Naha	26 12 6	127 40 0
Bonin	27 4 11	139 45 45

The value of the Horizontal Force on the top of Fuji-yama is about 5 per cent smaller than would be the case at a normal station at its base. In the absence of measurements of Dip it is impossible to interpret the result. The value for Sapporo is distinctly smaller than the value obtained by us; but it would hardly be safe to draw any conclusion from their comparison. The other stations lie quite outside the region surveyed by us. We may however compare the measurements made with the values given by means of the mean formulae which have been calculated. The results are given in the following table.

	Dip.	Horizontal Force.	Declination.
Kagoshima........	45° 55′.4	.3176	3° 47′.2
Naha	40 13 1	.3396	2° 11′.5
Bonin		.3262	1° 43′.6

If we compare these calculated values with the observed values given above, we see that Naha stands the test best, at least so far as the Horizontal Force and Declination are concerned. Especially as regards the latter, Naha fits in fairly well into the general system.

The circuit of Stations at which Mr. Schütt made his observations in 1880 may be said to touch our route in only one place—and that a very exceptional place, namely, Hakone. A general comparison, however, of the mean values for the district is possible. Thus the Dips give a mean of 49° 41′—about half a degree greater than what is indicated by the isoclinic lines as shown on our chart (Plate VIII) The mean of the declinations is 4° 19′—about 10′ smaller than the value as given by the mean formula. These differences are of a magnitude in no way extraordinary, seeing that the district is so highly volcanic. Hence how much of these differences may be referable to secular variation during eight years, it is quite impossible to say. The

mean Horizontal Force obtained by Mr. Schütt for the same region is .30717, the mean of two determinations for Tōkyō being .30743. This value is so much larger than all other values ever observed by different experimentalists, that we are almost forced to regard it as erroneous, and to refer the discrepancy to some uncorrected instrumental error. No other measurement of the Horizontal Force in or near Tōkyō has given a value greater than .3—usually indeed from 1 to 2 per cent smaller.

So far, then, we have no sure evidence of any marked secular variations. It remains to compare the results of the present survey with those obtained by Messrs. Sekino and Kōdari during the winter months of 1882 and 1883. As already mentioned their Survey was not conducted with a due regard to a fair distribution of Stations over the whole country. Many of our Stations accordingly lie quite outside the routes pursued by them. Indeed of our 81 stations only 27 can be regarded as coincident with any of theirs. A few of these are only roughly coincident, but near enough to allow of a comparison being instituted. The following is the list of these 27 Stations. It will be noticed that after some of these, bracketted names are inserted. These bracketted names give Mr. Sekino's stations which are near enough to ours to warrant a comparison being made.

List of Common Stations.

Hakodate, Aomori, Nobechi, Gonohe, Morioka, Hanamaki, Yamagata, Yonezawa, Niigata, Ebisu, Ishinomaki, Shiogama, Shiraishi, Ishibashi, Shimmachi [Takasaki], Shimizu [Okitsu, Shizuoka], Nagoya, Hyōgo [Kōbe], Kōchi, Hiroshima, Fukuoka [Koyanose, Yamae], Saganoseki, Hososhima, Miyazaki, Nagasaki, Yatsushiro [Miyanobama], Shioya [Komegawaki].

The method of comparison was the simplest that could be imagined. Sekino's values of the magnetic elements at any station were subtracted from the corresponding values obtained on the present Survey. This gave a series of differences for each set of elements. The algebraic sum of these differences may then be assumed to give some hint as to the *existence*, at any rate, of a secular variation.

As regards the Dip there is a distinct tendency for the differences to be negative. That is, our values are on the whole smaller than Sekino's. Thus, out of the 27 Stations, there are only four, for which our values of the dip are greater than his. Dividing the algebraic sum of all the differences by the number of stations we get for the average difference $-8'.8$ or $-7'.0$, according as we include or neglect Hakodate, for which the difference (as much as $-1° 10'.4$) is peculiarly large. We may therefore regard this $-7'.0$ as an indication that the dip is subject to an annual diminution of about $2'$ per year. This does not agree with the results of the observations made at the Naval Observatory,* which hint rather at a rate of increase.

As regards the Horizontal Forces, again, there is a marked tendency for the differences to be positive, six only out of the twenty-seven being negative. The mean difference is + .00091 ; or, in other words, our values are greater than Sekinos by fully one-third per cent. This means an annual rate of increase of .00023. This result is in agreement with the result already indicated by the observations made at the Naval Observatory.

The differences obtained by a comparison of the Total Forces are very nearly as often positive as negative. The general drift, however, is in the negative direction ; so that the Total Force seems to be subject to a mean diminution of $-.00012$ per year.

* As this paper was passing through the press, this Observatory was incorporated with the University, and is now known as the Tōkyō Observatory.

In discussing the Declinations we come face to face with the objection already touched upon (see Section I), namely, the lack in the earlier survey of any attempt at obtaining a really good mean daily value at every station. If we take the values as they are given in Mr. Sekino's lists, we shall find, on the whole, absolutely no indication of a secular variation. The mean difference is $-0'.07$, an altogether insignificant quantity. If, however, we apply probable corrections to the observations, so as to reduce them to the daily mean, the mean difference will be $+ 0'.8$,—that is, an increase of $0'.2$ per year. As regards the declination, then, we may conclude that, within the period beginning 1883 and ending 1887, there is practically no change, or if there is, it is a very small change indeed. It looks almost as if we were just passing through a time of maximum declination.

Finally, it may be remarked that a comparison of our results with those of other experimenters leads to the conclusion that within the present decade there are but small evidences of secular change in the magnetic elements. There is a suggestion that the Dip is diminishing and the Horizontal Force increasing; but the Declination seems to have reached a stationary point.

Appendix A.

Inō Tadayoshi, the Japanese Surveyor and Cartographer.

Inō (originally Jimbō) Kageyu* was born in 1744 in a small village called Sawaramura in the province of Shimōsa, Japan. Inō was the name he acquired by marrying into a family, in accordance with the very usual Japanese custom.

His father-in-law was a *sake* brewer, conducting a business which had descended from father to son for many generations. On his death, affairs were found to be in a very bad state. Inō thereupon applied himself diligently to the business, and through his untiring efforts, combined with strict economy, he gradually amassed considerable wealth. In his fiftieth year, that is about 1794, he transferred the whole business to his son and began his scientific career.

Astronomy was the study to which he devoted himself. The books at his disposal were all in Chinese and contained many obscure passages which he in vain tried to understand. Ultimately he made his way to Yedo, and sat at the feet of the Takahashis, father and son, astronomers to the Shōgun. The elder Takahashi died in 1804, and it was with the younger Takahashi that Inō had most to do. Certain letters written to him by Inō still exist, and their style is such as would naturally be used by one addressing a former teacher.

In 1800 Inō set out by permission of the Government, to survey the Island of Ezo† at his own expense. In the following year he

* Inō (伊能) is the *myōji* (名字) or family name, and Kageyu (勘解由) the *tsūshō* (通稱) or common name. His *jitsu-mei* or *na-nori* (實名, 名乘), which means literally *real name* and was used only on important occasions, was Tadayoshi (忠敬) or Chūkei, as some pronounce it.

† In the earlier pages of this paper, the more familiar spelling Yezo is inadvertently used. Ezo, however, is better phonetically, and is the spelling sanctioned by the Romanization Society of Japan. As the former name of Tōkyō is quite obsolete, it is best to adhere to the western historical spelling Yedo—although it too should be Edo.

was instructed to survey all the coasts and islands of Japan. The survey of the north-eastern coast was finished in 1804, and by 1818 his labours in the field were completed. It should be mentioned, perhaps, that certain parts of the coast were surveyed very imperfectly —such as the eastern and the north-western coasts. Exactly when he died is not known certainly, but for some time after the completion of the survey he seems to have been engaged in the construction of his maps.

The instruments which Inō employed in the survey were destroyed by fire; but in 1828 two instruments, said to be exact copies of the original ones, were made by Ōno Yasaburo, the father of the late engineer who constructed the Mint at Ōsaka. These are now in the possession of the Meteorological Office. A compass-needle, made and used by Inō, has however been preserved by his family.

Ōno's instruments are two, one for measuring azimuths and the other for measuring altitudes. The former is simply a horizontal circular disc of copper 19 inches in diameter, graduated by radial lines into degrees. Seven concentric circles are traced near the extremity of the disk at such distances apart that, when a straight line is engraved joining the point where the inmost circle cuts a given radial line to the point where the outmost circle cuts the next radial line, this so-called diagonal gives by its intersections with the intermediate circles angular intervals corresponding to 10′ or one-sixth of a degree. The graduated circular disc rests on three legs provided with levelling screws. From its centre rises an upright wooden pillar which is surmounted by a tube (or perhaps a telescope) for sighting distant objects. The levelling of the circle is accomplished by means of a brass "plummet" hanging down one side of the upright pillar. The pillar rotates freely, and carries with it a horizontal rod resting on the graduated circle. The position of this rod indicates at once the angle to be read.

The instrument for measuring altitudes is a brass quadrant, 19 inches in radius, with a telescope fixed to one of the straight limbs. The whole is mounted on an upright, wooden pillar resting on three legs. The telescope and quadrant, which move together in a vertical plane about a pivot passing approximately through the centre of gravity, can be clamped in any required position. From the angle of the quadrant a "plummet-line," in the form of a brass rod, hangs. The position of this rod, as it hangs just free of the quadrant arc, indicates the angle to be read. The quadrant is graduated in a manner very similar to the azimuth circle, only to a finer degree of division. The radial lines measure to thirds of a degree; and by means of the "diagonal-scale" arrangement, angles might be read to half-minutes. On the azimuth circle again it would be difficult if not impossible to read to minutes even.

With such instruments did Inō carry out his survey. About 1135 direct measurements of latitudes were taken by means of the quadrant. The distances between successive stations were measured by three distinct methods. Ropes were used as our land surveyors use chains; also a kind of wheel or roller, the number of revolutions of which measured the distance travelled. Then with the azimuth instrument in combination with the compass a triangulation by means of prominent hills and land-marks was carried out. From the distances so obtained, the longitudes seem to have been calculated.

The results of Inō's labours are given in the "Dai Nippon Enkai-jis-soku-roku," or, the Record of the True Survey of the Coasts of Japan (1821, 14 volumes). This treatise existed simply in manuscript till 1870 (Meiji, 3), when it was published in proper book form by the Tōkyō University (Hitotsu-bashi)—at that time known as the Daigaku Nankō. Three kinds of maps were constructed, the largest consisting of 30 different sheets, the medium sized of two,

and the smallest of one. These maps have been the basis of all subsequent ones; and for many places in Japan Inō's measurements of latitude (and longitude) are the only ones which have as yet been made.

On completion of the survey, Takahashi published an epitome of the results in a book having the title, "Inō's Table of Latitudes and Longitudes." From certain remarks in the preface to this work, it would appear that Inō rather doubted the truth of the magnetic variation, and was inclined to refer its appearance in Europe to carelessness either in the construction or handling of the compass needle. There can be little doubt, however, as to the accuracy of Inō's own observation that in Japan at that time the mean direction of magnetic north coincided practically with the direction of geographical north.

According to Inō the mean length of one degree of latitude is 28.2 *ri*. From a copy of the standard *shaku* used by Inō—the original seems to have been lost by fire—this distance has been estimated as equivalent to 110.7 kilometres. The true value is 111 kilometres. The lengths of a degree of longitude in latitudes 35°, 40°, 44° are given as 23.1 *ri*, 21.6 *ri* and 20.285 *ri* respectively. Reduced to kilometres, these are 90.7, 84.8 and 79.66. The true values are 91.08, 85.18, 79.99, differing in no case from Inō's values by as much as one-half per cent.

When we consider the age at which Inō began his scientific career—an age at which most men are thinking of retiring from the busy field of life—and when further we call to mind the rude instruments with which he did his work, we cannot but feel that we have here a man worthy of a high place amongst the scientific leaders of the last generation. In these days of candid criticism, his work has stood the severest tests and remains a grand monument of his perseverance, patience and accuracy. His greatness is now fully appreciated,

and some six or seven years ago received Imperial recognition. The rank of Shō-shi-i (正四位), or Senior 4th class, was at that time conferred on him. Excepting nobles, very few held that rank in the days when Inō flourished, although it is common enough nowadays. Such posthumous honours are, besides, very rare. A stone monument in his honour is very soon to be put up in Tōkyō.

In preparing this short biography of Inō, I have been fortunate in the hearty assistance of Mr. Arai, Superintendent of the Meteorological Office, and of Professor Yamagawa and Mr. Nagaoka of the Imperial University, without whose aid indeed I could have done little or nothing.

Appendix B.

Complete Lists of Observations made.

The following lists contain all the observations made during the survey, together with the dates and hours at which they were taken. The first list contains the Dips, the second the Horizontal Forces, and the third the Declinations.

The only point which calls for remark is in connection with the tabulation of the Horizontal Forces. It will be noticed that for the stations of the Northern Party, an extra column headed M_0 is added. This gives the magnetic moment of the magnet reduced to 0° C. The reductions were made in accordance with the table of corrections supplied by the Kew authorities along with the magnetometer. A glance down the column will show that during the survey the magnet has been subject to a gradual diminution in moment. This is no doubt due largely to the jolting experienced during travelling. On two occasions a somewhat sudden change seems to have occurred—namely between Hakodate and Sapporo, and at Ebisu. In the former case the magnetism of the steamer may have had a permanent effect; while in the latter the cause is no doubt to be traced to the exceptionally high temperature at which the experiments were made. In the circumstances of the case, it is quite probable that the temperature coefficients may have become altered during the survey; and that the table of corrections drawn up at Kew may no longer apply. At Ueda, the value of M_0 differs by one-half per cent. from the values at contiguous stations—much too large a discrepancy to be accounted on any other ground than that of a bad observation. Fortunately Ueda, being in a very hilly and volcanic district, cannot be regarded as an important station.

List I. The Dips.

Station.	Dip.	Date and Hour.		
Ishibashi	50° 7'.3	June	23	7.00 a.m.
Yabuki	50° 57'.3	,,	25	6.42 p.m.
Matsukawa	51° 13'.8	,,	26	6.35 a.m.
Shiraishi	51° 20'.9	,,	28	7.37 a.m.
Shiogama	51° 50'.1	,,	29	7.54 a.m.
	51° 46'.6	,,	,,	2.38 p.m.
	51° 49'.5	,,	30	8.03 a.m.
Ishinomaki	51° 11'.9 ?	July	1	7.14 a.m.
	51° 56'.2	,,	,,	3.26 p.m.
Ichinoseki	52° 32'.0	,,	3	6.58 a.m.
Hanamaki	53° 17'.7	,,	4	8.26 a.m.
Morioka...	53° 10'.9	,,	6	8.10 a.m.
Miyako	53° 36'.0	,,	8	5.35 p.m.
Kuji	54° 7'.4	,,	10	3.15 p.m.
Hachinohe	54° 23'.3	,,	12	7.38 a.m.
Gonohe	54° 39'.4	,,	,,	4.48 p.m.
Nobechi...	54° 44'.1	,,	13	5.20 p.m.
Aomori	54° 47'.1	,,	15	7.43 a.m.
Hakodate	55° 35'.5	,,	16	3.27 p.m.
Sapporo...	56° 41'.5	,,	19	4.38 p.m.
Kiitup	56° 48'.4	,,	21	5.33 p.m.
Nemuro...	57° 7'.1	,,	25	4.28 p.m.
Hirosaki	54° 24'.6	,,	29	5.21 p.m.
Ōdate	54° 12'.6	,,	31	7.45 a.m.
Nōshiro...	54° 17'.8	August	1	5.38 p.m.
Akita	53° 37'.6	,,	3	7.15 a.m.
Kariwano	53° 30'.5	,,	,,	6.50 p.m.
Yokote	53° 5'.0	,,	4	afternoon.
Innai	52° 51'.1	,,	5	4.05 p.m.

Station.	Dip.	Date and Hour.		
Shinjō	52° 37′.2	August	7	6.30 a.m.
Sakata	52° 51′.9	,,	8	6.49 a.m.
Yamagata	52° 1′.6	,,	9	5.20 p.m.
Yonezawa	51° 31′.6	,,	11	4.58 p.m.
Oguni	51° 47′.2	,,	13	7.20 a.m.
Nakajō	51° 49′.2	,,	14	7.15 a.m.
Ebisu	52° 3′.7	,,	15	11.15 a.m.
Niigata	51° 56′.9	,,	16	7.15 a.m.
Kashiwazaki	51° 55′.4	,,	17	5.25 p.m.
Sekiyama	50° 55′.7	,,	18	6.15 p.m.
Ueda	50° 3′.3	,,	20	6.32 p.m.
Takanomachi	49° 40′.6	,,	21	6.15 p.m.
Nagasawa	49° 8′.8	,,	22	6.35 p.m.
Kōfu...	50° 5′.7	,,	23	5.43 p.m.
Hara	48° 33′.3	,,	25	8.01 a.m.
Hakone	47° 25′.7	,,	,,	6.31 p.m.
Ōtsu...	48° 33′.8	,,	26	6.15 p.m.
Hōjō...	48° 32′.6	,,	28	7.33 a.m.
Katsuura	48° 30′.6	,,	30	6.53 a.m.
Tōgane	49° 2′.5	,,	31	8.13 a.m.
Chōshi	48° 43′.1	September	1	8.15 a.m.
Kioroshi...	49° 32′.5	,,	2	8.18 a.m.
Shimmachi	49° 56′.0	,,	25	5.43 p.m.
Ōmiya	49° 39′.0	,,	26	11.02 a.m.
Shimoda	47° 57′.4	June	23	8.45 a.m.
	47° 59′.6	,,	,,	1.15 p.m.
	48° 1′.5	,,	,,	night.
Shimizu...	48° 39′.8	,,	25	5.28 p.m.
	48° 43′.5	,,	26	11.15 a.m.
	48° 40′.2	,,	,,	8.46 p.m.
	48° 41′.8	,,	27	10.18 a.m.

Station.	Dip.	Date and Hour.		
Nagoya	48° 58'.7	June	29	5.57 p.m.
	48° 57'.7	„	30	10.30 a.m.
	48° 58'.3	„	„	5.12 p.m.
Kamiyashiro	48° 16'.0	July	4	8.03 a.m.
	48° 17'.2	„	„	3.35 p.m.
	48° 17'.2	„	„	7.08 „
Nagahama	49° 15'.6	„	6	11.21 a.m.
	49° 19'.0	„	„	4.30 p.m.
	49° 18'.3	„	7	8.20 a.m.
Hyōgo	48° 42'.9	„	8	3.49 p.m.
	48° 43'.1	„	„	9.22 „
	48° 41'.0	„	9	10.48 a.m.
Tokushima	48° 0'.8	„	10	4.49 p.m.
	48° 1'.3	„	11	8.53 a.m.
	48° 0'.0	„	„	11.10 „
	47° 59'.6	„	„	1.04 p.m.
Kōchi	47° 32'.2	„	17	1.45 a.m.
	47° 33'.2	„	„	6.27 „
	47° 30'.6	„	„	10.25 „
Minabe	47° 46'.8	„	21	7.47 a.m.
	47° 45'.4	„	22	7.33 „
	47° 45'.8	„	„	8.28 „
	47° 46'.3	„	„	9.31 „
	47° 46'.0	„	„	10.30 „
	47° 46'.7	„	„	11.29 „
	47° 46'.5	„	„	0.31 p.m.
	47° 45'.5	„	„	1.28 „
	47° 47'.0	„	„	2.36 „
	47° 46'.2	„	„	3.20 „
	47° 47'.3	„	„	4.27 „
	47° 45'.9	„	„	5.27 „
	47° 46'.5	„	„	7.54 „

Station.	Dip.	Date and Hour.			
Okayama … … … … … …	48° 47.′5	July	26	6.48	a.m.
	48° 47.′3	,,	,,	0.42	p.m.
	48° 48.′9	,,	,,	4.38	,,
Hiroshima … … … … … …	48° 39.′4	,,	28	11.29	a.m.
	48° 40.′4	,,	,,	5.03	p.m.
	48° 40.′4	,,	30	8.05	a.m.
	48° 38.′0	,,	,,	11.25	,,
Wakwan (Korea) … … …	50° 10.′1	August	6	7.33	a.m.
	50° 9.′3	,,	,,	11.42	,,
	50° 8.′5	,,	,,	4.33	p.m.
Meho (Korea) … … … …	50° 27.′0	,,	10	7.14	p.m.
	50° 28.′9	,,	11	8.29	a.m.
	50° 26.′1	,,	,,	11.53	,,
	50° 26.′0	,,	,,	3.48	p.m.
Pusan (Korea) … … … …	50° 0.′6	,,	13	10.36	a.m.
	50° 1.′7	,,	,,	1.04	p.m.
	50° 1.′4	,,	,,	4.58	,,
Ōmizutare (Korea)… … …	50° 3.′1	,,	14	5.37	p.m.
Kurosaki (Korea) … … …	50° 7.′8	,,	15	9.50	a.m.
	50° 6.′6	,,	,,	0.13	p.m.
Kurosaki Cape (Korea)… …	49° 55.′3	,,	,,	3.32	p.m.
Shiinokijima Cape (Korea)…	51° 2.′7	,,	,,	5.34	p.m.
Shiinokijima … … … …	50° 7.′5	,,	,,	8.14	p.m.
Fukuoka … … … … … …	48° 8.′3	,,	22	8.16	a.m.
	48° 4.′0	,,	,,	11.31	,,
	48° 7.′1	,,	,,	4.21	p.m.
Nakatsu . … … … … …	48° 1.′6	,,	24	2.25	p.m.
	48° 1.′0	,,	,,	5.02	,,
	48° 3.′4	,,	25	9.20	a.m.
Saganoseki … … … … …	47° 21.′6	,,	28	7.09	a.m.

Station.	Dip.	Date and Hour.			
Saganoseki *(continued)* ...	47° 19'.3	August	28	0.04	p.m.
	47° 21'.7	„	„	4.19	„
Hichiyamura	46° 26'.6	„	29	4.39	p.m.
	46° 27'.9	„	30	9.06	a.m.
	46° 28'.6	„	„	11.22	„
Miyazaki	45° 50'.0	„	31	0.52	p.m.
	45° 51'.7	„	„	4.21	„
	45° 52'.1	September	2	8.16	a.m.
Yatsushiro	46° 39'.8	„	6	0.04	p.m.
	46° 40'.3	„	„	5.03	„
	46° 39'.9	„	7	10.26	a.m.
Nagasaki	46° 57'.1	„	10	10.33	a.m.
Hagi	48° 14'.6	„	13	11.39	a.m.
	48° 14'.0	„	„	1.37	p.m.
	48° 13'.6	„	„	5.03	„
Hamada	50° 12'.8	„	16	5.34	p.m.
	50° 11'.9	„	17	8.01	a.m.
	50° 11'.9	„	„	0.15	p.m.
Hamada Bridge	49° 50'.5	„	„		
Asaimura Ōte	49° 49'.9	„	„		
Kurokawa	50° 4'.8	„	„		
Sumiyoshiyama	49° 54'.7	„	„		
Matsue	50° 0'.9	„	20	2.47	p.m.
	50° 1'.6	„	„	5.52	„
	50° 3'.0	„	21	7.46	a.m.
Imaichi	50° 14'.2	„	22	9.35	a.m.
	50° 12'.0	„	„	0.17	p.m.
	50° 13'.1	„	„	5.50	„
Kanōmura	49° 42'.3	„	25	0.53	p.m.
	49° 42'.7	„	„	5.28	„
	49° 45'.3	„	26	8.38	a.m.

Station.	Dip.	Date and Hour.
Koyamamura	49° 54.1	September 26 2.12 p.m.
	49° 55.0	,, ,, 5.50 ,,
	49° 53.2	,, 27 8.34 a.m.
	49° 53.0	,, ,, 11.56 ,,
Maizuru	49° 24.9	,, 30 0.34 p.m.
	49° 24.3	,, ,, 5.33 ,,
	49° 25.2	October 1 8.54 a.m.
Obama	49° 30.7	,, 2 0.21 p.m.
	49° 30.4	,, ,, 5.03 ,,
	49° 30.2	,, 3 9.22 a.m.
Shioyaura	50° 31.1	,, 4 2.49 p.m.
	50° 31.5	,, 5 7.58 a.m.
	50° 31.8	,, ,, 9.17 ,,
	50° 32.6	,, ,, 9.44 ,,
	50° 32.0	,, ,, 10.27 ,,
	50° 33.6	,, ,, 11.14 ,,
	50° 32.9	,, ,, 11.50 ,,
	50° 30.0	,, ,, 0.51 p.m.
	50° 29.6	,, ,, 1.31 ,,
	50° 29.6	,, ,, 2.12 ,,
	50° 29.2	,, ,, 2.48 ,,
	50° 30.0	,, ,, 3.31 ,,
	50° 29.0	,, ,, 4.20 ,,
	50° 29.9	,, ,, 4.48 ,,
	50° 29.6	,, ,, 5.26 ,,
Nanao	51° 14.8	,, 8 1.54 p.m.
	51° 15.4	,, ,, 5.05 ,,
Tōkyō (W)...	49° 10.8	November 8 11.09 a.m.
	49° 11.1	,, ,, 2.35 p.m.
,, (E)	49° 13.1	,, 10 11.19 a.m.
	49° 12.9	,, ,, 2.31 p.m.

List II. The Horizontal Forces, Temperatures, and Magnetic Moments.

Station.	Horiz. Force.	M.	Temp.	M₀.	Date & Hour.		
Ishibashi	.29143	937.15	21°.0 c.	944.37	June	23	6.00 a.m.
Yabuki	.28952	938.60	20°.4	945.54	,,	25	8.00 a.m.
Matsukawa	.28785	937.78	20°.0	944.60	,,	26	7.00 a.m.
Shiraishi	.28830	936.72	20°.1	943.58	,,	28	7.30 a.m.
Shiogama	.28755	935.41	22°.6	943.21	,,	29	9.00 a.m.
	.28715	933.79	22°.7	941.59	,,	30	6.00 ,,
Ishinomaki	.28777	936.53	21°.2	943.82	July	1	8.00 a.m.
	.28728	937.28	22°.3	944.50	,,	,,	5.30 p.m.
Ichinoseki	.28426	936.20	18°.0	942.65	,,	3	7.30 a.m.
Hanamaki	.28032	938.04	18°.4	944.28	,,	5	7.30 a.m.
Morioka	.28180	937.26	20°.8	944.39	,,	6	8.00 a.m.
Miyako	.27876	934.70	23°.2	942.76	,,	8	4.00 p.m.
Kuji	.27983	935.56	23°.4	943.67	,,	10	4.00 p.m.
Hachinohe	.27687	937.02	22°.2	944.69	,,	12	8.00 a.m.
Gonohe	.27615	933.75	24°.3	942.24	,,	,,	5.00 p.m.
Nobechi	.27576	934.52	24°.1	942.91	,,	13	6.00 p.m.
Aomori	.27727	935.88	22°.7	943.73	,,	15	8.00 a.m.
Hakodate	.27318	935.75	22°.3	943.19	,,	16	5.00 p.m.
Sapporo	.26766	931.52	25°.2	940.29	,,	19	5.30 p.m.
Kiitup	.26667	933.83	23°.1	941.82	,,	24	4.30 p.m.
Nemuro	.26025	931.86	26°.3	941.07	,,	25	4.00 p.m.
Hirosaki	.27819	932.97	26°.3	942.19	,,	29	4.30 p.m.
Ōdate	.27743	935.34	20°.3	942.24	,,	31	6.00 a.m.
Nōshiro	.27854	930.85	31°.0	941.89	Aug.	1	4.00 p.m.
Akita	.28275	932.18	25°.2	941.00	,,	3	6.15 a.m.
Kariwano	.28224	934.50	26°.4	943.78	,,	4	6.30 a.m.
Yokote	.28309	932.91	26°.4	942.17	,,	5	7.00 a.m.
Innai	.28226	931.47	28°.0	941.35	,,	,,	4.30 p.m.

Station.	Horiz. Force.	M.	Temp.	M_{0}.	Date & Hour.		
Shinjō … … … …	.28524	931.43	27°.4 c.	941.07	Aug.	7	7.00 p.m.
Sakata … … … …	.28465	931.94	27°.6	941.65	,,	8	7.00 a.m.
Yamagata … … …	.28651	932.80	25°.4	941.63	,,	10	7.00 a.m.
Yonezawa … … …	.28836	931.28	30°.3	942.04	,,	11	5.30 p.m.
Oguni … … … …	.28748	933.58	21°.8	941.02	,,	13	7.00 a.m.
Nakajō … … … …	.28762	930.23	28°.3	940.20	,,	14	7.30 a.m.
Ebisu … … … …	.28950	923.57	35°.0	936.16	,,	15	noon
Niigata … … …	.28699	928.32	28°.1	938.19	,,	16	7.30 a.m.
Kashiwazaki … …	.28878	927.66	29°.7	938.17	,,	17	6.00 a.m.
Sekiyama … … …	.29152	930.36	27°.1	939.83	,,	19	8.15 a.m.
Ueda … … … …	.29629	936.59	21°.8	944.06	,,	21	6.00 a.m.
Takanomachi … …	.29502	928.90	26°.6	938.19	,,	,,	6.00 p.m.
Kōfu … … … …	.29261	926.19	33°.4	938.40	,,	23	4.30 p.m.
Hara … … … …	.30150	929.69	25°.0	938.37	,,	25	6.35 a.m.
Hakone… … … …	.30490	929.16	25°.4	937.97	,,	,,	5.00 p.m.
Ōtsu … … … …	.29655	927.19	29°.2	937.45	,,	26	5.00 p.m.
Hōjō … … … …	.29639	927.85	27°.3	937.41	,,	28	6.00 a.m.
	.29621	926.75	29°.4	937.10	,,	,,	4.00 p.m.
Katsuura … … …	.29607	929.20	25°.2	937.95	,,	29	5.30 p.m.
Tōgane… … … …	.29533	929.22	22°.4	936.87	,,	31	6.15 a.m.
Chōshi … … … …	.29696	928.60	27°.0	938.02	Sept.	1	7.20 a.m.
Kioroshi … … …	.29377	930.46	23°.8	938.69	,,	2	7.00 a.m.
Shimmachi… … …	.29367	929.65	19°.0	935.86	,,	25	5.00 p.m.
Ōmiya … … … …	.29428	928.92	20°.1	935.73	,,	26	10.20 a.m.
Shimoda … … …	.29993	443.54	20°.7		June	23	10.30 a.m.
	.30001	443.42	21°.2		,,	,,	3.13 p.m.
	.29980	441.17	21°.8		,,	,,	8.21 ,,
Shimizu … … …	.30164	442.19	21°.3		,,	25	afternoon
	.30024	443.39	21°.4		,,	,,	,,
	.29983	443.19	26°.0		,,	26	2.10 p.m.
	.30013	441.46	25°.2		,,	,,	4.31 ,,

Station.	Horiz. Force.	M.	Temp.	M_o.	Date & Hour.
Shimizu (*continued*)	.29929	441.95	25°.1 c.		June 27 8.48 a.m.
Nagoya	.29874	442.91	26°.9		„ 30 9.02 a.m.
	.29849	442.67	29°.7		„ „ 11.44 „
	.29990	442.11	24°.2		„ „ 6.26 p.m.
Kamiyashiro	.30225	441.58	24°.7		July 4 8.46 a.m.
	.30175	441.89	26°.3		„ „ 11.25 „
	.30196	442.11	24°.8		„ „ 5.25 p.m.
Nagahama	.30050	440.18	26°.8		„ 6 0.53 p.m.
	.30017	441.05	23°.7		„ „ 6.03 „
	.30012	440.81	24°.9		„ 7 9.02 a.m.
Hyōgo	.30280	440.59	29°.3		„ 8 2.07 p.m.
	.30259	441.10	26°.5		„ „ 4.48 „
	.30266	442.545	26°.4		„ 9 9.10 a.m.
	.30373	439.59	31°.6		„ „ 1.20 p.m.
Tokushima	.30730	437.56	27°.8		„ 10 6.51 p.m.
	.30516	440.58	27°.3		„ 11 9.53 a.m.
	.30562	440.29	28°.2		„ „ 0.10 p.m.
Kōchi	.31098	436.69	27°.9		„ 17 0.16 a.m.
	.31066	436.72	29°.0		„ „ 7.48 „
	.30855	439.08	31°.7		„ „ 11.55 „
Minabe	.30476	439.64	31°.6		„ 21 9.57 a.m.
	.30509	439.25	31°.5		„ „ 11.30 „
	.30494	438.86	32°.8		„ „ 2.55 p.m.
	.30496	439.48	30°.7		„ „ 5.56 „
Okayama	.30431	438.90	31°.4		„ 26 9.00 a.m.
	.30461	438.45	34°.2		„ „ 1.28 p.m.
	.30474	438.60	33°.0		„ „ 5.07 „
Hiroshima	.30732	438.76	34°.15		„ 28 0.41 p.m.
	.30678	438.56	35°.2		„ „ 4.13 „
	.30747	441.08	25°.0		„ 29 6.32 a.m.
	.30722	440.95	26°.9		„ „ 7.35 „

Station.	Horiz. Force.	M.	Temp.	M_0.	Date & Hour.
Hiroshima (*continued*)	.30688	440.19	29°.9 c.		July 29 8.37 a.m.
	.307185	439.17	34°.15		,, ,, 9.32 ,,
	.30730	438.34	35°.75		,, ,, 10.34 ,,
	.30719	437.96	37°.0		,, ,, 11.35 ,,
	.30731	438.12	36°.0		,, ,, 0.32 p.m.
	.30709	438.11	36°.2		,, ,, 1.38 ,,
	.30724	437.96	35°.75		,, ,, 2.32 ,,
	.30726	438.48	35°.1		,, ,, 3.32 ,,
	.30682	439.18	33°.3		,, ,, 4.35 ,,
	.30712	438.945	30°.5		,, ,, 5.32 ,,
	.30691	439.96	29°.0		,, ,, 6.47 p.m.
Wakwan (Korea) ...	.30447	439.68	31°.9		Aug. 6 10.06 a.m.
	.30470	438.87	32°.7		,, ,, 1.39 p.m.
	.30485	440.07	27°.6		,, ,, 5.19 ,,
Meho (Korea)	.30694	440.52	26°.8		,, 11 6.34 a.m.
	.30603	439.33	32°.8		,, ,, 11.01 ,,
	.30691	437.50	36°.1		,, ,, 2.08 p.m.
Pusan (Korea)	.30909	438.63	29°.2		,, 13 10.56 a.m.
	.30923	436.81	34°.3		,, ,, 2.23 p.m.
	.30891	438.18	29°.2		,, ,, 4.06 ,,
Kurosaki	.30616	437.56	32°.6		,, 15 11.24 a.m.
Shiinokijima	.30518	438.57	28°.3		,, ,, 6.40 p.m.
Fukuoka	.31049	438.14	30°.5		,, 22 7.44 a.m.
	.31008	437.29	32°.5		,, ,, 10.40 ,,
	.30999	436.69	34°.0		,, ,, 3.24 p.m.
Nakatsu	.30992	436.27	35°.8		,, 24 1.26 p.m.
	.31002	436.78	34°.9		,, ,, 4.23 ,,
	.31006	438.43	28°.8		,, 25 8.27 a.m.
Saganoseki	.31124	437.98	30°.7		,, 28 8.28 a.m.
	.31138	437.80	32°.5		,, ,, 11.10 ,,
	.31148	437.68	30°.4		,, ,, 3.37 p.m.

Station.	Horiz. Force.	M.	Temp.	M_0.	Date & Hour.
Hichiyamura	.31392	437.48	31°.4 c.		Aug. 29 3.49 p.m.
	.31349	439.11	27°.8		,, 30 8.07 a.m.
	.31361	437.44	34°.0		,, ,, 10.45 ,,
Miyazaki	.31608	438.035	31°.4		Sept. 1 8.53 a.m.
	.31575	437.41	32°.9		,, ,, 9.31 ,,
	.31567	437.07	34°.8		,, ,, 10.12 ,,
	.31541	436.945	34°.6		,, ,, 10.51 ,,
	.31523	437.01	34°.8		,, ,, 11.31 ,,
	.31530	437.01	34°.8		,, ,, 0.11 p.m.
	.31590	437.07	34°.0		,, ,, 1.30 ,,
	.31589	436.14	34°.5		,, ,, 2.10 ,,
	.31549	436.96	33°.8		,, ,, 2.50 ,,
	.31545	437.19	33°.35		,, ,, 3.31 ,,
	.31603	437.05	33°.15		,, ,, 4.12 ,,
	.31567	437.58	31°.6		,, ,, 4.49 ,,
	.31557	437.94	29°.6		,, ,, 5.29 ,,
	.31523	438.44	28°.35		,, ,, 6.09 ,,
Yatsushiro...	.31503	437.26	32°.6		,, 6 10.48 a.m.
	.31504	437.55	31°.8		,, ,, 4.20 p.m.
	.31463	437.16	31°.5		,, ,, 9.33 ,,
Nagasaki	.31392	437.055	33°.9		,, 10 9.43 a.m.
Hagi	.30873	437.16	32°.2		,, 13 11.20 a.m.
	.30905	437.77	29°.4		,, ,, 0.48 p.m.
	.30881	438.74	26°.1		,, ,, 4.18 ,,
Hamada	.30034	438.88	26°.0		,, 16 4.32 p.m.
	.30039	440.80	18°.9		,, 17 7.12 a.m.
	.30063	437.42	31°.2		,, ,, 11.21 ,,
Matsue...	.30001	438.70	26°.5		,, 20 1.04 p.m.
	.30002	438.65	26°.0		,, ,, 4.55 ,,
	.30016	440.99	19°.9		,, 21 7.00 a.m.
Imaichi...	.29999	438.88	24°.2		,, 22 8.53 a.m.

Station.	Horiz. Force.	M.	Temp.	M_0.	Date & Hour.
Imaichi (*continued*)	.30056	438.23	29°.3 c.		Sept. 22 11.40 a.m.
	.30010	439.03	23°.7		„ „ 5.14 p.m.
Kanōmura	.30161	439.88	21°.9		„ 25 0.09 p.m.
	.30154	440.05	20°.9		„ „ 1.42 „
	.30116	441.96	14°.7		„ 26 7.29 a.m.
Koyamamura	.29978	438.95	24°.3		„ „ 1.51 p.m.
	.29959	439.195	21°.9		„ „ 5.07 „
	.30023	439.48	21°.3		„ 27 7.54 a.m.
	.29976	437.90	28°.7		„ „ 10.26 „
Maizuru	.30092	439.01	25°.3		„ 30 11.49 a.m.
	.30094	439.66	21°.5		„ „ 4.45 p.m.
	.30105	441.08	16°.8		Oct. 1 7.07 a.m.
Obama	.30001	438.87	24°.7		„ 2 11.34 a.m.
	.30039	439.56	21°.2		„ „ 4.33 p.m.
	.30014	441.40	16°.8		„ 3 8.00 a.m.
Shioyaura	.29542	438.56	26°.9		„ 4 0.02 p.m.
	.29549	438.50	26°.0		„ „ 4.53 „
	.29575	439.74	20°.6		„ 5 7.26 a.m.
Nanao	.29379	441.36	16°.8		„ 8 9.05 a.m.
	.29401	440.87	19°.2		„ „ 0.59 p.m.
	.29416	440.00	20°.6		„ „ 4.33 „
Tōkyō (W)	.29682	438.87	23°.4		Nov. 8 11.52 a.m.
	.29670	438.45	25°.4		„ „ 1.39 p.m.
	.29633	440.49	17°.1		„ „ 4.20 „
„ (E)	.29622	439.73	19°.7		„ 10 9.29 a.m.
	.29638	439.40	22°.1		„ „ 1.01 p.m.
	.29506	441.04	16°.8		„ „ 4.15 „

List III. The Declinations.

Station.	Declin.	How taken.	Date & Hour.
Ishibashi	4° 1′.6	Polaris	June 23 10.31 p.m.
	3° 59′.7	Jupiter	,, ,, 10.15 ,,
Yabuki	4° 29′.5	Sun	,, 25 7.24 a.m.
Matsukawa	4° 29′.6	Polaris	,, ,, 11.09 p.m.
Shiraishi...	4° 33′.7	Sun	,, 28 10.14 a.m.
Shiogama	5° 9′.0	Jupiter	,, 29 9.29 p.m.
	5° 5′.5	Polaris	,, ,, 9.42 ,,
	5° 10′.2	Sun	,, 30 6.37 a.m.
Ishinomaki	4° 56′.5	Sun	June 30 6.00 ,,
	5° 3′.3	,,	July 1 9.00 a.m.
	5° 5′.0	Spica	,, ,, 8.54 p.m.
Hanamaki...	5° 25′.2	Polaris	,, 4 8.50 p.m.
Morioka	4° 59′.9	Polaris	,, 5 8.40 p.m.
	5° 5′.0	Sun	,, ,, 5.20 ,,
Miyako	5° 51′.0	Polaris	,, 8 9.42 p.m.
	5° 46′.0	Sun	,, ,, 5.01 ,,
	5° 44′.0	,,	,, ,, 7.05 a.m.
Kuji	4° 38′.5	Sun	,, 10 5.36 p.m.
	4° 36′.8	Venus	,, ,, 8.24 ,,
Hachinohe	4° 32′.6	Polaris	,, 11 11.10 p.m.
Gonohe	4° 57′.7	Polaris	,, 12 9.32 p.m.
	5° 12′.9	Sun	,, ,, 3.32 ,,
Nobechi	5° 31′.7	Polaris	,, 13 8.55 p.m.
Aomori	5° 22′.2	Polaris	,, 14 9.20 p.m.
Hakodate	5° 33′.7	Sun	,, 17 5.13 p.m.
	5° 31′.3	Polaris	,, 21 10.12 ,,
Sapporo	5° 59′.0	Sun	,, 20 6.36 a.m.
Kiitup...	4° 46′.0	Sun	,, 24 4.03 p.m.
Nemuro	4° 21′.3	Polaris	,, 25 9.31 p.m.

Station.	Declin.	How taken.	Date & Hour.		
Hirosaki	5° 23'.3	Polaris	July	29	8.11 p.m.
Ōdate	5° 15'.7	Polaris	,,	30	8.09 p.m.
Nōshiro	5° 32'.3	Polaris	August	1	9.13 p.m.
Akita	5° 9'.3	Polaris	,,	2	9.34 p.m.
Kariwano	5° 9'.2	Polaris	,,	3	8.40 p.m.
Yokote	5° 23'.1	Polaris	,,	4	8.17 p.m.
Innai	5° 9'.7	Sun	,,	6	7.08 a.m.
Shinjō	4° 56'.9	Sun	,,	7	8.09 a.m.
Sakata	5° 11'.0	Polaris	,,	7	8.45 p.m.
Yamagata	4° 32'.6	Polaris	,,	9	9.06 p.m.
Yonezawa	4° 32'.6	Polaris	,,	11	7.56 p.m.
Oguni	4° 51'.4	Polaris	,,	12	8.19 p.m.
Nakajō	5° 0'.3	Polaris	,,	13	9.56 p.m.
Ebisu	5° 4'.3	Sun	,,	15	11.47 a.m.
Niigata	5° 9'.4	Polaris	,,	,,	9.05 p.m.
Kashiwazaki	5° 3'.4	Polaris	,,	17	10.14 p.m.
Sekiyama	4° 36'.2	Polaris	,,	19	3.36 a.m.
Takanomachi	4° 3'.8	Sun	,,	21	3.53 p.m.
Kōfu	4° 32'.4	Polaris	,,	23	8.24 p.m.
Hara	4° 29'.4	Polaris	,,	24	8.00 p.m.
Hakone	2° 24'.0	Sun	,,	25	4.13 p.m.
Ōtsu	4° 19'.2	Polaris	,,	26	8.40 p.m.
Hōjō	4° 23'.4	Polaris	,,	27	8.18 p.m.
Katsuura	4° 1'.5	Polaris	,,	29	8.18 p.m.
Tōgane	4° 13'.6	Polaris	,,	30	8.31 p.m.
Chōshi	4° 19'.2	Polaris	,,	31	10.45 p.m.
Kioroshi	4° 25'.9	Polaris	September	1	10.55 p.m.
Shimmachi	4° 45'.7	Polaris	,,	25	7.28 p.m.
Ōmiya	4° 35'.9	Sun	,,	26	9.51 a.m.
Shimoda	4° 14' 8″		June	23	11.30 a.m.
	4° 17' 13″		,,	,,	1.33 p.m.

Station.	Declin.	Date & Hour.
Shimoda (*continued*) ...	4° 13′ 5″	June 23 5.40 p.m.
Shimizu	4° 0′ 58″	,, 25 6.08 p.m.
	3° 59′ 30″	,, 27 7.55 a.m.
	4° 1′ 58″	,, ,, 10.43 ,,
Nagoya	4° 31′ 58″	,, 29 8.03 p.m.
	4° 26′ 38″	,, 30 8.04 a.m.
	4° 26′ 53″	,, ,, 11.00 ,,
	4° 34′ 8″	,, ,, 0.17 p.m.
	4° 35′ 45″	,, ,, 2.55 ,,
	4° 36′ 33″	,, ,, 7.25 ,,
Kamiyashiro	4° 19′ 29″	July 4 9.01 a.m.
	4° 22′ 52″	,, ,, 10.28 ,,
	4° 27′ 2″	,, ,, 0.25 p.m.
	4° 19′ 22″	,, ,, 6.07 ,,
	4° 24′ 2″	,, ,, 11.00 ,,
Nagahama	4° 49′ 48″	,, 6 1.23 p.m.
	4° 48′ 45″	,, ,, 2.52 ,,
	4° 42′ 58″	,, ,, 7.59 ,,
	4° 42′ 46″	,, 7 8.41 a.m.
	4° 46′ 1″	,, ,, 10.11 ,,
	4° 47′ 26″	,, ,, 10.49 ,,
Hyōgo...	4° 33′ 46″ ?	,, 8 1.08 p.m.
	4° 33′ 51″	,, ,, 7.24 ,,
	4° 33′ 36″	,, ,, 8.51 ,,
	4° 33′ 34″	,, 9 10.05 a.m.
	4° 38′ 52″	,, ,, 2.07 p.m.
	4° 38′ 22″	,, ,, 4.16 ,,
Tokushima	4° 26′ 52″	,, 10 5.36 p.m.
	4° 24′ 9″	,, 11 9.24 a.m.
	4° 28′ 57″	,, ,, 11.29 ,,
	4° 28′ 34″	,, ,, 4.09 p.m.

Station.	Declin.	Date & Hour.
Kōchi	4° 19′ 44″	July 16 11.20 p.m.
	4° 18′ 31″	„ 17 9.03 a.m.
	4° 24′ 59″	„ „ 0.16 p.m.
Minabe	4° 30′ 35″	„ 21 9.13 a.m.
	4° 34′ 10″	„ „ 10.44 „
	4° 34′ 2″	„ „ 2.26 p.m.
	4° 30′ 27″	„ „ 6.24 „
	4° 27′ 20″	„ 22 6.56 a.m.
	4° 27′ 2″	„ „ 7.48 „
	4° 27′ 10″	„ „ 8.40 „
	4° 29′ 10″	„ „ 9.45 „
	4° 31′ 20″	„ „ 10.43 „
	4° 33′ 5″	„ „ 11.43 „
	4° 34′ 20″	„ „ 0.48 p.m.
	4° 35′ 00″	„ „ 1.42 „
	4° 33′ 45″	„ „ 2.51 „
	4° 32′ 27″	„ „ 3.44 „
	4° 30′ 50″	„ „ 4.40 „
	4° 30′ 37″	?
	4° 31′ 2″	„ „ 7.34 „
Okayama	4° 36′ 28″	„ 25 5.42 p.m.
	4° 37′ 28″	„ „ 7.06 „
	4° 35′ 6″	„ 26 7.14 a.m.
	4° 36′ 1″	„ „ 10.02 „
	4° 31′ 31″	„ „ 11.35 „
	4° 39′ 3″	„ „ 3.51 p.m.
	4° 36′ 51″	„ „ 5.33 „
Hiroshima	4° 31′ 17″	„ 28 11.57 a.m.
	4° 29′ 54″	„ „ 3.50 p.m.
	4° 28′ 32″	„ „ 9.35 „
	4° 27′ 22″	„ 29 5.33 „

Station.	Declin.	Date & Hour.		
Hiroshima (*continued*)	4° 23′ 39″	July	29	7.57 a.m.
	4° 25′ 37″	,,	,,	9.52 ,,
	4° 30′ 14″	,,	,,	11.54 ,,
	4° 31′ 57″	,,	,,	1.54 p.m.
	4° 30′ 39″	,,	,,	2.53 ,,
	4° 28′ 9″	,,	,,	4.53 ,,
	4° 28′ 37″	,,	,,	7.07 ,,
Wakwan	4° 20′ 47″	August	6	8.07 a.m.
	4° 20′ 37″	,,	,,	8.31 ,,
	4° 26′ 28″	,,	,,	10.32 ,,
	4° 30′ 33″	,,	,,	0.05 p.m.
	4° 31′ 23″	,,	,,	2.03 ,,
	4° 26′ 8″	,,	,,	4.34 ,,
	4° 24′ 33″	,,	,,	6.02 ,,
	4° 24′ 43″	,,	,,	9.51 ,,
Mēho	4° 42′ 9″	,,	10	7.32 p.m.
	4° 41′ 52″	,,	11	5.50 a.m.
	4° 38′ 42″	,,	,,	7.59 ,,
	4° 38′ 32″	,,	,,	8.45 ,,
	4° 44′ 2″	,,	,,	10.41 ,,
	4° 48′ 37″	,,	,,	1.29 p.m.
	4° 46′ 47″	,,	,,	2.30 ,,
	4° 44′ 34″	,,	,,	4.07 ,,
Pusan	3° 34′ 30″	,,	13	9.04 a.m.
	3° 38′ 52″	,,	,,	11.18 ,,
	3° 40′ 20″	,,	,,	1.19 p.m.
	3° 40′ 18″	,,	,,	1.46 ,,
	3° 38′ 45″	,,	,,	2.42 ,,
	3° 36′ 20″	,,	,,	5.16 ,,
	3° 36′ 58″	,,	,,	9.00 ,,
Kurosaki	4° 35′ 15″	,,	15	11.39 a.m.

Station.	Declin.	Date & Hour.		
Kurosaki (*continued*)...	4° 36′ 15″	August	15	1.19 p.m.
Shiinokijima	4° 2′ 40″	,,	,,	7.04 p.m.
Fukuoka	4° 18′ 39″	,,	21	9.37 p.m.
	4° 17′ 42″	,,	,,	10.09 ,,
	4° 14′ 47″	,,	22	7.16 a.m.
	4° 14′ 49″	,,	,,	8.28 ,,
	4° 17′ 26″	,,	,,	9.25 ,,
	4° 20′ 44″	,,	,,	11.00 ,,
	4° 22′ 42″	,,	,,	1.57 p.m.
	4° 21′ 27″	,,	,,	2.52 ,,
	4° 18′ 54″	,,	,,	4.33 ,,
Nakatsu	4° 27′ 35″	,,	24	11.51 a.m.
	4° 27′ 28″	,,	,,	1.50 p.m.
	4° 26′ 23″	,,	,,	2.40 ,,
	4° 24′ 55″	,,	,,	3.54 ,,
	4° 23′ 50″	,,	,,	9.54 ,,
	4° 19′ 55″	,,	25	7.31 a.m.
	4° 19′ 13″	,,	,,	8.50 ,,
	4° 19′ 50″	,,	,,	9.54 ,,
Saganoseki	4° 8′ 21″	,,	28	7.30 a.m.
	4° 8′ 31″	,,	,,	8.44 ,,
	4° 12′ 46″	,,	,,	10.08 ,,
	4° 16′ 9″	,,	,,	11.29 ,,
	4° 17′ 11″	,,	,,	1.43 p.m.
	4° 15′ 14″	,,	,,	3.07 ,,
	4° 12′ 36″	,,	,,	4.36 ,,
Hichiya	4° 1′ 20″	,,	29	2.54 p.m.
	3° 59′ 30″	,,	,,	4.45 ,,
	3° 58′ 23″	,,	,,	10.58 ,,
	3° 57′ 15″	,,	30	6.26 a.m.
	3° 56′ 13″	,,	,,	7.32 ,,

Station.	Declin.	Date & Hour.
Hichiya (*continued*) ...	3° 58′ 48″	August 20 8.38 a.m.
	4° 1′ 38″	„ „ 9.22 „
	4° 4′ 23″	„ „ 11.38 „
Miyazaki	4° 3′ 21″	„ 31 11.34 a.m.
	4° 3′ 8″	„ „ 0.05 p.m.
	4° 2′ 9″	„ „ 1.39 „
	3° 59′ 26″	„ „ 2.59 „
	3° 58′ 26″	„ „ 4.00 „
	3° 57′ 34″	„ „ 4.36 „
	3° 58′ 4″	„ „ 6.15 „
	3° 55′ 34″	September 1 5.59 a.m.
	3° 56′ 44″	„ „ 6.36 p.m.
	3° 55′ 36″	„ 2 7.37 a.m.
	3° 55′ 51″	„ „ 8.30 „
	3° 57′ 16″	„ „ 9.05 „
	3° 59′ 38″	„ „ 9.43 „
	4° 1′ 16″	„ „ 10.30 „
	4° 2′ 6″	„ „ 11.13 „
Yatsushiro	4° 4′ 2″	„ 6 9.52 a.m.
	4° 5′ 15″	„ „ 10.20 „
	4° 6′ 20″	„ „ 11.07 „
	4° 5′ 38″	„ „ 0.14 p.m.
	4° 3′ 43″	„ „ 1.18 „
	4° 1′ 50″	„ „ 2.17 „
	3° 59′ 38″	„ „ 3.29 „
	3° 58′ 33″	„ „ 4.37 „
	3° 58′ 43″	„ „ 5.20 „
	3° 56′ 1″	„ 7 7.55 a.m.
	3° 56′ 25″	„ „ 8.26 „
	3° 57′ 55″	„ „ 9.08 „
	4° 3′ 5″	„ „ 10.46 „

Station.	Declin.	Date & Hour.			
Nagasaki	3° 31′ 54″	September	10	9.12	a.m.
	3° 35′ 36″	,,	,,	10.04	,,
	3° 37′ 13″	,,	,,	10.48	,,
	3° 37′ 58″	,,	,,	11.24	,,
Hagi	4° 29′ 37″	,,	14	5.52	p.m.
	4° 30′ 32″	,,	,,	7.56	,,
	4° 30′ 5″	,,	15	0.22	a.m.
	4° 28′ 27″	,,	,,	4.09	,,
	4° 28′ 3″	,,	,,	6.15	,,
	4° 27′ 42″	,,	,,	6.45	,,
	4° 27′ 42″	,,	,,	7.24	,,
	4° 27′ 27″	,,	,,	7.52	,,
	4° 27′ 37″	,,	,,	8.32	,,
	4° 29′ 3″	,,	,,	9.15	,,
	4° 30′ 23″	,,	,,	9.44	,,
	4° 32′ 0″	,,	,,	10.16	,,
	4° 33′ 8″	,,	,,	10.46	,,
	4° 33′ 38″	,,	,,	11.15	,,
	4° 33′ 40″	,,	,,	11.47	,,
	4° 33′ 40″	,,	,,	0.14	p.m.
	4° 33′ 30″	,,	,,	0.43	,,
	4° 32′ 30″	,,	,,	1.15	,,
	4° 31′ 20″	,,	,,	2.09	,,
*	4° 31′ 30″	,,	,,	2.17	,,
(a stone removed) *	4° 31′ 10″	,,	,,	2.24	p.m.
	4° 29′ 58″	,,	,,	2.56	,,
	4° 29′ 38″	,,	,,	3.15	,,
	4° 29′ 33″	,,	,,	3.43	,,
	4° 29′ 20″	,,	,,	4.12	,,
	4° 29′ 35″	,,	,,	4.43	,,
	4° 30′ 13″	,,	,,	5.18	,,

Station.	Declin.	Date & Hour.			
Hagi (*continued*)	4° 30′ 48″	September	15	7.04	p.m.
	4° 30′ 45″	„	„	7.55	„
	4° 30′ 15″	„	„	11.03	„
Hamada	4° 34′ 58″	„	16	3.34	p.m.
	4° 35′ 5″	„	„	4.52	„
	4° 36′ 8″	„	„	5.48	„
	4° 37′ 8″	„	„	7.27	„
	4° 35′ 40″	„	17	7.29	a.m.
	4° 36′ 15″	„	„	8.15	„
	4° 37′ 10″	„	„	8.51	„
	4° 39′ 43″	„	„	9.40	„
	4° 41′ 5″	„	„	10.25	„
	4° 42′ 20″	„	„	10.42	„
	4° 43′ 42″	„	„	11.45	„
	4° 43′ 7″	„	„	0.28	p.m.
	4° 42′ 5″	„	„	1.10	„
	4° 40′ 42″	„	„	1.42	„
	4° 39′ 45″	„	„	2.08	„
	4° 38′ 37″	„	„	2.40	„
	4° 37′ 56″	„	„	3.12	„
	4° 37′ 37″	„	„	3.41	„
	4° 37′ 20″	„	„	4.08	„
	4° 37′ 31″	„	„	4.39	„
	4° 37′ 37″	„	„	5.09	„
	4° 38′ 5″	„	„	5.38	„
	4° 39′ 2″	„	„	6.30	„
	4° 39′ 35″	„	„	7.06	„
	4° 39′ 0″	„	„	7.42	„
	4° 39′ 0″	„	„	8.12	„
	4° 38′ 50″	„	„	8.42	„
	4° 38′ 57″	„	„	9.25	„

Station.	Declin.	Date & Hour.		
Hamada (*continued*) ...	4° 38′ 5″	September	18	2.02 a.m.
	4° 36′ 42″	,,	,,	5.53 ,,
	4° 36′ 17″	,,	,,	6.18 ,,
	4° 36′ 16″	,,	,,	6.39 ,,
Matsue	4° 56′ 3″	,,	20	0.21 p.m.
	4° 55′ 52″	,,	,,	0.32 ,,
	4° 55′ 19″	,,	,,	1.25 ,,
	4° 54′ 22″	,,	,,	2.24 ,,
	4° 53′ 7″	,,	,,	3.02 ,,
	4° 51′ 52″	,,	,,	3.48 ,,
	4° 51′ 2″	,,	,,	4.28 ,,
	4° 51′ 9″	,,	,,	5.13 ,,
	4° 51′ 17′	,,	,,	6.02 ,,
	4° 51′ 22″	,,	,,	7.55 ,,
	4° 51′ 12″	,,	21	0.35 a.m.
	4° 50′ 9″	,,	,,	5.57 ,,
	4° 49′ 49″	,,	,,	6.41 ,,
	4° 49′ 22″	,,	,,	7.18 ,,
	4° 48′ 39″	,,	,,	7.58 ,,
	4° 48′ 41″	,,	,,	8.28 ,,
	4° 50′ 16″	,,	,,	9.25 ,,
	4° 51′ 24″	,,	,,	9.56 ,,
Imaichi	4° 45′ 51″	,,	22	8.29 a.m.
	4° 46′ 53″	,,	,,	9.10 ,,
	4° 48′ 41″	,,	,,	9.50 ,,
	4° 50′ 1″	,,	,,	10.23 ,,
	4° 51′ 7″	,,	,,	10.53 ,,
	4° 52′ 1″	,,	,,	11.16 ,,
	4° 53′ 21″	,,	,,	noon
	4° 53′ 31″	,,	,,	0.32 p.m.
	4° 52′ 58″	,,	,,	0.55 ,,

Station.	Declin.	Date & Hour.
Imaichi (*continued*) ...	4° 52′ 16″	September 22 1.58 p.m.
	4° 51′ 26″	„ „ 2.49 „
	4° 50′ 56″	„ „ 3.26 „
	4° 49′ 23″	„ „ 3.59 „
	4° 48′ 26″	„ „ 4.42 „
	4° 47′ 51″	„ „ 5.32 „
	4° 48′ 11″	„ „ 6.05 „
	4° 48′ 33″	„ „ 7.28 „
	4° 48′ 18″	„ „ 8.01 „
	4° 48′ 18″	„ „ 9.23 „
	4° 48′ 6″	„ 23 0.40 a.m.
	4° 47′ 18″	„ „ 6.09 „
	4° 46′ 56″	„ „ 7.24 „
	4° 46′ 43″	„ „ 7.47 „
Kanōmura...	5° 9′ 15″	„ 25 11.11 a.m.
	5° 9′ 50″	„ „ 11.44 „
	5° 10′ 9″	„ „ 0.32 p.m.
	5° 9′ 54″	„ „ 1.07 „
	5° 9′ 39″	„ „ 1.52 „
	5° 9′ 0″	„ „ 2.38 „
	5° 8′ 8″	„ „ 3.23 „
	5° 7′ 58″	„ „ 4.03 „
	5° 8′ 48″	„ „ 5.04 „
	5° 8′ 34″	„ „ 5.46 „
	5° 7′ 40″	„ „ 7.31 „
	5° 8′ 19″	„ „ 10.06 „
	5° 8′ 15″	„ 26 5.55 a.m.
	5° 10′ 42″	„ „ 6.45 „
	5° 12′ 41″	„ „ 7.43 „
	5° 12′ 35″	„ „ 8.23 „
	5° 12′ 56″	„ „ 8.56 „

Station.	Declin.	Date & Hour.			
Koyamamura	5° 12′ 33″	September	26	0.41	p.m.
	5° 8′ 54″	,,	,,	1.16	,,
	5° 5′ 58″	,,	,,	2.08	,,
	5° 7′ 3″	,,	,,	2.37	,,
	5° 6′ 57″	,,	,,	3.25	,,
	5° 7′ 25″	,,	,,	4.08	,,
	5° 4′ 42″	,,	,,	4.36	,,
	5° 1′ 43″	,,	,,	5.24	,,
	5° 2′ 30″	,,	,,	6.10	,,
	5° 4′ 28″	,,	,,	7.28	,,
	5° 1′ 35″	,,	,,	8.40	,,
	5° 3′ 10″	,,	,,	9.30	,,
	5° 1′ 43″	,,	,,	11.15	,,
	5° 3′ 20″	,,	27	5.47	a.m.
	5° 3′ 0″	,,	,,	6.20	,,
	5° 2′ 55″	,,	,,	6.54	,,
	5° 3′ 28″	,,	,,	7.16	,,
	5° 3′ 37″	,,	,,	7.30	,,
	5° 3′ 55″	,,	,,	8.12	,,
	5° 4′ 28″	,,	,,	8.48	,,
	5° 4′ 24″	,,	,,	9.24	,,
	5° 4′ 25″	,,	,,	9.58	,,
	5° 5′ 27″	,,	,,	10.44	,,
	5° 7′ 4″	,,	,,	11.22	,,
	5° 6′ 43″	,,	,,	11.54	,,
Maizuru	5° 1′ 54″	,,	30	11.21	a.m.
	5° 2′ 25″	,,	,,	0.10	p.m.
	5° 1′ 53″	,,	,,	0.46	,,
	5° 1′ 49″	,,	,,	1.35	,,
	5° 1′ 11″	,,	,,	2.15	,,
	4° 59′ 53″	,,	,,	3.01	,,

Station.	Declin.	Date & Hour.			
Maizuru (*continued*) ...	4° 59′ 12″	September	30	3.39	p.m.
	4° 58′ 23″	,,	,,	4.18	,,
	4° 58′ 4″	,,	,,	5.05	,,
	4° 57′ 21″	,,	,,	5.47	,,
	4° 58′ 16″	,,	,,	7.18	,,
	4° 59′ 21″	,,	,,	8.16	,,
	4° 58′ 51″	,,	,,	9.50	,,
	4° 58′ 47″	October	1	0.15	a.m.
	4° 58′ 44″	,,	,,	3.22	,,
	4° 58′ 4″	,,	,,	4.47	,,
	4° 58′ 29″	,,	,,	6.22	,,
	4° 57′ 54″	,,	,,	7.30	,,
	4° 58′ 13″	,,	,,	8.16	,,
	4° 58′ 21″	,,	,,	8.57	,,
	4° 59′ 3″	,,	,,	9.36	,,
	4° 59′ 53″	,,	,,	10.01	,,
	5° 0′ 36″	,,	,,	10.18	,,
Obama...	4° 54′ 16″	,,	2	9.39	a.m.
	4° 54′ 51″	,,	,,	10.14	,,
	4° 55′ 25″	,,	,,	10.56	,,
	4° 55′ 58″	,,	,,	11.56	,,
	4° 56′ 41″	,,	,,	0.34	p.m.
	4° 56′ 51″	,,	,,	1.13	,,
	4° 56′ 18″	,,	,,	1.51	,,
	4° 55′ 26″	,,	,,	2.52	,,
	4° 54′ 10″	,,	,,	3.50	,,
	4° 53′ 43″	,,	,,	4.52	,,
	4° 53′ 42″	,,	,,	5.26	,,
	4° 54′ 47″	,;	,,	6.44	,,
	4° 54′ 31″	,,	,,	7.22	,,
	4° 54′ 40″	,,	,,	8.51	,,

Station.	Declin.	Date & Hour.
Obama (*continued*) ...	4° 54′ 36″	October 2 10.53 p.m.
	4° 53′ 51″	,, 3 2.19 a.m.
	4° 52′ 46″	,, ,, 6.34 ,,
	4° 52′ 16″	,, ,, 7.26 ,,
	4° 52′ 42″	,, ,, 8.23 ,,
	4° 52′ 55″	,, ,, 9.05 ,,
	4° 53′ 47″	,, ,, 10.02 ,,
Shioya...	4° 59′ 26″	,, 4 10.58 a.m.
	5° 0′ 54″	,, ,, 11.30 ,,
	5° 2′ 49″	,, ,, 0.22 p.m.
	5° 2′ 38″	,, ,, 1.33 ,,
	5° 1′ 20″	,, ,, 2.23 ,,
	4° 59′ 36″	,, ,, 3.16 ,,
	4° 58′ 21″	,, ,, 4.22 ,,
	4° 58′ 31″	,, ,, 5.13 ,,
	4° 59′ 19″	,, ,, 6.09 ,,
	4° 59′ 28″	,, ,, 9.01 ,,
	4° 59′ 34″	,, ,, 9.50 ,,
	4° 58′ 29″	,, 5 7.00 a.m.
	4° 57′ 14″	,, ,, 8.15 ,,
	4° 56′ 41″	,, ,, 9.04 ,,
	4° 56′ 46″	,, ,, 9.54 ,,
	4° 57′ 49″	,, ,, 10.41 ,,
	5° 0′ 10″	,, ,, 11.27 ,,
	5° 1′ 49″	,, ,, 0.31 p.m.
	5° 2′ 8″	,, ,, 1.00 ,,
	5° 1′ 53″	,, ,, 1.47 ,,
	5° 0′ 16″	,, ,, 2.59 ,,
	4° 58′ 29″	,, ,, 4.16 ,,
	4° 58′ 30″	,, ,, 4.55 ,,
	4° 58′ 49″	,, ,, 5.39 ,,

Station.	Declin.	Date & Hour.
Shioya (*continued*) ...	4° 59′ 46″	October 4 6.36 p.m.
Nanao...	5° 5′ 38″	,, 7 10.43 p.m.
	5° 4′ 9″	,, 8 0.52 a.m.
	5° 4′ 56″	,, ,, 4.31 ,,
	5° 5′ 5″	,, ,, 7.35 ,,
	5° 5′ 21″	,, ,, 8.21 ,,
	5° 4′ 34″	,, ,, 9.24 ,,
	5° 5′ 18″	,, ,, 10.09 ,,
	5° 5′ 49″	,, ,, 10.42 ,,
	5° 6′ 8″	,, ,, 11.41 ,,
	5° 6′ 11″	,, ,, 11.45 ,,
	5° 6′ 50″	,, ,, 0.35 p.m.
	5° 7′ 16″	,, ,, 1.18 ,,
	5° 7′ 15″	,, ,, 2.09 ,,
	5° 7′ 21″	,, ,, 3.08 ,,
	5° 6′ 45″	,, ,, 4.01 ,,
	5° 5′ 46″	,, ,, 5.16 ,,
	5° 5′ 6″	,, ,, 6.05 ,,
	5° 5′ 6″	,, ,, 6.53 ,,
	5° 5′ 9″	,, ,, 7.29 ,,
	5° 5′ 34″	,, ,, 9.32 ,,
	5° 5′ 25″	,, ,, 9.50 ,,
Tōkyō (W)	4° 18′ 56″	November 8 10.23 a.m.
	4° 19′ 43″	,, ,, 11.29 ,,
	4° 20′ 27″	,, ,, 0.13 p.m.
	4° 20′ 15″	,, ,, 1.17 ,,
	4° 19′ 15″	,, ,, 2.20 ,,
	4° 18′ 35″	,, ,, 3.10 ,,
	4° 18′ 12″	,, ,, 4.11 ,,
	4° 17′ 17″	,, ,, 6.05 ,,
	4° 17′ 36″	,, ,, 7.38 ,,

Station.	Declin.	Date & Hour.		
Tōkyō (W) (*continued*)	4° 17′ 54″	November	8	8.30 p.m.
	4° 18′ 52″	,,	,,	9.25 ,,
	4° 18′ 30″	,,	,,	10.17 ,,
	4° 17′ 18″	,,	9	0.14 a.m.
	4° 17′ 54″	,,	,,	6.31 ,,
	4° 19′ 30″	,,	,,	8.04 ,,
	4° 22′ 4″	,,	,,	8.49 ,,
	4° 19′ 45″	,,	,,	9.49 ,,
	4° 19′ 15″	,,	,,	10.28 ,,
	4° 19′ 23″	,,	,,	11.21 ,,
	4° 19′ 43″	,,	,,	1.02 p.m.
	4° 19′ 28″	,,	,,	2.05 ,,
	4° 17′ 49″	,,	,,	3.45 ,,
	4° 18′ 4″	,,	,,	5.38 ,,
Tōkyō (E)	4° 24′ 17″	,,	10	9.55 a.m.
	4° 24′ 34″	,,	,,	10.51 ,,
	4° 24′ 14″	,,	,,	11.35 ,,
	4° 25′ 1″	,,	,,	0.44 p.m.
	4° 24′ 46″	,,	,,	2.09 ,,
	4° 23′ 8″	,,	,,	3.10 ,,
	4° 22′ 18″	,,	,,	4.37 ,,
	4° 23′ 28″	,,	,,	4.54 ,,
	4° 24′ 13″	,,	,,	6.34 ,,
	4° 22′ 3″	,,	,,	7.26 ,,
	4° 23′ 13″	,,	,,	8.10 ,,
	4° 23′ 31″	,,	,,	8.57 ,,
	4° 24′ 41″	,,	,,	9.53 ,,
	4° 23′ 9″	,,	,,	10.33 ,,
	4° 23′ 51″	,,	11	1.00 a.m.
	4° 23′ 41″	,,	,,	7.06 ,,
	4° 22′ 43″	,,	,,	8.08 ,,

Station.	Declin.	Date & Hour.		
Tōkyō (E) (*continued*)	4° 22′ 51″	November	11	9.02 a.m.
	4° 23′ 9″	,,	,,	10.02 ,,
	4° 23′ 38″	,,	,,	11.05 ,,
	4° 24′ 17″	,,	,,	noon
	4° 24′ 11″	,,	,,	1.57 p.m.

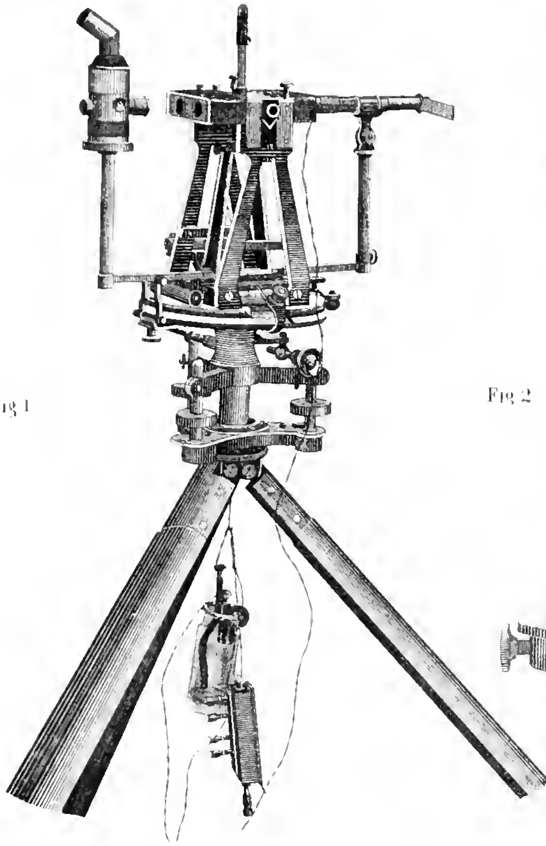


Fig 1

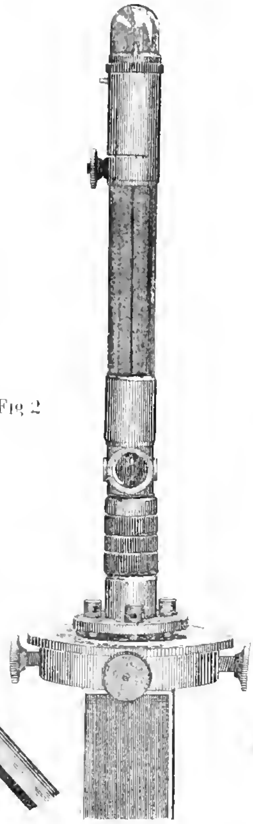


Fig 2

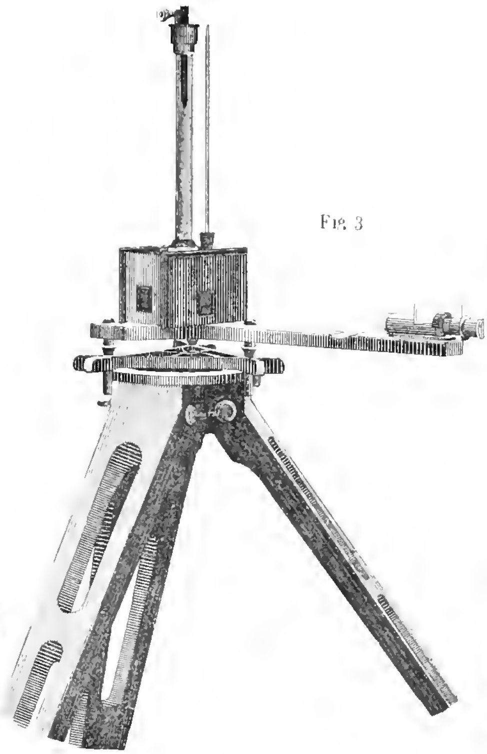


Fig 3

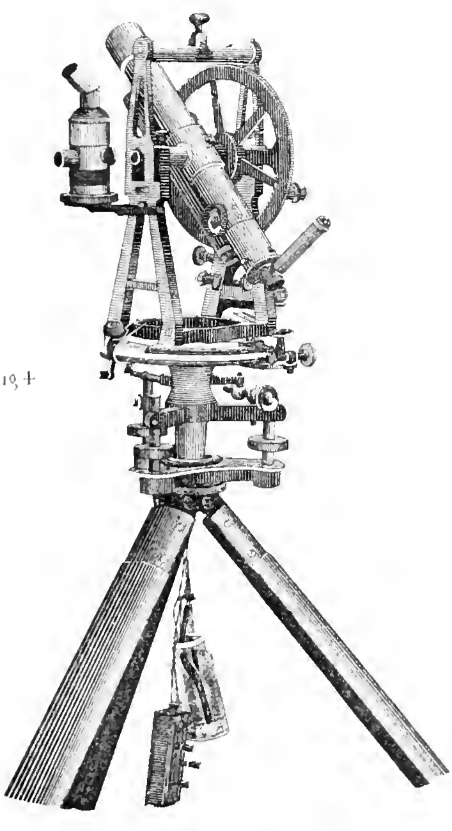


Fig 4

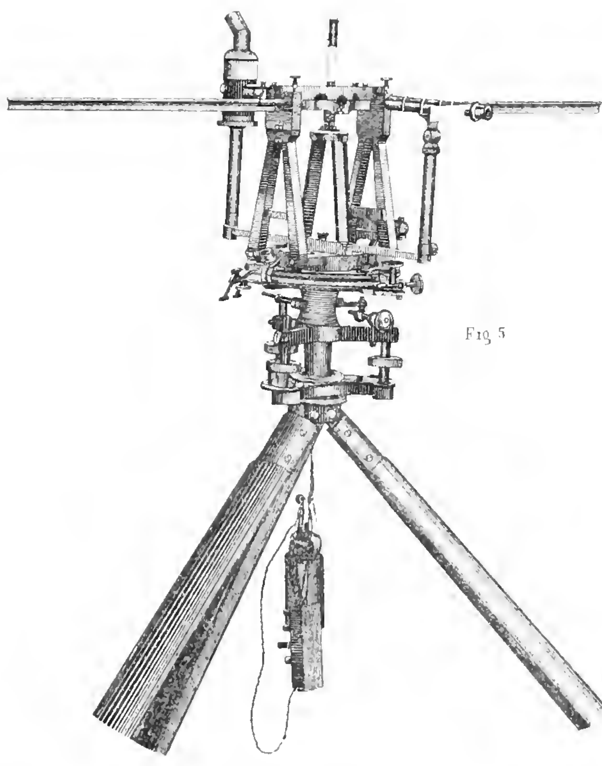
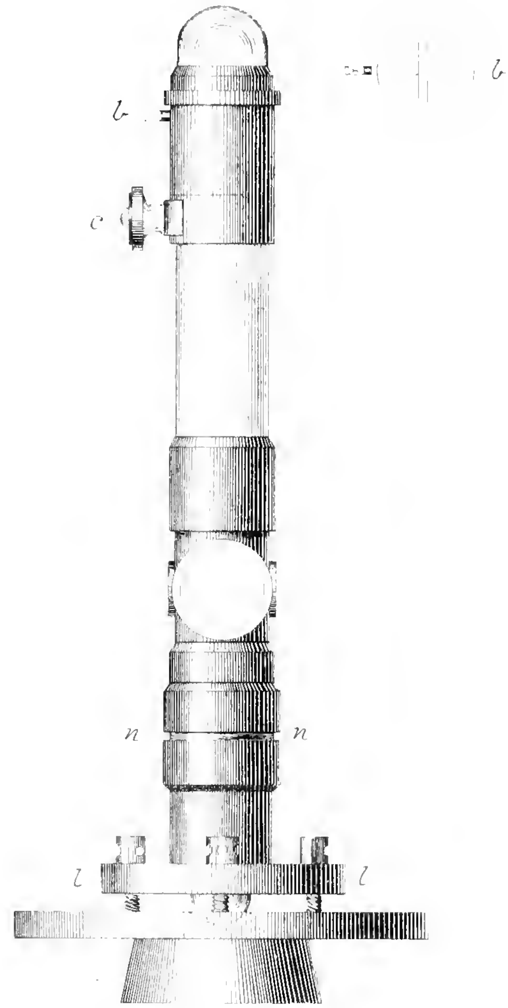


Fig 5

Fig. 1.

Fig. 2.

Fig. 3.



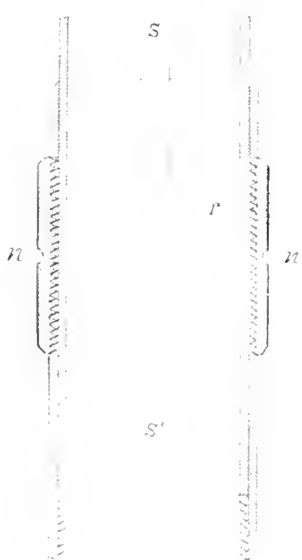


Fig. 4.

Fig. 5.

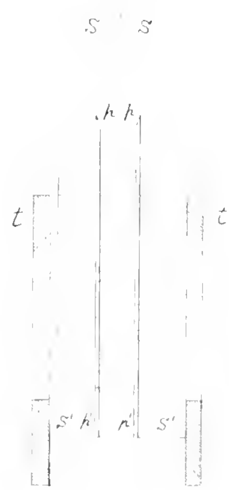
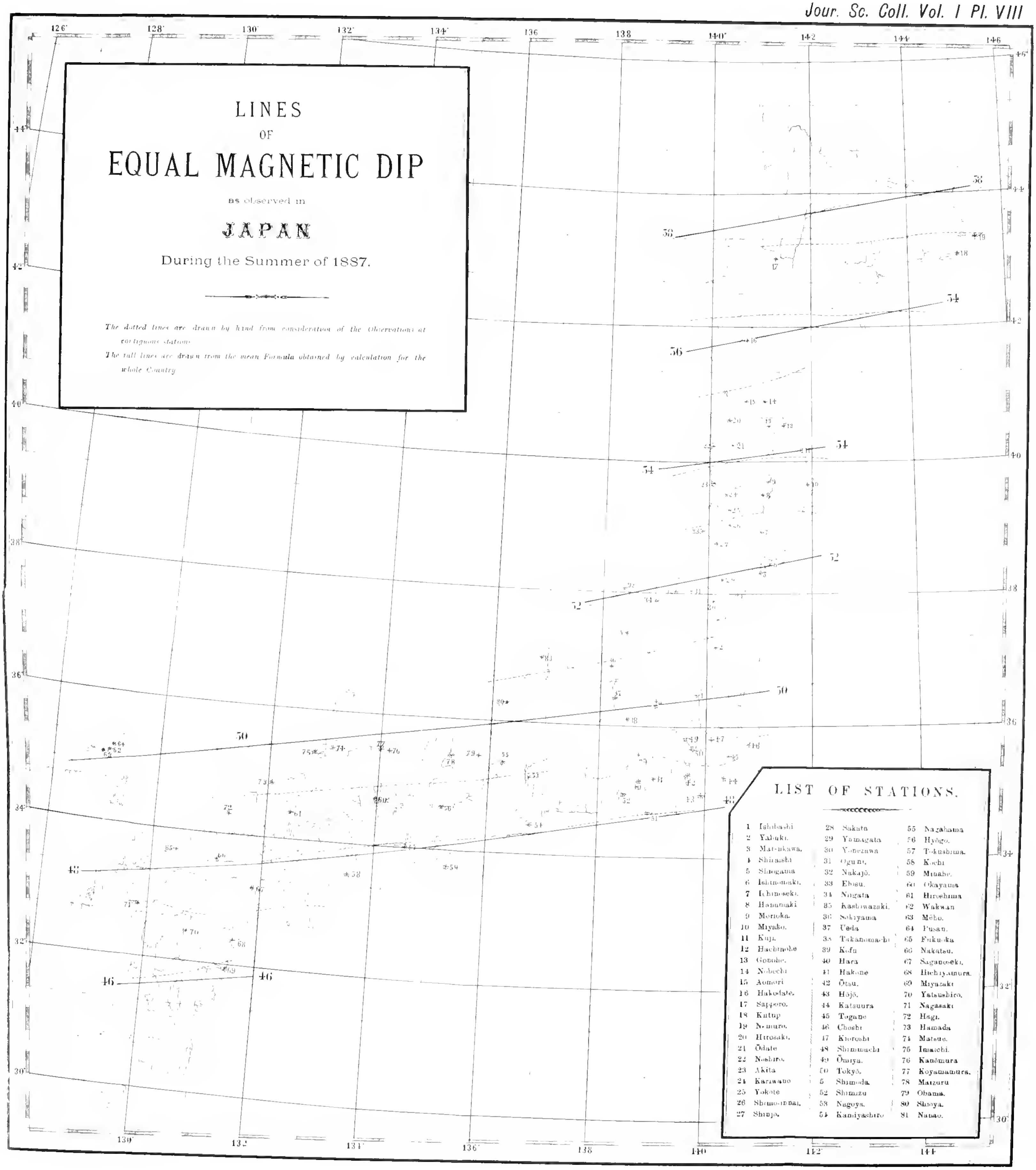
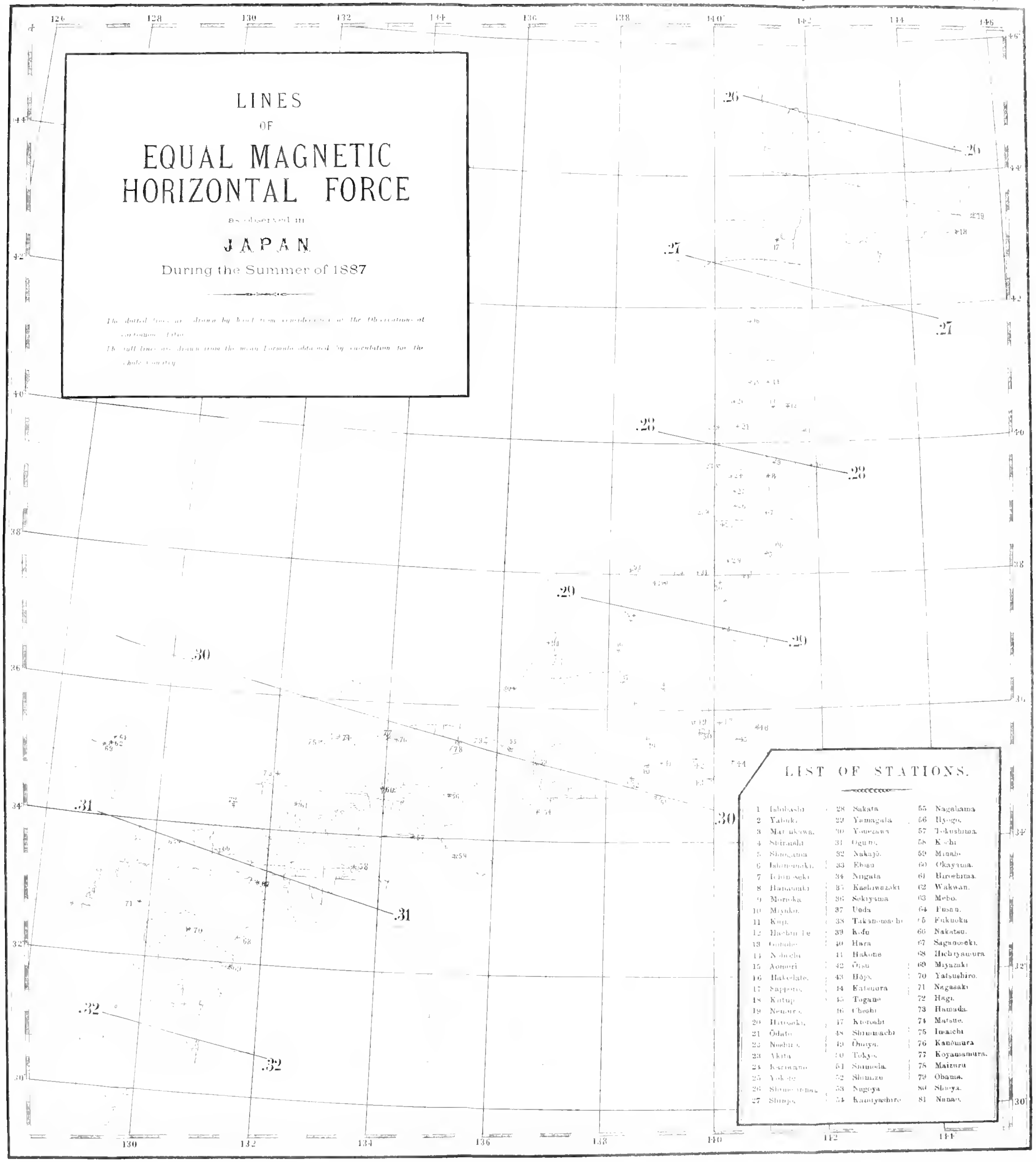
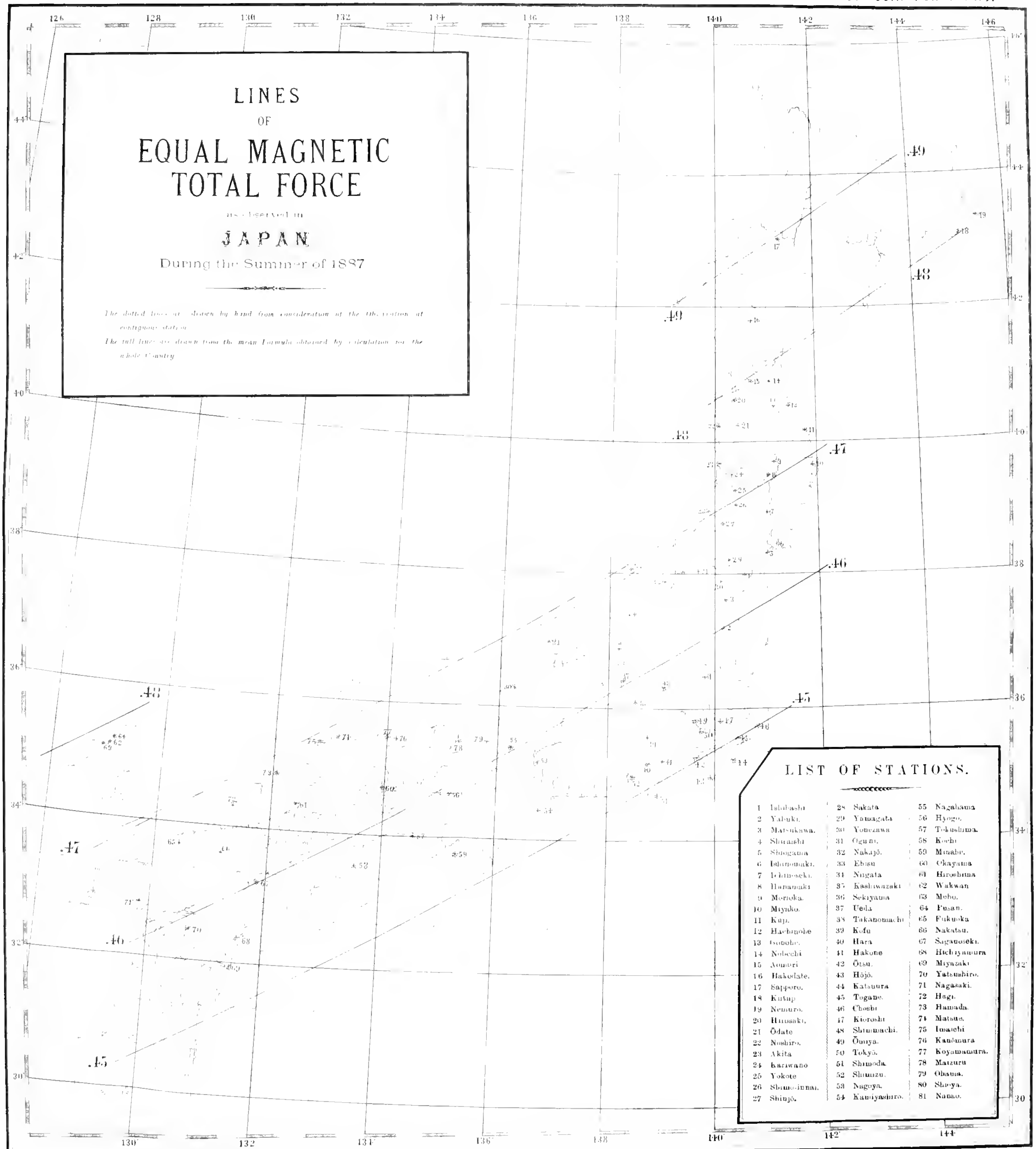


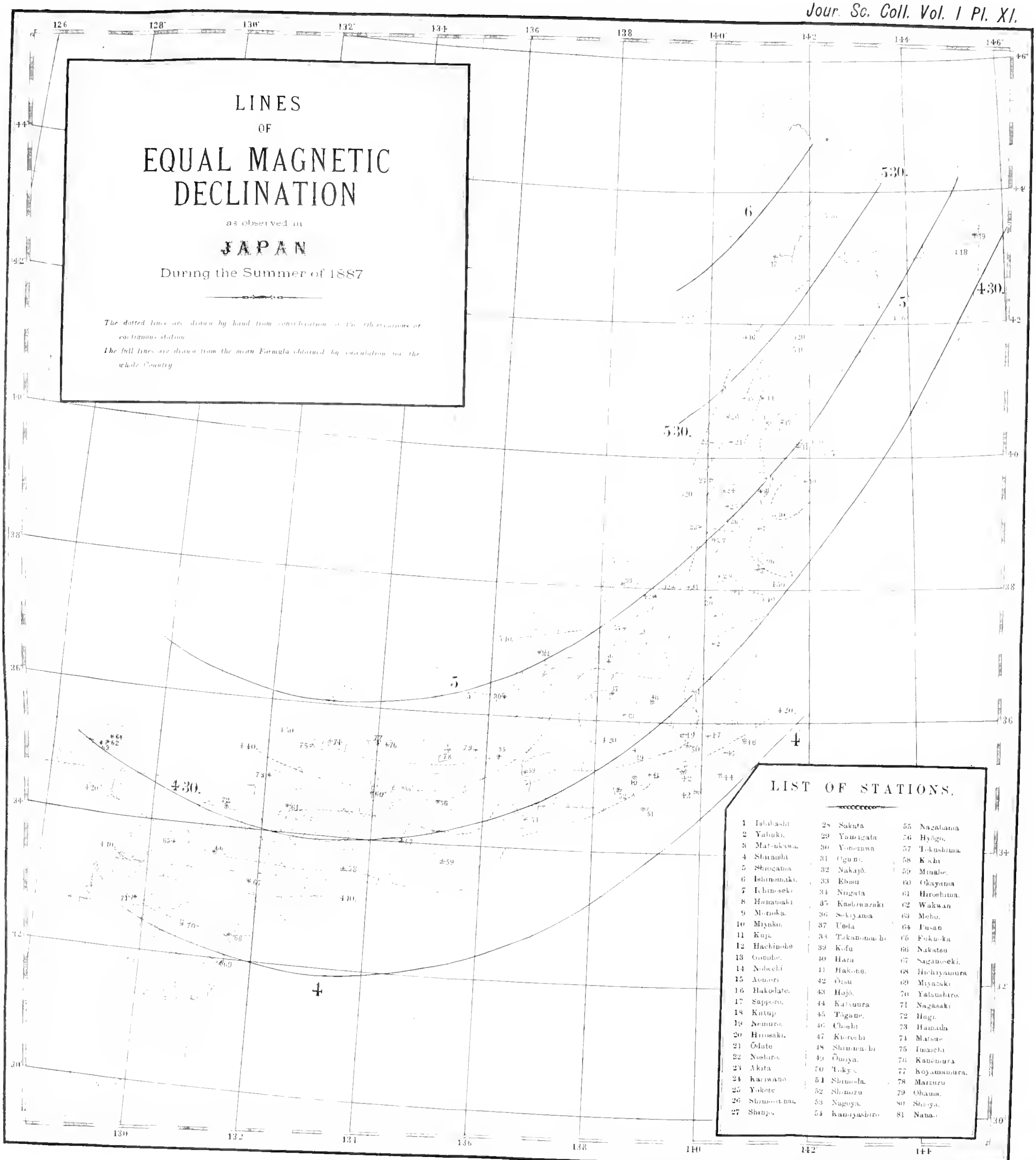
Fig. 6.


LINES
OF
EQUAL MAGNETIC DIP
as observed in
JAPAN
During the Summer of 1887.
The dotted lines are drawn by hand from consideration of the Observations at contiguous stations.
The full lines are drawn from the mean Formula obtained by calculation for the whole Country.
LIST OF STATIONS.
1 Ishibashi
2 Yabuki.
3 Matsukawa.
4 Shiraishi
5 Shiogama
6 Ishinomaki.
7 Ichinoseki.
8 Hanamaki
9 Morioka.
10 Miyako.
11 Kuji.
12 Hachinohe
13 Gonohe.
14 Nobechi
15 Aomori
16 Hakodate.
17 Sapporo.
18 Kutup
19 Nemuro.
20 Hirosaki.
21 Ōdate
22 Noshiro.
23 Akita
24 Kariwano
25 Yokote
26 Shimo-innai.
27 Shinjo.
28 Sakata
29 Yamagata
30 Yonezawa
31 Oguni.
32 Nakajō.
33 Ebisu.
34 Niigata
35 Kashiwazaki.
36 Sekiyama
37 Ueda
38 Takanomachi
39 Kofu
40 Hara
41 Hakone
42 Ōtsu.
43 Hōjō.
44 Katsuura
45 Togane
46 Choshi
47 Kioroshi
48 Shimmachi
49 Ōmiya.
50 Tokyō.
5 Shimoda.
52 Shimizu
53 Nagoya.
54 Kamiyashiro
55 Nagahama
56 Hyōgo.
57 Tokushima.
58 Kochi
59 Minabe.
60 Okayama
61 Hiroshima
62 Wakwan
63 Mōho.
64 Fusan.
65 Fukuoka
66 Nakatsu.
67 Saganoseki.
68 Ichiyamura.
69 Miyazaki
70 Yatsushiro.
71 Nagasaki
72 Hagi.
73 Hamada
74 Matsue.
75 Imaichi.
76 Kanōmura
77 Koyamamura.
78 Maizuru
79 Obama.
80 Shioya.
81 Nanao.
126°
128°
130°
132°
134°
136
138
140°
142
144
146
46°
44°
42°
40°
38°
36°
34°
32°
30°
58
56
54
52
50
48
46

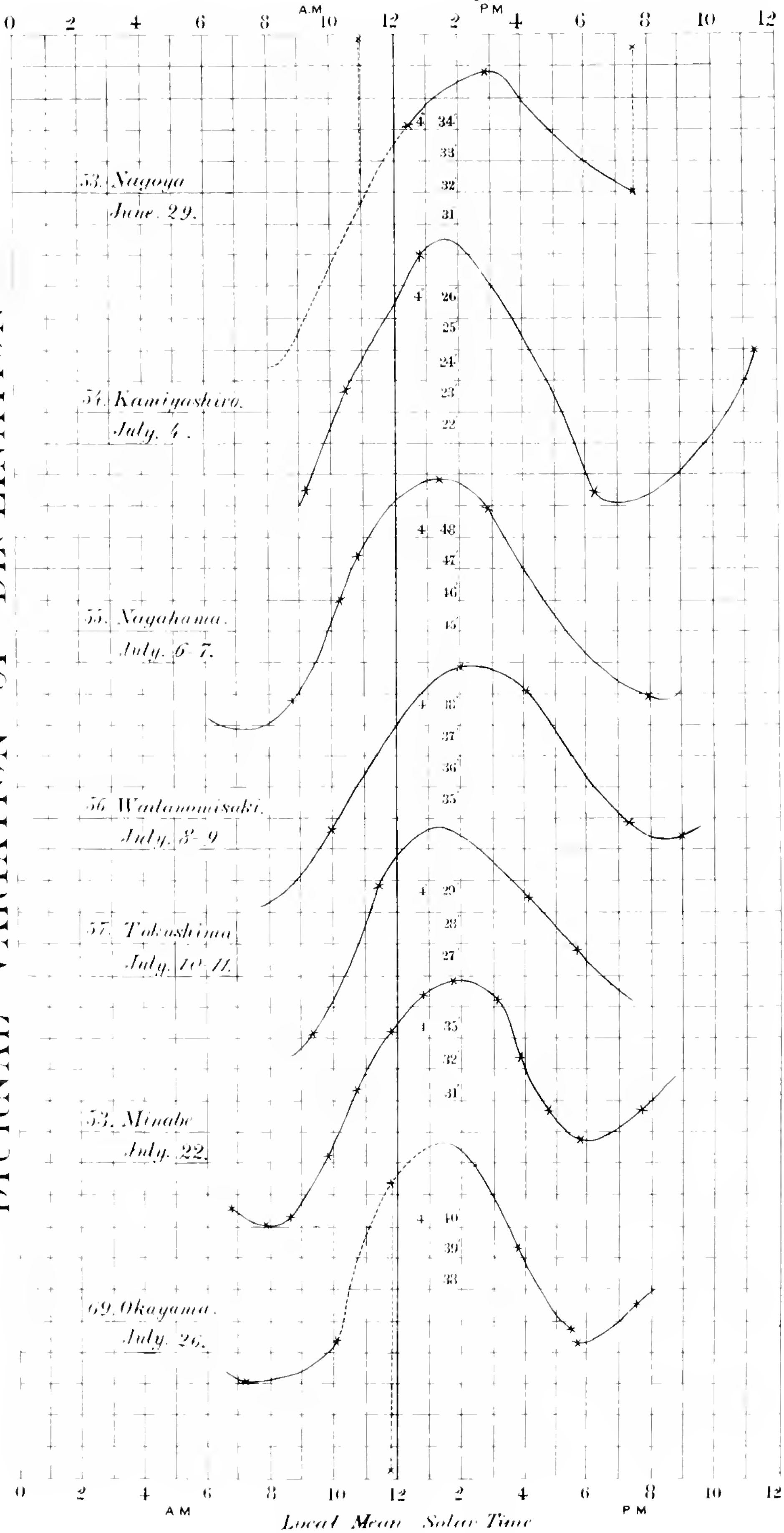

LINES
OF
EQUAL MAGNETIC
HORIZONTAL FORCE
as observed in
JAPAN
During the Summer of 1887
LIST OF STATIONS.
1 Ishibashi
2 Yabuki
3 Matsukawa.
5 Shiogama
6 Ishinomaki.
7 Ichinoseki
8 Hanamaki
9 Morioka
10 Miyako.
11 Kuji.
12 Hachinohe
15 Aomori
16 Hakodate.
17 Sapporo.
18 Kutup
19 Nemuro.
20 Hirosaki.
21 Ōdate
22 Noshiro.
23 Akita
25 Yokote
27 Shinjo.
28 Sakata
29 Yamagata
30 Yonezawa
31 Oguni.
32 Nakajō.
33 Ebisu
34 Niigata
35 Kashiwazaki
36 Sekiyama
37 Ueda
38 Takanomachi
39 Kofu
40 Hara
41 Hakone
42 Ōiso
43 Hōjo.
44 Katsuura
45 Togane
46 Choshi
48 Shimmachi
49 Ōmiya.
50 Tokyo.
51 Shimoda.
52 Shimizu
53 Nagoya
54 Kamiyashiro
55 Nagahama
56 Hyogo.
57 Tokushima.
58 Kochi
59 Minabe
60 Okayama.
61 Hiroshima.
62 Wakwan.
63 Mebo.
64 Fusan.
65 Fukuoka
66 Nakatsu.
67 Saganoseki.
68 Hichiyamura
69 Miyazaki
70 Yatsushiro.
71 Nagasaki
72 Hagi.
73 Hamada.
74 Matsue.
75 Imaichi
76 Kanōmura
77 Koyamamura.
78 Maizuru
79 Obama.
80 Shioya.
81 Nanao.
.26
.27
.28
.29
.30
.31
.32
126
128
130
132
134
136
138
140
142
144
146
30
32
34
36
38
40
42
44
46


LINES
OF
EQUAL MAGNETIC
TOTAL FORCE
as observed in
JAPAN
During the Summer of 1887
The dotted lines are drawn by hand from consideration of the observation at contiguous station.
The full lines are drawn from the mean formula obtained by calculation for the whole Country.
.49
.48
.47
.46
.45
LIST OF STATIONS.
1 Ishibashi
2 Yabuki.
3 Matsukawa.
4 Shiraishi
5 Shiogama
6 Ishinomaki.
7 Ichinoseki.
8 Hanamaki
9 Morioka.
10 Miyako.
11 Kuji.
12 Hachinohe
13 Gonohe.
14 Nobechi
15 Aomori
16 Hakodate.
17 Sapporo.
18 Kutup
19 Nemuro.
20 Hirosaki.
21 Ōdate
22 Noshiro.
23 Akita
24 Kariwano
25 Yokote
26 Shimo-innai.
27 Shinjō.
28 Sakata
29 Yamagata
30 Yonezawa
31 Oguni.
32 Nakajō.
33 Ebisu
34 Niigata
35 Kashiwazaki
36 Sekiyama
37 Ueda
38 Takanomachi
39 Kofu
40 Hara
41 Hakone
42 Ōtsu.
43 Hōjō.
44 Katsuura
45 Togane.
46 Choshi
47 Kioroshi
48 Shinmachi.
49 Ōmiya.
50 Tokyō.
51 Shimoda.
52 Shimizu.
53 Nagoya.
54 Kamiyashiro.
55 Nagahama
56 Hyogo.
57 Tokushima.
58 Kochi
59 Minabe.
60 Okayama
61 Hiroshima
62 Wakwan
63 Mobo.
64 Fusan.
65 Fukuoka
66 Nakatsu.
67 Saganoseki.
68 Hichiyamura
69 Miyazaki
70 Yatsushiro.
71 Nagasaki.
72 Hagi.
73 Hamada.
74 Matsue.
75 Imaichi
76 Kanōmura
77 Koyamamura.
78 Maizuru
79 Obama.
80 Shioya.
81 Nanao.

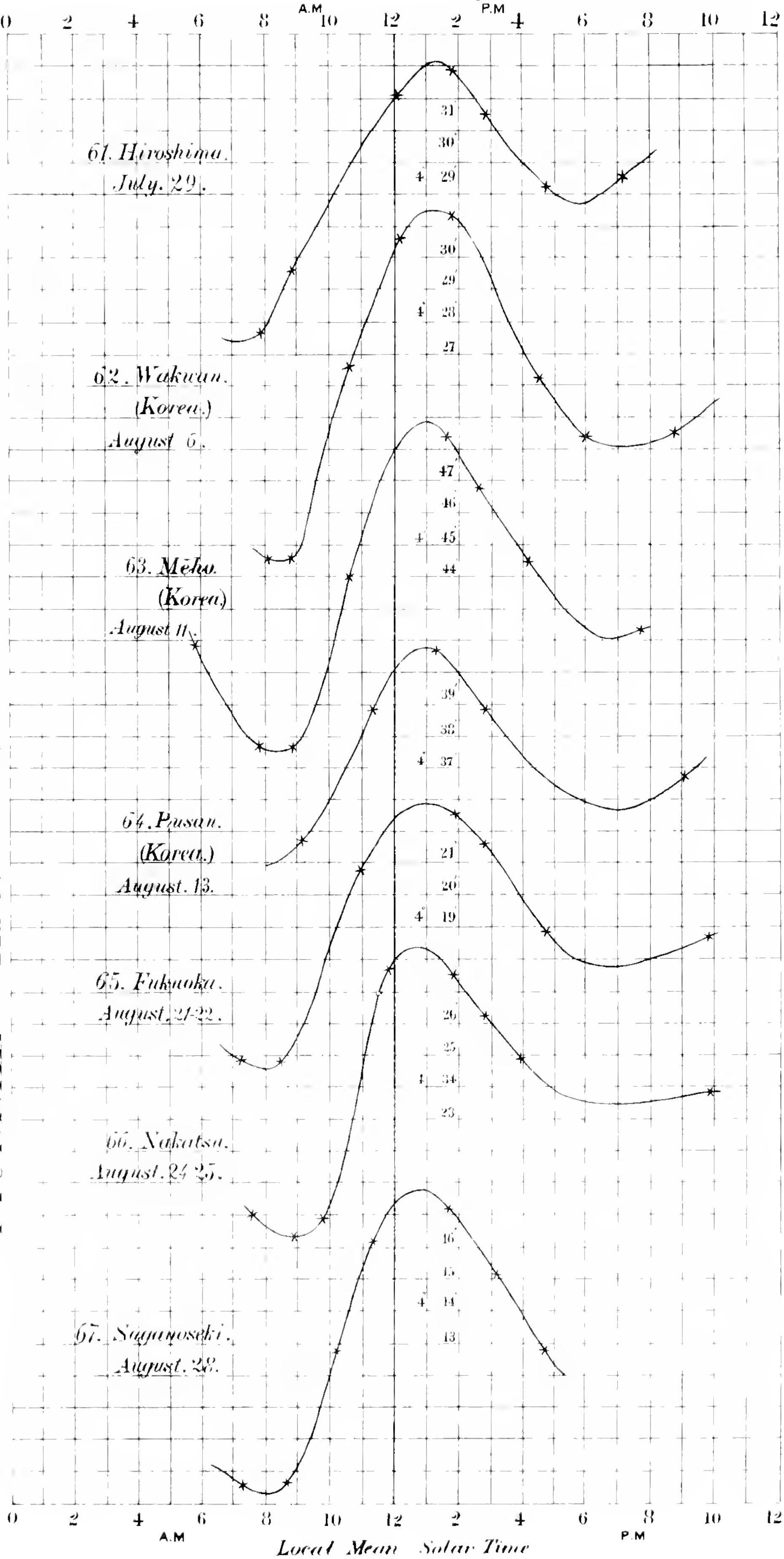
Jour. Sc. Coll. Vol. I Pl. XI.

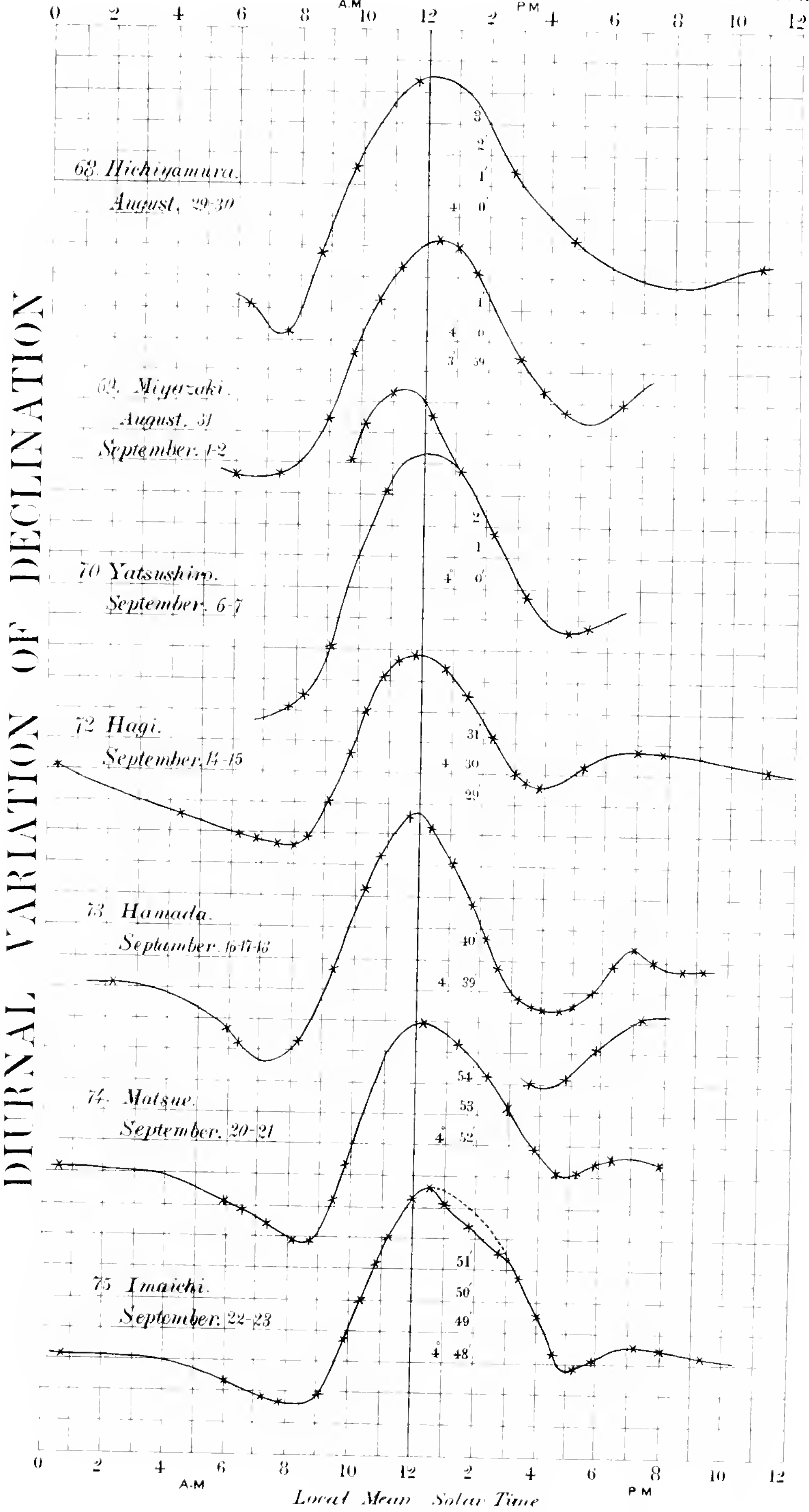


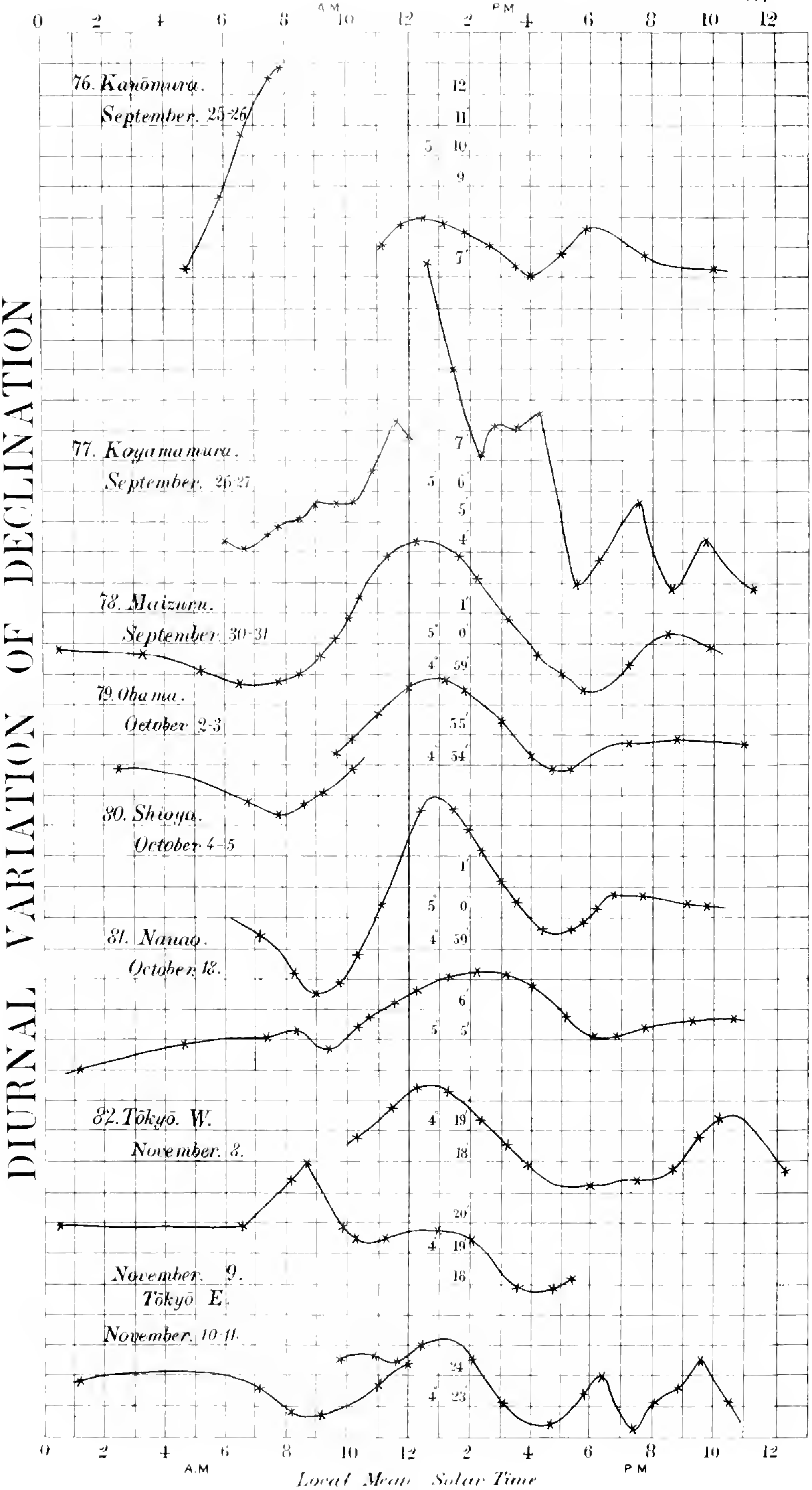
DIURNAL VARIATION OF DECLINATION



DIURNAL VARIATION OF DECLINATION




DIURNAL VARIATION OF DECLINATION
68. Hichiyamura. August, 29-30
69. Miyazaki. August. 31 September. 1-2
70 Yatsushiro. September. 6-7
72 Hagi. September. 14-15
73 Hamada. September. 16-17-18
74. Matsue. September. 20-21
75 Imaichi. September. 22-23
A.M
P.M
Local Mean Solar Time


DIURNAL VARIATION OF DECLINATION
76. Kanōmura.
September. 25-26
77. Koyamamura.
September. 26-27
78. Maizuru.
September. 30-31
79. Obama.
October 2-3
80. Shioya.
October 4-5
81. Nanao.
October 18.
82. Tōkyō. W.
November. 8.
November. 9.
Tōkyō E.
November. 10-11.
A.M
P.M
Local Mean Solar Time

ERRATA IN VOLUME II.

Page 176, 16th line from top

for "$k\,I\,v/r$" read "$k\,I\,r/r^3$"

" 178, 5th line from bottom

for "(1884-6)" read "(1883-4)"

" 185, in foot-note

for "(1884-6)" read "(1883-4)"

Table facing page 190, 4th line from top

for "30°.03 c." read "30°.30 C."

8th line from top on the right hand

for "211°" read "221°"

8th line from bottom

for "$\frac{\chi}{10}$" read "$\frac{\alpha}{16}$"

Page 195, 16th line from top

for "121" read "144"

In Table I. page 193; on page 225, 6th line from bottom; and in the Lists at the end

for "Nobechi" read "Nohechi"

Page 214, 3rd line from bottom

for "Foreign settlement" read "Japanese settlement."

Determination of the Thermal Conductivity of Marble.

Kenjirō Yamagawa, Ph. B.

Professor of Physics, Imperial University.

If a solid sphere at a given temperature be immersed into a water-bath at another temperature, and the water be stirred vigorously, we may assume that, after a short time, the surface of the sphere is at the same temperature as the bath, provided that the substance constituting the sphere be a very poor conductor of heat, such as stone, wood, &c. In fact, most determinations of the conductivities of these substances are based on this assumption. The method used in the experiments to be described in the present paper also assumes this fact. A stone sphere of a convenient size is immersed into a bath of a constant temperature for a certain time, and is then suddenly taken out, and dipped into another bath at another constant temperature for the same length of time; then again into the first bath, and so forth. After a certain number of cycles, the temperature of any point in the interior of the sphere will be subject to a steady oscillation about a definite mean. The determination of the thermal conductivity of the substance may be effected by observing the temperature-variation at any such point, say, for simplicity the centre.

Through the kindness of Mr. Nakano of the Physical Laboratory of the Kōkwa Daigaku (Engineering College), the stone-spheres used by Professors Ayrton and Perry in their determination of the heat-conductivity of stone were placed at my disposal. After working

with them, however, I had to reject them, and get a new stone-sphere constructed.

The porphyritic stone-spheres of Professors Ayrton and Perry were found to increase in weight, if left long enough in water, especially when water was hot. At first, it seemed natural to attribute this to soaking in through the surface*; and yet it was hard to believe that soaking, and soaking only could explain such a great amount of increase as 12 per cent. of weight, inasmuch as 12 per cent. in weight was equivalent to some 30 per cent. in volume. Besides, the hole in the sphere, which contained a thermoelectric junction, was always found to be full of water—a fact, hardly explicable by surface soaking. Various devices were tried to prevent this supposed soaking, but without any success. But when the balls were repolished, very fine cracks were discovered. These cracks, of course, fully explain the increase of weight, and the presence of water in the hole. But cracked balls, it was obvious, could not be used, and the construction of a new ball was necessary. It is to be hoped, that the cracks did not exist at the time when Professors Ayrton and Perry experimented upon these balls.

A sphere of 10.46 c.m. in radius was cut out of a block of saccharoidal marble (crystalline limestone, $CaCO_3$) of density 2.71. A small hole 6 m.m. in diameter was bored into the sphere radially towards the centre. Into this a nickel-iron junction enclosed in a fine glass tube 4.2 m.m. in diameter was inserted. The junction itself protruded out of the end of the glass tube so as to be in contact with a drop of mercury at the centre of the sphere. The end of the glass tube was completely closed by a cement of Japanese varnish *(urushi)*, which at the same time fixed the glass and junction-wires.

*Professors Ayrton and Perry seem to hint at this possibility in their paper (see Phil. Mag. 5th Series Vol. V. p. 257, last sentence of Section VII.)

The wires used for the junction were .5 m. m. in diameter; coarser wires having been found to produce thermal effects by direct conduction from parts outside the sphere. Even the glass-tube itself was found to be a similar source of disturbance, so that it was necessary to protect the upper part of the tube from direct contact with the water. This was effected by means of a slightly larger metal tube. This metal tube was provided with three metallic strips of a quadrantal form, which, fitting close over the upper half of the sphere, were screwed to three similar strips from below. Where the tube met the three quadrantal strips, it expanded into a disc, which fitted well on the sphere near the junction-hole. After the insertion of the glass-tube containing the junction, the space between it and the wall of the hole was filled with a paste of zinc sulphate, minium, and linseed oil. A thin coating of the same paste was spread over the under surface of the disc, which when the strips were screwed tight, prevented any water from passing into the hole from the outside. By these arrangements, the heat conducted directly through wires or their connections from the part outside the sphere, was diminished to such a degree as to be inappreciable. These precautions were found to be absolutely essential.*

Another thing to be looked to carefully was the stirring of the water in the baths. The more vigorous the stirring of the water in the hot and cold baths, the greater the range between the maximum and minimum temperatures. The reason was that when the agitation was not sufficient, the surface of the sphere was not at the same temperature as the bath itself. With increased agitation, however,

*Although Professors Ayrton and Perry speak of very fine wire, the wires I found in their balls could hardly be so designated; the copper was 1.3 m.m in diameter; the iron .7 m.m. Further the copper wire, simply coated with gutta-percha and cotton, seemed to have been directly exposed to the bath, and the iron does not seem to have had any covering at all. As Professors Ayrton and Perry speak of careful insulation of wires from water and from one another, probably the wires I found were inserted by some body else afterward.

the surface temperature more nearly approached the temperature of the bath. There seemed to be a limit to this effect of stirring as indeed there ought to be, so that after a certain degree of agitation further increase caused no appreciable change in maximum and minimum temperatures. At this limit, it may be supposed that the surface temperature was same as that of the bath. The experiments were performed always with this limiting vigor of agitation.

The heating bath was made of copper of about 30 cubic decimeters in capacity, and was heated by means of a charcoal fire. The water contained in the bath was maintained always boiling. In order to accomplish this, two tubes from separate boilers were dipped into the bath so as to have their muzzles near the sphere, when the latter was in position. The steam from these tubes kept the water in a constant state of violent agitation, and this combined with the bubbling of the water itself from the botton of the bath produced stirring enough to warrant us in assuming that the temperature of the surface of the sphere to be equal to that of the bath.

The cold bath was of about the same capacity as the hot bath, and contained water mixed with a large quantity of pounded ice. Over the bath, a tripod of wooden poles was placed, from which the sphere with its accessories was suspended. A man sitting near kept the whole apparatus constantly shaking to and fro, so that the water in contact with the sphere was constantly changing.

The stone sphere rested on a horizontal ring, which was suspended by four stout wires from the thick wooden board, which served as a cover for the bath. The board had a sliding door, which was closed, when the sphere was in the hot bath, and opened when in the cold bath. This was found necessary, as it was difficult to maintain the water in a boiling state without a cover; and then, on the other hand, when the sphere and its belongings were removed to the cold bath, the

heated board and ring seemed to retard the cooling of the ball.

At first, it was attempted to determine the temperature of the centre by the method of compensation; that is to say, the electromotive force in the thermometric circuit was balanced by an equal and opposite electromotive force produced by a second junction inserted into the circuit. The bath into which this circuit was put was heated or cooled until the galvanometer gave no current. The temperature of this second junction would then be the same as temperature at the centre of the sphere. It was found, however, very difficult to manipulate cooling or heating of this external bath so as to keep pace with the changing temperature at the centre of the ball. The results obtained were quite irregular. The second method tried was to balance the electromotive force of the circuit (the junction not in the ball being always kept at 0° C.) by a portion of the electromotive force of a Daniel's cell. A long wire was stretched to and fro on a board a considerable number of times, and, with an additional resistance of 40 ohms, was put in circuit with the cell. The electromotive force of the thermo-junction could then be balanced by the difference of potentials between the two extremities of a portion of the wire. The wire was gauged immediately afterwards, so that the temperatures at the centre could be at once deduced from the measured lengths of the portions of wire. One of the specimens given below was carried out by this method.

The third method used was that of deflection; that is to say, the external junction was maintained at a constant temperature; a sensitive galvanometer was put in the circuit; the position of the spot of light in the galvanometer-scale was read from time to time, the change of the zero point being also observed. Directly after an experiment, the galvanometer was gauged, so that the temperature corresponding to any particular galvanometer reading

could be easily found. This would be an exceedingly valuable method, if one could work far from any disturbing sources. In the present case, every precaution was taken to remove any movable piece of iron and other strongly magnetic substance from the neighborhood of the galvanometer. Also to diminish torsional set, the galvanometer-mirror was suspended by a real spider line; nevertheless, the zero-point moved as much as 3 divisions in 30 minutes. Below is given the result obtained by this method.

Owing to the necessity of using small wires for junctions, the resistance of the circuit was considerable, amounting to more than 6 ohms, so that the galvanometer was not so sensitive as might have been desired. The difference of a degree in the two junctions gave a little over 10 divisions of deflection. The thermometer used was graduated to fifths of a degree centigrade. The total range of the temperature in the second experiment was something below 45° C. and the total range of the galvanometer reading was a little below 480; we can only be sure of one division in reading; so the accuracy of the result can only extend to something like one or two tenths of a degree.

When the temperatures of points equally distant from the centre of a sphere are equal, the differential equation to be satisfied is

$$\frac{\partial u}{\partial t} = \frac{K}{c\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{2}{x}\frac{\partial u}{\partial x}\right)$$

$u =$ temperature

$c =$ specific heat

$\rho =$ density

$K =$ conductivity

$t =$ time

$x =$ variable radius

The well-known solution of this equation applicable to the present case is *

* See Thomson's Art. on Heat, in Ency. Brit., or Thomson's Math. and Phy. Papers, Vol. II. page 41. In the former ν is put equal to n k, a misprint, which is corrected in the latter.

$$u = \frac{1}{x} \sum C_i \left(\varepsilon^{-\nu_i^{\frac{1}{2}} x} \sin (2n_i t - \nu_i^{\frac{1}{2}} x + D_i) - \varepsilon^{\nu_i^{\frac{1}{2}} x} \sin (2n_i t + \nu_i^{\frac{1}{2}} x + D_i) \right)$$

where C, n, and D are constants, and

$$\nu = \frac{n}{k}^{*} = \frac{nc\rho}{K}.$$

The constants must be determined so as to satisfy the surface condition. In the present case, for the first half of the cycle or period, the temperature of the surface is $\frac{1}{2}a$, and during the last half of the period, it is $-\frac{1}{2}a$, where a is the difference of temperature between the two baths, and the mean temperature is adopted as zero. The surface-condition is then satisfied by

$$f(t) = \frac{2a}{\pi} \left\{ \frac{1}{1} \sin \frac{2\pi}{T} t + \frac{1}{3} \sin 3 \frac{2\pi}{T} t + \cdots\cdots\cdots \right.$$
$$\left. \cdots\cdots\cdots + \frac{1}{i} \sin i \frac{2\pi}{T} t + \cdots\cdots\cdots \right\}$$

T being the period, and i any odd number.

Now if we take

$$A_i = \sqrt{\varepsilon^{-2\nu_i^{\frac{1}{2}} x} + \varepsilon^{+2\nu_i^{\frac{1}{2}} x} - 2 \cos 2\nu_i^{\frac{1}{2}} x}$$

$$B_i = \tan^{-1} \frac{\varepsilon^{-\nu_i^{\frac{1}{2}} x} \sin (D_i - \nu_i^{\frac{1}{2}} x) - \varepsilon^{\nu_i^{\frac{1}{2}} x} \sin (D_i + \nu_i^{\frac{1}{2}} x)}{\varepsilon^{-\nu_i^{\frac{1}{2}} x} \cos (D_i - \nu_i^{\frac{1}{2}} x) - \varepsilon^{\nu_i^{\frac{1}{2}} x} \cos (D_i + \nu_i^{\frac{1}{2}} x)}$$

Then

$$u = \frac{1}{x} \sum C_i A_i \sin (2n_i t + B_i)$$

Putting $x = X$, the radius of the sphere, we have

$$u_{x=X} = f(t).$$

If we denote what becomes of A_i and B_i when x in them is made equal to X, by $\mathbb{A}_i$ and $\mathbb{B}_i$, then, comparing the two values of $f(t)$, we must have

$$\frac{C_i \mathbb{A}_i}{X} = \frac{2a}{i\pi}; \quad n = \frac{i\pi}{T}; \quad \mathbb{B}_i = 0,$$

i being any integer, and C_i vanishing for even terms. Hence

$$C_i = \frac{2aX}{i\pi \sqrt{\varepsilon^{-2\nu_i^{\frac{1}{2}} X} + \varepsilon^{2\nu_i^{\frac{1}{2}} X} - 2 \cos 2\nu_i^{\frac{1}{2}} X}}$$

* k is what Thomson calls diffusivity of heat.

and D_i is to be determined by

$$\mathbb{B}_i = 0.$$

Hence the general solution is

$$u = \frac{1}{x}\sum_{i=1}^{i=\infty}\frac{2aX}{i\pi\sqrt{\varepsilon^{-2\nu_i^{\frac{1}{2}}X}+\varepsilon^{2\nu_i^{\frac{1}{2}}X}-2\cos 2\nu_i^{\frac{1}{2}}X}}\Big\{\varepsilon^{-\nu_i^{\frac{1}{2}}x}\sin\Big(\frac{2i\pi}{T}t-\nu_i^{\frac{1}{2}}x+D_i\Big)$$
$$-\varepsilon^{\nu_i^{\frac{1}{2}}x}\sin\Big(\frac{2i\pi}{T}t+\nu_i^{\frac{1}{2}}x+D_i\Big)\Big\}$$

This solution, however, is only true, when the cyclical process of heating and cooling has been repeated for an infinite number of times, so as to efface completely the trace of the initial distribution of temperature. If by repetition of the cyclical process, the initial effect be completely destroyed, then the temperature distribution would depend on the surface condition only. But the solution satisfies the surface condition, and the differential equation ; therefore the value of u corresponding to any value of t and x must give the temperature at x and at t. Hence the solution is unique.

When x is made equal to zero, the value of u will represent the temperature at the centre. Denoting by u_0, the value of u for $x=0$, we obtain

$$u_0 = -\frac{4a}{\pi}\sum_{i=1}^{i=\infty}\frac{\nu_i^{\frac{1}{2}}X}{i\sqrt{\varepsilon^{-2\nu_i^{\frac{1}{2}}X}+\varepsilon^{2\nu_i^{\frac{1}{2}}X}-2\cos 2\nu_i^{\frac{1}{2}}X}}\Big\{\sin\Big(\frac{2\pi i}{T}t+D_i\Big)$$
$$+\cos\Big(\frac{2\pi i}{T}t+D_i\Big)\Big\}$$

or changing D_i to $\beta_i - \dfrac{\pi}{4}$,

$$u_0 = -\sqrt{32}\,\frac{a}{\pi}\sum_{i=1}^{i=\infty}\frac{\nu_i^{\frac{1}{2}}X}{i\sqrt{\varepsilon^{-2\nu_i^{\frac{1}{2}}X}+\varepsilon^{2\nu_i^{\frac{1}{2}}X}-2\cos 2\nu_i^{\frac{1}{2}}X}}\sin\Big(\frac{2i\pi}{T}t+\beta_i\Big)$$

which might be written

$$u_0 = \alpha_1\sin\Big(\frac{2\pi}{T}t+\beta_1\Big)+\alpha_3\sin\Big(3\,\frac{2\pi}{T}t+\beta_3\Big)+\ \&c.$$

Now if we determine the values of u_0 for different values of time, the α's and β's might be determined. As the series is a rapidly converging one, we may stop at 4th or 5th term, and then with a large number of determination of u_0 for different times, we may determine the values of the α's, and β's by the method of least squares. There is however another, and far simpler method, which in this particular case, at least is no less accurate. The above equation might be put

$$u_0 = \alpha_1 \cos\beta_1 \sin\frac{2\pi}{T}t + \alpha_1 \sin\beta_1 \cos\frac{2\pi t}{T}$$

$$+ \alpha_3 \cos\beta_3 \sin 3\frac{2\pi}{T}t + \alpha_3 \sin\beta_3 \cos 3\frac{2\pi t}{T}$$

$$+ \text{\&c. \&c.}$$

If we multiply this by

$$\sin\frac{2i\pi}{T}t\,dt \text{ or } \cos\frac{2i\pi}{T}t\,dt,$$

and integrate from $t=0$, to $t=T$, all the terms of the right-hand member of the equation will in the usual way vanish, excepting one, which is the term of the ith order. The equation is then reduced to

$$\int_0^T u_0 \sin\frac{2i\pi}{T}t\,dt = \frac{T}{2}\alpha_i \cos\beta_i \qquad \text{or}$$

$$\int_0^T u_0 \cos\frac{2i\pi}{T}t\,dt = \frac{T}{2}\alpha_i \sin\beta_i$$

The first members of these equations are simply the areas included between the t-axis and the curve.

$$y = u_0 \frac{\sin}{\cos}\left(\frac{2i\pi}{T}t\right)$$

between the limits $t=0$, and $t=T$. Now from the values of u_0 and corresponding t, different values of y corresponding to t are to be found, and the curve carefully drawn on section-paper. By means of a planimeter, the quadrature can be easily effected. If

$$\frac{T}{2}\alpha_i \cos\beta_i = A_i\,;\ \frac{T}{2}\alpha_i \sin\beta_i = A_i'$$

then

$$\alpha_i = \frac{2}{T}\sqrt{A_i^2 + A_i'^2}$$

$$\tan \beta_i = \frac{A_i'}{A_i}$$

By this method, α's and β's can be found with an accuracy equal to that of the experiment.

The determination of the amplitude α_i of any term of the series would enable us to calculate the value of the conductivity. If we were to determine all the α's, the value of K calculated from each would be same, provided that the experiment had been performed, so as to satisfy all the theoretical conditions. But, from necessary imperfections of experiment, the values would almost certainly all differ from one another. Inasmuch, however, as the first term is by far the most important, we may safely assume that the value of K calculated from it cannot be far from the truth.

In the same way, different β's will give different values of K; but for the reason just given the one obtained from β_1 will probably be a better value than that deducible from any of the other β's. In the following calculation, the values were obtained from α_1 and β_1 only.

The value of β_i enables us to calculate D_i, and D_i is related to the other quantities by

$$\mathbb{B}_i = 0$$

Hence

$$\tan \mathbb{B}_i = 0 = \frac{\varepsilon^{-\nu_i^{\frac{1}{2}} X} \sin (D_i - \nu_i^{\frac{1}{2}} X) - \varepsilon^{\nu_i^{\frac{1}{2}} X} \sin (D_i + \nu_i^{\frac{1}{2}} X)}{\varepsilon^{-\nu_i^{\frac{1}{2}} X} \cos (D_i - \nu_i^{\frac{1}{2}} X) - \varepsilon^{\nu_i^{\frac{1}{2}} X} \cos (D_i + \nu_i^{\frac{1}{2}} X)}$$

or $$\varepsilon^{-\nu_i^{\frac{1}{2}} X} \sin (D_i - \nu_i^{\frac{1}{2}} X) = \varepsilon^{\nu_i^{\frac{1}{2}} X} \sin (D_i + \nu_i^{\frac{1}{2}} X)$$

putting $$\nu_i^{\frac{1}{2}} X = S_i$$

we find $$\sin (D_i - S_i) = \varepsilon^{2S_i} \sin (D_i + S_i)$$

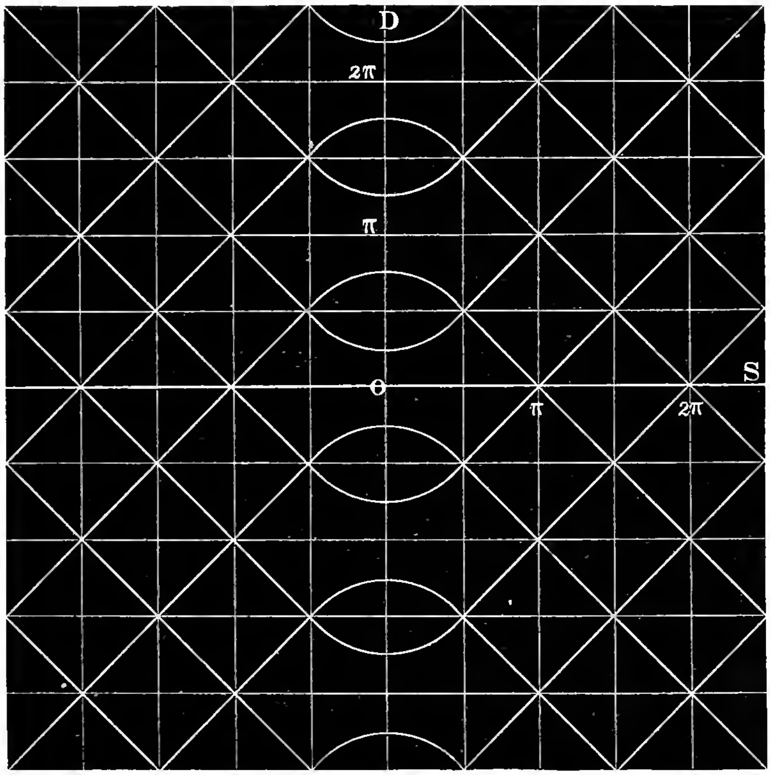
or $$\tan D_i = \pm \frac{\varepsilon^{2S_i} + 1}{\varepsilon^{2S_i} - 1} \tan S_i$$

And when S_i is large,

$$\tan D_i = \pm \tan (n\pi \pm S_i)$$

The following diagram shows the relation between S and D.

Curve showing relation between D and S.



From the diagram, it is easy to see that for a value of D, there are infinite number of values of S. But we are enabled to select the proper value by comparison with that obtained from the value of α.

Out of many experiments, the following two are taken as specimens. In the first, the period was 50 minutes or 3,000 seconds, and the readings were taken during the 8th period from the beginning of the cyclical operation. It was found, that after the 6th period, the mean value of the temperature at a point remained sensibly constant, or in other words the trace of the initial distribution of temperature was completely effaced, so far at least as experiment can tell us. The experiment was conducted according to the potentiometer method.

Experiment on May 7—1888. The temperatures were determined by the method of potentiometer.

First Specimen Experiment. Period 3000 secs. 8th Cycle
Zero of temperaturs — 50° c.

Potentiometer Reading.	Corresponding Temperature	Time in Seconds.	Potentiometer Reading.	Corresponding Temperature.	Time in Seconds.
94.4	−22.95	7	268.8	24.65	1762
84.2	−25.65	75	273.3	25.75	1826
76.0	−27.90	146	275.2	26.31	1875
64.0	−31.30	314	275.3	26.33	1928
66.1	−30.75	365	266.0	23.90	2037
75.4	−28.10	478	253.9	20.65	2157
99.4	−21.65	640	220.8	11.85	2322
112.1	−18.15	711	208.0	8.40	2376
117.3	−16.65	717	206.0	7.85	2394
132.8	−12.25	833	182.0	+ 1.20	2497
148.7	− 7.75	911	163.5	− 3.75	2590
161.9	− 4.25	982	153.6	− 6.40	2637
178.0	+ 0.12	1071	146.1	− 8.50	2676
187.4	2.70	1140	136.1	−11.22	2731
193.7	1.40	1180	123.1	−15.05	2802
204.2	7.40	1242	117.6	−16.60	2845
219.4	11.50	1345	106.3	−19.75	2914
231.3	14.72	1426	103.0	−20.60	2942
236.8	16.17	1528	96.8	−22.25	2982

The second column was calculated by means of the following table obtained by ganging the wire.

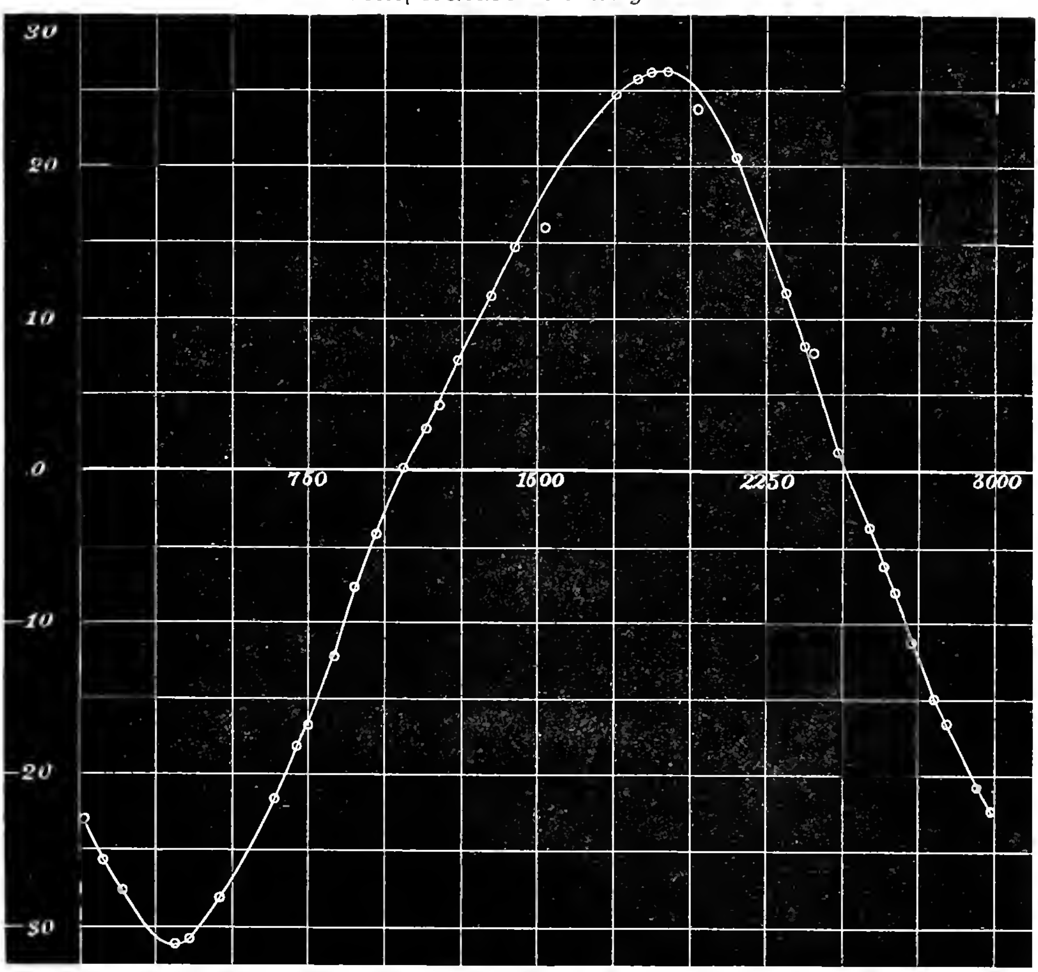
Potentiometer Reading and Temperature.

Potentiometer Reading.	Corresponding Temperature.	Potentiometer Reading.	Corresponding Temperature.
299.5	32.70	139.6	10.50
250.3	19.92	111.1	− 18.30
216.1	10.80	75.7	− 28.00
184.0	+ 1.85	39.7	38.35

A curve was carefully drawn from the numbers in the second table, and the temperatures corresponding to the numbers in the second column of the first table were carefully interpolated.

The curve representing the temperature-variations at the centre is here given.

Temperature Variation at Centre.
Period = 3000 Sec.; 8th Cycle.
Temperature in Centigrade.



From this curve, temperatures corresponding to every 41.6 seconds or to $t = T\frac{5^\circ}{2\pi}$ were calculated, and the curves

$$y = u_0 \sin 2\pi \frac{t}{T}$$

$$u_0 \cos 2\pi \frac{t}{T}$$

were drawn ; careful measurements gave

$$A_1 = -24787.$$

$$A_1' = -32294.6.$$

whence

$$\beta_1 = 52^\circ - 29' - 34''$$

$$\alpha_1 = -27.136$$

Referring to the curve showing the relation between D and S, we at once see that the value of S_1 must be nearly π. Now assuming that S_1 is near π, ε^{-2S_1} would be exceedingly small, so that it is negligible in comparison with ε^{+2S_1} ; and with an assumed approximate value for $2\cos 2S_1$, the value of α_1 gives

$$S_1 = 2.990$$

If S_1 be near π, $\frac{\varepsilon^{2S_1}+1}{\varepsilon^{2S_1}-1}$ may be taken equal to unity for a first approximation ; then

$$\tan D_1 = \pm \tan (n\pi \pm S_1)$$

Hence

$$S_1 = 3.272 \text{ or } = 3.011$$

If we calculate α_1 from these values, we obtain respectively

$$\alpha_1 = -22.38 \text{ and } \alpha_1 = -26.14$$

Therefore the second value must be the first approximation to the true value.

As $\frac{\varepsilon^{2S_1}+1}{\varepsilon^{2S_1}-1}$ varies very slowly when S_1 is large, to calculate it,

we take $S_1 = 3$, and we obtain

$$S_1 = 3.012$$

Again making $S_1 = 3.012$ in $\frac{\varepsilon^{2S_1} + 1}{\varepsilon^{2S_1} - 1}$, we get the same value

$$S_1 = 3.012$$

so that this which is the calculated value of S_1 from β_1 must be correct to a unit in the last figure. But

$$K = \frac{\pi c \rho}{T S_1^2} X^2$$

and the mean value of two specific heats for marble obtained by Regnault is .21287.* Assuming this value and the mean value of S_1 as determined from α_1 and $\beta_1 = 3.001$, we obtain

$$K = .00734 \text{ } c.g.s. \text{ } centigrade.$$

Experiment on May 14—1888. The temperatures were determined by the method of deflection.

Second Specimen Experiment Zero point —347.
Period = 40 minutes 14th Cycle
Zero of Temperature = 50° c.

Galvanometer Reading.	Corresponding Temperature.	Time in Minutes.	Galvanometer Reading.	Corresponding Temperature.	Time in Minutes.
— 56	— 14.44	.5	— 144	— 24.97	6.75
— 68	— 15.90	1.0	— 143	— 24.80	7.0
— 80	— 17.30	1.5	— 140	— 24.47	7.5
— 90	— 18.50	2.0	— 135	— 23.85	8.0
— 101	— 19.80	2.5	— 129	— 23.20	8.5
— 111	— 20.94	3.0	— 121	— 22.17	9.0
— 119	— 21.90	3.5	— 113	— 21.24	9.5
— 128	— 23.07	4.0	— 61	— 15.07	12.0
— 135	— 23.85	4.5	— 58	— 14.70	12.5
— 140	— 24.47	5.0	— 47	— 13.35	13.0
— 142	— 24.77	5.25	— 33	— 11.70	13.5
— 144	— 24.97	5.5	— 21	— 10.34	14.0
— 145	— 25.07	5.75	— 10	— 9.07	14.5
— 145	— 25.07	6.0	+ 3	— 7.54	15.0
— 145.5	— 25.10	6.25	16	— 6.07	15.5
— 145	— 25.07	6.5	29	— 4.54	16.0

* See Clark's Constants of Nature, Specific Heat Table.

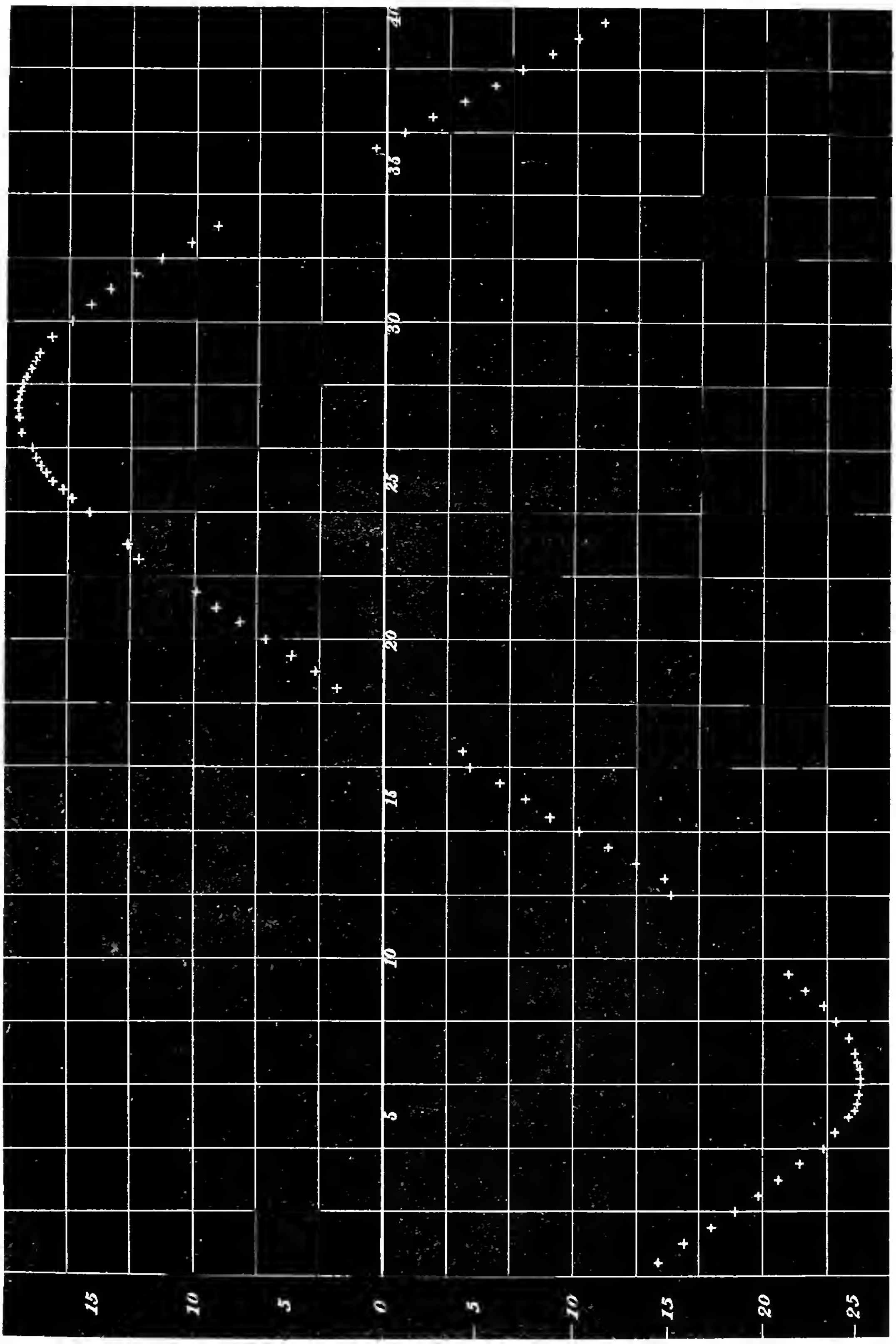
Galvanometer Reading.	Corresponding Temperature.	Time in Minutes.	Galvanometer Reading.	Coraesponding Temperature.	Time in Minutes.
32	− 4.17	16.5	242.5	19.26	27.5
89	+ 2.33	18.5	241	19.10	27.75
100	3.56	19.0	240.5	19.03	28.0
111	4.83	19.5	239	18.86	28.25
123	6.23	20.0	237	18.60	28.5
134	7.16	20.5	235	18.43	28.75
146	8.86	21.0	233	18.23	29.0
155	9.93	21.5	225	17.40	29.5
168	11.23	22.0	217	16.60	30.0
177	12.53	22.5	208	15.56	30.5
188	13.40	23.0	197	11.43	31.0
199	14.63	23.5	185	13.13	31.5
207	15.50	24.0	174	11.86	32.0
216	16.50	24.5	159	10.23	32.5
221	16.90	24.75	146	8.86	33.0
225	17.40	25.0	74	+ .56	35.5
228	17.73	25.25	60	− 1.00	36.0
231	18.10	25.5	47	− 2.40	36.5
234	18.33	25.75	33	− 4.10	37.0
237	18.60	26.0	19	− 5.70	37.5
239	18.86	26.25	+ 6	− 7.20	38.0
240.5	19.03	26.5	− 8	− 8.77	38.5
242	19.23	27.0	− 19	− 10.10	39.0
242.5	19.26	27.25	− 32	− 11.60	39.5

Galvanometer Reading and Corresponding Temperature.

Zero point of the galvanometer = −347.

Galvanometer Reading.	Corresponding Temperature.	Galvanometer Reading.	Corresponding Temperature.
279	23.10	− 1	− 7.98
217	16.60	− 43	− 12.90
195	14.20	− 98	− 19.10
142	8.40	−140	− 24.50
95	+ 3.05	−157	− 26.60
+ 50	− 2.05	−206	− 32.20

The second column of the first table was obtained by interpolation from the curve drawn from the second table. The curve showing the temperature variation at the centre is



Temperature Variation at Centre
Period 40 minutes; 14th Cycle.
Temperature in Centigrade.

The curve is even more free from irregularities than the curve for the first experiment. The same process of calculation was performed in order to obtain the values of A_1 and A_1'; and they are

$$A_1 = -21942.9; \; A_1' = -12547.6$$

And consequently

$$\alpha_1 = -21.064; \; \beta_1 = 29^\circ - 15' - 14''$$

Calculating from α_1 we gets for S_1 the value

$$3.358$$

Calculating from β_1 is in the same way as before, we get

$$S_1 = 3.407$$

If we take the mean value of the two, we obtain

$$S_1 = 3.383$$

Then

$$K = \frac{\pi c \rho}{T S_1^2} X^2 = .00722.$$

The results from the two experiments do not differ from one another by more than $1\frac{3}{5}$ per cent, and the mean value of K from the two experiments is

$$K = .00728$$

The value, however, differs much from values obtained by previous experimenters, as may be seen from the appended table.

G. Forbes......	White marble..............................	.00115
Péclet.........	Marble......................................	.0018
Depretz	Fine-grained marble, (density=2.68)	.0097
Ditto..........	Sugar-white, coarse-grained marble (density = 2.77)	.0077*
Herschel	Marbles, limestone, etc...............	.0047 to .0056

The second marble experimented upon by Depretz seems to have been very much like the marble used in my experiments, and the value

* See Everett's Physical Constants.

is very much like the one obtained by me. We may therefore assume that the value $K = .0073$ is not very far removed from the true value of the thermal conductivity of white coarse-grained marble.

The present mode of experimenting seems to be open to criticism in one respect only—namely, the method of determining the temperature at the centre. With a distinct improvement in this particular, the accuracy of the final result would be greatly enhanced. It is hoped that such improved experiments may be undertaken at some future time.

The experiments here described were all performed by Messrs. K. Ikeda and M. Ogawa, chemical students, while working in the Physical Laboratory, under my direction.

Combined Effects of Torsion and Longitudinal Stress on the Magnetization of Nickel.

By

H. Nagaoka, *Rigakushi,*

of the

Imperial University.

With Plates XVI–XIX.

The effect of torsion in altering the induced magnetism of iron has long engaged the attention of many physicists. Among the experimenters in this field of research may be named Wertheim, Wiedemann, Thomson, and Tomlinson (whose latest work on such subjects I have not yet seen.) The experiments of Wiedemann were made by twisting and untwisting the wire, which was placed horizontally in a magnetizing solenoid, till the changes of magnetism became cyclic. But it was Thomson who first investigated the effect of torsion on the magnetism of iron, the wire being at the same time subject to definite longitudinal stresses. In his experiments, the soft iron wire was placed vertically in the earth's field. No one seems to have made similar experiments in different magnetizing fields. Scanty though this kind of investigation has been for iron, it is still more scanty for nickel. Indeed, so far as I am aware, the effect of torsion on the magnetism of nickel wire under various longitudinal stresses has not hitherto been investigated. This accordingly was the problem

I resolved to attack; and the results that have been obtained will, I believe, be found to contain distinct novelties.

It is well-known from the results of various experimenters that by the application of longitudinal stress, the magnetism of iron increases up to a certain critical load, while that of nickel always diminishes. Thus we should naturally expect that the effect of torsional stress on nickel will be opposite to that on iron. According to Thomson's experiments, the effect of torsion on the magnetism of iron is to increase it at first, but when the twist exceeds a certain angle, it tends to diminish, while on untwisting it increases and attains its former value, and similar things take place when the wire is twisted in the opposite direction. So with nickel, it seemed quite likely that there will be decrease of magnetism till the twist reaches a certain value, and beyond this, the magnetism of nickel will increase, which on untwisting again decreases to its former value. In fact, this is exactly reproduced in one of Wiedemann's experiments,* the curve obtained being just the reverse of one given by Thomson.† But Wiedemann's result was obtained by simply clamping the wire in a horizontal position, so that the wire was subject to a weak longitudinal stress only. On this account, the combined effect of pull and torsion on the magnetism of nickel was still a matter to be determined.

In the following experiments, I have examined these points, and have found that the longitudinal stress produced a singular effect. For weak stresses, the changes of magnetism came out as was to be expected, but when the load exceeds a certain limit, this is no longer the case. The changes of magnetism become gradually altered, and beyond a critical value of the longitudinal stress, one end of the nickel wire acquires the two opposite kinds of magnetism during the

*Wiedemann's Annalen, Bd. 26, S. 376, 1886

†Philosophical Transactions, 1879, p. 72

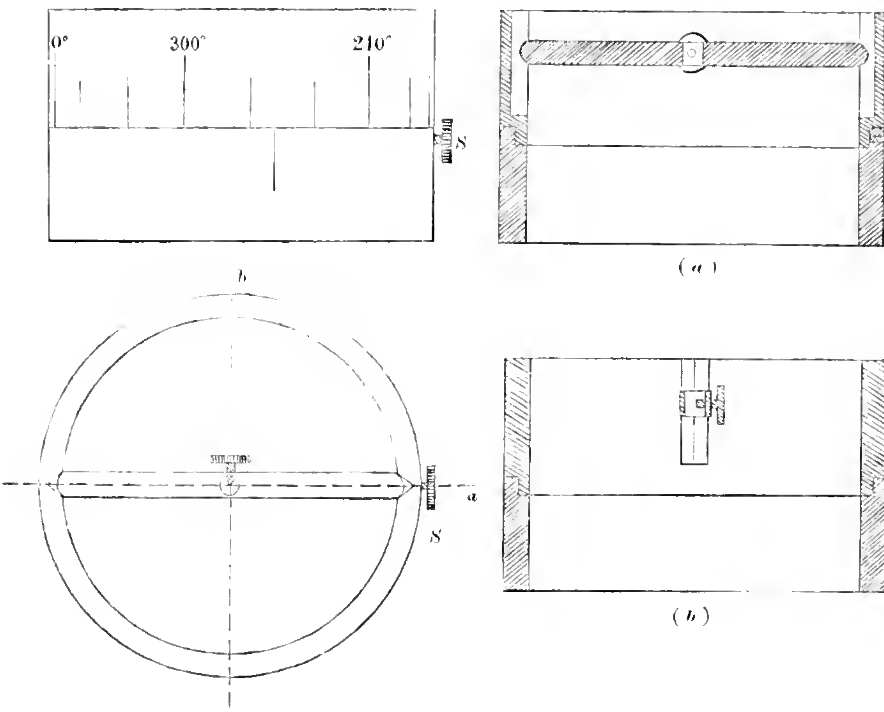
torsion and detorsion, notwithstanding the absolute constancy of the magnetizing force both in direction and magnitude. This critical value of the load seems to vary with the strength of the magnetizing field, becoming greater as the field is increased. All these points will be described in the following pages.

I must here express my thanks to Dr. C. G. Knott, for his kind suggestions during the course of experiments.

The intensity of magnetization was measured by a direct magnetometric method. The magnetometer consisted of a small mirror hung by a spider thread 11 cms long. This was geometrically fixed in position on a wooden plank according to Thomson's method of the hole, slot, plane. Levelling was effected by three base-screws. In front of the magnetometer a lamp was placed, and the image of the slit was reflected on a circular scale. Its radius was 1 metre, and it was so placed that the magnetometer was just at its centre. The wire to be examined was set vertically due east of the magnetometer. The upper end of the wire was level with the centre of the magnetometer mirror. To each end of the wire, a short stout brass wire was brazed. The lower of these was bent into a hook, so that a pan holding the weight could be hung from it. The upper one was riveted to a strong brass rod projecting from the middle of the side of a table, which rested on stone piers. The nickel wire* was surrounded by a magnetizing coil 45 cms. long. The resistance of the coil was 19.6 Ohms, and the strength of the field for a current of one ampere was 138.4 C. G. S. units. The magnetizing current was sent from 12 Daniell cells, and its strength was adjusted by means of a liquid slide, and measured by a tangent galvanometer.

* This wire contained 1.7 per cent. of iron, besides small quantities of carbon as impurities.

The twisting apparatus is shown in the subjoined cut.



(a)

(b)

It consisted of two hollow cylinders of 2.5 cms radius. The lower cylinder was fixed to a tripod stand, while the upper one fitted into it, and was movable. The latter was graduated on its external cylindrical surface at intervals of 20°, and by means of a pointer which was cut on the corresponding surface of the fixed cylinder, the amount of twist was easily read off. The screw S attached to the lower cylinder served to fix the upper one at any desired angle of twist. On the inner surface of the movable cylinder, two vertical V grooves were cut opposite each other. On these the two ends of the thick brass diametral rod were made to slide. This rod had a small hole at its centre which was just large enough to allow the passage of the brass wire attached to the lower end of the nickel wire. In order to secure the axial position of this hole with reference to the twisting

cylinder. the rod was fixed between the V's and bored on a lathe by turning it together with the cylinder. A small clamping screw served to pin the wire fast against the side of the rod, so that the wire and cylinder rotated together.

It might at first sight appear that this arrangement might prevent the longitudinal stress being applied uniformly for various angles of twist, because of the friction of the rod against the V's. To test this point, the wire was fixed by the screw, and the longitudinal pull applied by known loads hung on below. The upper end of the wire was fastened to a spring balance, by means of which any variations of stress could at once be detected. With a given load, the wire was twisted though various angles; but scarcely any sensible variation of longitudinal stress was indicated.

The magnetic experiments were conducted in the following manner. At first, a constant current was made to pass through the magnetizing coil, and the magnetometer zero was determined. The wire was then placed in position, and was first twisted through 180° in what we call the positive direction—although it may as well be stated once for all that the positive direction mean the *first chosen* direction, whether that is, so to speak, with the magnetizing current or against it. Then after a complete revolution in the opposite direction, it was brought back to its original position. This process was repeated till the changes became nearly cyclic. Then at every successive 20° of twist, the deflection of the magnetometer magnet as given by the scale reading was taken and noted, the reading being observed by means of a telescope. Each complete set of experiments was made in a constant magnetic field, while the wire was subjected to gradually increasing longitudinal stresses. The results thus obtained are given in C. G. S. electromagnetic units, though such reduction would have been unnecessary in experiments of this kind. The

observed values of the intensity $\mathfrak{I}$ for successive 60° of twist are given in the appendix. In the figures showing the changes of magnetization, the abscissæ denote the amount of twist, while the ordinates represent the intensity of magnetization $\mathfrak{I}$.

The first experiment was made with a nickel wire 40 cms long, and 1 mm. thick. The wire was deprived of its initial magnetism by heating it red hot. It was then placed vertically and so came under the influence of a magnetic field of .34 C. G. S. units. With a steady load of .64 kgs. the wire was subjected to repeated twisting and untwisting. After 6 such operations, the changes became nearly cyclic, and the following readings of deflections were taken at the 7th cycle.

Twist (Positive.)	Readings.	Twist (Negative.)	Readings.
0°	24.2	0°	3.4
20°	35.8	20°	− .2
40°	45.5	40°	− 2.0
60°	55.8	60°	− 2.4
80°	64.4	80°	− 2.6
100°	71.0	100°	2.4
120°	76.3	120°	− 1.9
140°	79.8	140°	− 1.2
160°	82.2	160°	− 0.5
180°	83.8	180°	0.0
160°	81.9	160°	0.0
140°	78.8	140°	− .2
120°	73.5	120°	.3
100°	65.2	100°	1.0
80°	51.9	80°	1.0
60°	35.9	60°	1.9
40°	22.0	40°	7.8
20°	10.8	20°	17.3
0°	3.4	0°	28.7

The above readings reduced to absolute units are shown plotted in Fig. 1. The amount of load per sq. cm. was 82 kgs., and the twist of 180° corresponded to that of .0785 radian per cm.

Examining the figure we see that the first effect of twisting is to increase the magnetization. The rate of increase is rapid at first, but gradually falls off as the magnetization attains its greatest value at the maximum twist. During the process of untwisting, the magnetization diminishes more rapidly than it increased during twisting, so that for every position of twist, the magnetization during twisting is greater than the magnetization during untwisting. The diminution of magnetization goes on even after the wire has passed the original position from which the twisting was begun, until the apparent magnetization is at length reduced to zero. This happens very soon after the original position of the untwisted wire has been reached, as the process of untwisting is continued as twisting in the negative direction. But now as this negative twisting is continued, the *polarity of the wire changes sign*, a very striking fact indeed. As the twisting is continued on towards $-180°$, this negative magnetization passes through an arithmetical maximum, becoming finally almost zero. As the wire is being brought back to the position from which it started, the magnetization gradually recovers nearly its original value as shown in the figure.

Now reasoning from analogy, we should expect to obtain by such twisting and untwisting a curve of the form given in Fig. 5, since the behaviour of nickel with regard to the effect of stress in magnetization seems to be just opposite to that of iron (see Sir. W. Thomson's figures for iron. *Philosophical Transactions* 1879). But in this experiment, the curve of magnetization seems to be no resemblance at all to any figured by Sir. W. Thomson; and then there is the very curious fact that the magnetization of nickel in a steady field can be made to

change sign by twisting. It first occurred to me that this very extraordinary result must be due to some defect in the arrangement, but careful examination discovered no flaw. The question naturally suggested itself, was this phenomenon a function of the load as well as of the twist? Hence as a next step, the weight was increased by .5 kg., and the experiment was performed in the usual manner.

The result is shewn in Fig. 2. Here the effect is quite similar to the former, the differences being only differences of detail. The march of magnetization with positive twisting is sensibly the same as in the former case; but during negative twisting, the opposite magnetization has increased to more than 10 times its amount in the first experiment. However, the rate of recovery has very much diminished, and even after a twist of 180°, the magnetization is far from reaching the former value.

The same experiment was then performed with the load increased to 5.14 kgs. The result is shown in Fig. 3. There again we find the opposite magnetization still more increased. Indeed, the two kinds of magnetization do not differ much in intensity at the two extreme twists, although the initial or positive magnetization is somewhat predominant. In the two former experiments, there was a distinct tendency towards recovery of positive magnetization as the wire was twisted more and more in the negative direction. Here, however, no such tendency shows itself except in the diminished rate of growth of negative magnetization.

Still increasing the load, we see that the curve (Fig. 4) becomes nearly symmetrical with respect both to the line of zero magnetization, and the line of zero twisting. In this case we find further that the range of the change of magnetization has considerably diminished. The slight excess of the initial magnetization over the other still shows itself, a fact which is probably to be referred to the direction of the magnetizing force.

The general conclusion from these experiments is that in a weak field of .34 units, the increase of load makes the manner of change of magnetization in nickel under the influence of cyclic twisting depart more and more from any slight resemblance which at small loads it seemed to bear to the manner of change for iron. For still smaller load, then it might be possible to obtain the magnetization curve just opposite to that of iron.

In the second series of experiments, the strength of the field was raised to 2.47 units. The load at first applied was only the weight of the brass wire attached to the lower end of the nickel wire, and a brass rod which gripped the wire during the process of twisting. This load was .02 kg., that is, a tension of 2.6 kgs. weight per sq. cm. With this amount of longitudinal stress, the successive twistings and untwistings were performed 7 times, and the following readings of deflection were taken :—

Twist (Positive.)	Readings.	Twist (Negative.)	Readings
0°	118.7	0	118.0
20°	166.4	20	167.4
40°	181.5	40	183.5
60°	193.2	60	195.3
80°	201.4	80°	202.9
100°	205.9	100	208.0
120°	209.0	120°	210.0
140°	210.3	140°	211.4
160	210.9	160	211.5
180°	211.0	180°	211.6
160°	207.4	160°	207.9
140	201.8	140	202.0
120°	192.5	120	193.4
100°	177.2	100°	178.3
80°	157.3	80°	157.6
60°	136.6	60	137.5
40°	126.9	40°	127.0
20	133.0	20°	132.2
0	118.0	0	118.0

The curve thus obtained (see Fig. 5) is nearly perfectly symmetrical with respect to the line of zero twisting. Also, the magnetization remains positive throughout the whole cycle. It is moreover interesting to observe, that the curve is exactly the reverse for that of iron as obtained by Thomson. This shows that the behaviour of nickel twisted in a magnetic field under feeble loads is opposite to that of iron.

When the stress was increased to 145 kgs. weight per sq. cm., the magnetization curve lost its symmetry and became as shown in Fig. 6. The intensity of magnetization became greatly diminished, but still remained positive throughout the whole cycle of operations. The features to be noted are that, although suggestive of the symmetrical form given in Fig. 5, the curve is now distinctly one-sided with respect to the line of zero twisting, and indicates much greater magnetization for positive than for negative twists. These peculiarities are still more pronounced when the stress is increased to 209 kgs. as shown in Fig. 7.

This experiment is of special interest, illustrating as it does, the manner in which the magnetization cycle changes character just as the extraordinary phenomenon of change of sign is about to show itself.

As usual the first effect of twisting the wire is to increase the intensity of magnetization, while the effect of untwisting is to decrease it. Then as the untwisting is continued as negative twisting, the magnetization tends to recover its former value. But for the maximum negative twist of two right angles, the magnetization does not nearly recover its former value. Thus it is evident that in such a strength of field, the increase of longitudinal stress tends to make the increase of magnetization during negative twisting gradually less and less until finally for a certain load the increase does not take place at

all for the particular range of twist. This is shown in Fig. 8, which is the curve for a tension of 782 kgs. weight per sq. cm.

A study of these from curves (Figs. 5, 6, 7, 8,) shows the character of the changes wrought in the cycles as the load is increased. The symmetry is first lost, the negative loop becoming smaller and smaller. Then, as shown in Fig. 7, it ceases to be a loop, the course of the return curve from greatest negative twist lying above the other, and never cutting it. Then as in Fig. 8, the *upward* course of the curve on the negative side of zero twist vanishes away altogether; while at the same time the phenomenon of reversal of magnetic polarity shows itself. Thus the double-looped curve for low tensions passes gradually into a single-looped curve as the tension is increased. And after this single-looped curve is obtained, the phenomenon of the reversed polarity begins to appear. The passage from the double looped to the single looped curve betokens a peculiar alteration in the lagging-effect in nickel—an alteration which has no analogue in the case of iron.

To study more carefully the law of *hysteresis* in nickel—to use Professor Ewing's word—the experiments were repeated in stronger fields.

Figs. 9, 10, 11, 12 show the march of events in a field of 1.94 units, the tensions increasing from 100. For smaller tensions the curves are of the approximately symmetrical form shown already in Fig. 5, and do not call for special remark. Here then we see how the already diminished loop for negative twisting, as shown in Fig. 9, has vanished altogether in Fig. 10. At the same time, the curve begins to cross the lines of zero magnetization into the negative region. Fig. 11 is a further development in the same direction. In both of these curves (10 and 11), the quantity $\frac{d\Im}{d\tau}$, the rate of change of magnetization with twist, still changes sign at particular twists.

The negative *tail*—as it might be called—so evident in Fig. 10, has disappeared in Fig. 11; while at the same time, the negative magnetization has numerically increased. But now passing to the higher load (see Fig. 12), we see that the various changes discussed above have their final end in a simple single-looped curve, with a large portion in the region of negative magnetization, and with no true minimum points for $\mathfrak{J}$. The twist for which, in the first three cases, the value of $\frac{d\,\mathfrak{J}}{d\,\tau}$ changes sign, works towards the left as the load is increased. Thus

$$\begin{array}{llllll} \text{for} & W = 400 \;, & d\mathfrak{J}/d\tau & = 0 & \text{at} & +\ 8 \\ ,, & ,, = 527 \;, & ,, & = 0 & \text{at} & -\ 20^\circ \\ ,, & ,, = 655 \;, & ,, & = 0 & \text{at} & -\ 60^\circ \end{array}$$

For $W = 1910$, we may regard this critical twist as being too great to be included in the range of greatest twist applied.

It was remarked while discussing the experiment made in the earth's vertical field, that the ratio of the two opposite magnetizations gradually tends to unity as the load is increased; but this does not seem to be generally the case. There is a certain limit beyond which there is an opposite tendency. The following calculations shows that this must be the case. Let $+\mathfrak{J}$ and $-\mathfrak{J}$ be the greatest magnetization during positive and negative twists respectively, then in the field $= 1.94$ units, and

$$\begin{array}{l} W = \ \ 655: \ +\mathfrak{J} = 188.6, \ -\mathfrak{J} = \ 49.9, \ +\mathfrak{J} - \mathfrak{J} = 3.78 \\ W = \ \ 910: \ +\mathfrak{J} = 152.1, \ -\mathfrak{J} = 151.3, \ +\mathfrak{J} - \mathfrak{J} = 1.00 \\ W = 1270: \ +\mathfrak{J} = 119.3, \ -\mathfrak{J} = 113.1, \ +\mathfrak{J} - \mathfrak{J} = 1.05 \\ W = 1910: \ +\mathfrak{J} = \ \ 86.3, \ -\mathfrak{J} = \ \ 77.4, \ +\mathfrak{J} - \mathfrak{J} = 1.11 \end{array}$$

The following experiments in field $= 6.71$, show how this ratio depends on the strength of the field. The changes which the magnetization curve undergoes while it is changing its sign are quite

analogous to the preceding two cases, as a glance at Figs. 13, 14, and 15 will show. From Figs. 15, 16, 17, however, we see how the opposite magnetization becomes smaller as we increase the field. The ratio $+\mathfrak{J}/(-\mathfrak{J})$ is always a very large quantity and has a minimum value for a particular load. For loads greater than this particular load, the negative magnetization decreases, and at last completely vanishes. Thus for stress $W=1770$ kgs., there is no opposite magnetization therein agreeing with the curves for loads smaller than that for which the negative magnetization first appears. But there is the difference that the curve is a single loop in which $d\mathfrak{J}/\mathfrak{J}\tau$ changes sign during negative twisting and untwisting.

Taking into account all the series of experiments, we see that the greatest value in each series of the ratio $+\mathfrak{J}/-\mathfrak{J}$ tends to increase as the field is increased. The strength of field in which these last experiments were tried seems to be about the critical value for which we can get the two transitions of magnetization—namely the change of sign for particular load and the vanishing of this change for a higher load.

The peculiarities, which have just been the subject of discussion, do not however persist at all strength of fields. At still higher fields, a different order of things comes in. Take for example the next series of experiments with a field of 8.06, as shown in Figs. 18, 19, 20, 21, 22. First of all, for the lower tensions, the two-looped curve is more symmetrical than it can be obtained at lower fields (see Fig. 18.) In Fig. 19, for a tension of 782, a symmetry begins to show itself; but the diminution instead of taking place in the left hand or negative loop, takes place in the right hand or positive loop. Figs. 20, 21, show the gradual vanishing away of this right hand loop as the load is increased. Also, exactly as in the former sets of experiments, the curve dips below the zero magnetization line, as the right hand portion

loses its loop character. Now if we were to compare Fig. 21 with Figs 17, 12, and 8, 2, and take no account of the intermediate links of development, we should at once regard them as being of essentially opposite character. In the four earlier cases, positive twist increased the magnetization, and negative twist diminished it ; but in the present case, the effects are exactly opposite. In the same way, Fig. 20 present features quite opposite to those presented by Figs. 1, 7, 11, and 16. However, just as in the former sets of experiments, the twist at which $d\mathfrak{J}/d\tau$ vanishes was shown to shift gradually to the negative side as the load was increased ; so in the present case, the twist at which $d\mathfrak{J}/d\tau$ vanishes shifts gradually to the right as the load is increased. These very interesting reversals of effects clearly depend on the strength of the field. Then again the particular strength of field at which the reversal of these effects begins to show itself seems to be connected with the fact already discussed that for a particular field the ratio $+\mathfrak{J}/(-\mathfrak{J})$ becomes infinite. In other words, when the strength of the field is such that under sufficient loading the reversal of polarity vanishes away, this seems to be the signal for the new set of conditions to appear. Up to this critical strength of field, it is the left hand loop in the typical symmetrical curve that gradually diminishes under loading. But for strengths of fields higher than this critical value, it is the right hand loop that disappears when the load is great enough. This reversal of effects also seems to be accompanied by a lingering of the magnetization near the zero (see figures 18, and 19.)

It is not necessary to discuss in detail other combinations of field and stress which were experimented upon. There are certain minor differences depending on strength of the field ; but the principal features for fields higher than 8 are the same. The essential characteristics can

be gathered from figures, brief explanations of which I shall content myself with giving

Fig. 22 – 23.

These illustrate the changes of magnetization in field 11.9.

Fig. 24 – 27.

These were obtained in field 13.85. The curious transition curve Fig. 30 is specially worthy of note.

Fig. 28 – 31.

These show the gradual changes of magnetization for field 15.78.

Fig. 32 – 34.

were obtained in field 23.50.

Fig. 35 – 40.

were obtained in field 33.54.

This last was the highest strength of the field at which it was possible to notice the changes of magnetization. For the stronger the field the higher is the load necessary to bring out the curious changes. The critical load for fields higher than 33.54 is greater than the tenacity of the wire.

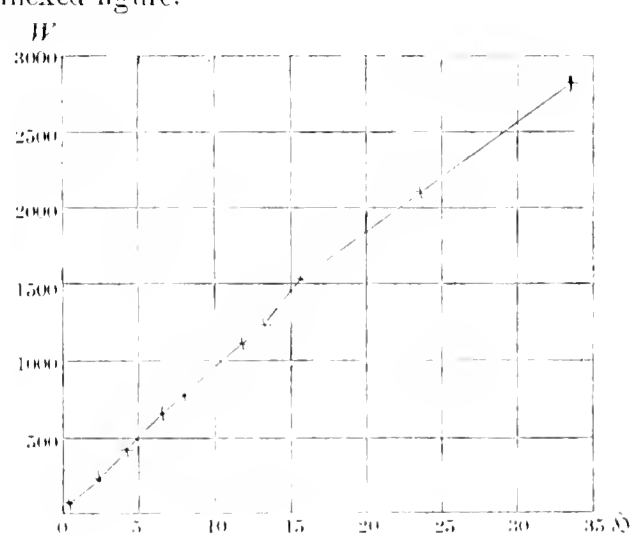
There is one other point that calls for remark. The range of the change of magnetization under feeble stresses begins gradually to diminish after a certain strength has been reached, so that for a field of 20 or 30, the changes of magnetization by twisting becomes almost inappreciable. This can be accounted for by the fact that nickel wire whether in the normal or twisted condition behaves practically the same as regards magnetization in higher fields. Indeed, nickel is more easily saturated than iron, so that when $\mathfrak{H} = 20$ or 30, it is already far beyond the saturation point. Consequently the difference in the susceptibilities of nickel in the normal and twisted conditions, to which this alteration of magnetic intensities must be ascribed, becomes less and less marked as the strength of the field is increased. Many of

the features of magnetization curve are quite simply accounted for by this consideration.

It is very difficult to determine the exact loading at which the reversed phenomenon of reverse polarity makes its appearance in various fields. Assuming, however, that within small ranges of load, the decrease in the intensities of magnetization is proportional to the amount of loading, I obtained the following values for the stress which must be applied so as to effect the reversal of polarity of the particular nickel wire placed in various fields :—

For $\mathfrak{H}$ =	.34 ,	W =	77	kgs. cm².
,, ,, =	2.47 ,	,, =	247	,,
,, ,, =	4.91 ,	,, =	421	,,
,, ,, =	6.74 ,	,, =	654	,,
,, ,, =	8.06 ,	,, =	783	,,
,, ,, =	11.92 ,	,, =	1120	,,
,, ,, =	13.85 ,	,, =	1220	,,
,, ,, =	15.78 ,	,, =	1530	,,
,, ,, =	23.50 ,	,, =	2140	,,
,, ,, =	33.54 ,	,, =	2830	,,

Plotting the curve with $\mathfrak{H}$ for abscissa and W for ordinate, we get the annexed figure.



This seems to show that for moderate strengths of field, the load at which the wire begins to show reversed polarity, is nearly directly proportional to the strength of the field. For very weak and strong fields, this rule does not seem to hold. It must be remembered of course that all these peculiarities are for one particular twist only, and that it is possible that quite a different series of effects might exist for other twists.

The general results of these experiments may be thus summarised.

In all magnetic fields with moderate loading the effect of twisting nickel wire is to increase the magnetization. But the increase depends on the strength of the field as well as the longitudinal stress applied. If the field is weak, and the longitudinal stress sufficiently great, the magnetization increases in one direction of twist, and decreases in the other. Eventually for a particular stress which is approximately proportional to the field, the wire begins to show opposite polarity; and the cyclic curve of magnetization passes gradually from a two-looped to a single looped form. For stronger fields similar effects exist. But in fields higher than a critical value, the increase and decrease of magnetization take place for reversed directions of twist, and at the same time, the course of the curve becomes reversed.

In a recent paper (Magnetische Untersuchungen, Wiedemann's Annalen, March 1886) Professor Wiedemann has described certain experiments on the combined effects of magnetization and twist in iron and nickel. He does not seem, however, to have investigated the effect of longitudinal stress in conjunction with these. His chief aim seems to have been to adduce facts in support of his theory of frictionally rotated molecules. Some of the results above described may be expressed in terms of his theory. Thus when the external magnetizing force is great, the magnetic molecules will be held in position more strongly and consequently, the change of magnetization due to

twisting will be diminished. This agrees with experiment. But again we saw that by sufficiently loading the wire, we could bring the apparent magnetization down to zero and eventually reverse its sign by mere twisting. Now if this be due to the frictional rotation of molecules, the molecules must, notwithstanding the directive force of 30 units, be rotated through more than a right angle from their first position, while the amount of mechanical twist amounts only to .079 radian per cm. in each direction. Admit it to be so; what effect then may we expect increased loading to produce on the rotation of the molecules? The magnetic molecules in strong fields are acted on only by a greater directive force, and consequently they must tend to remain more in the direction of the magnetizing force; but why they should assume nearly the position of magnetic neutrality when they are subjected to sufficient longitudinal stress, is a question which every supporter of the theory of frictionally rotated molecules is bound to answer.

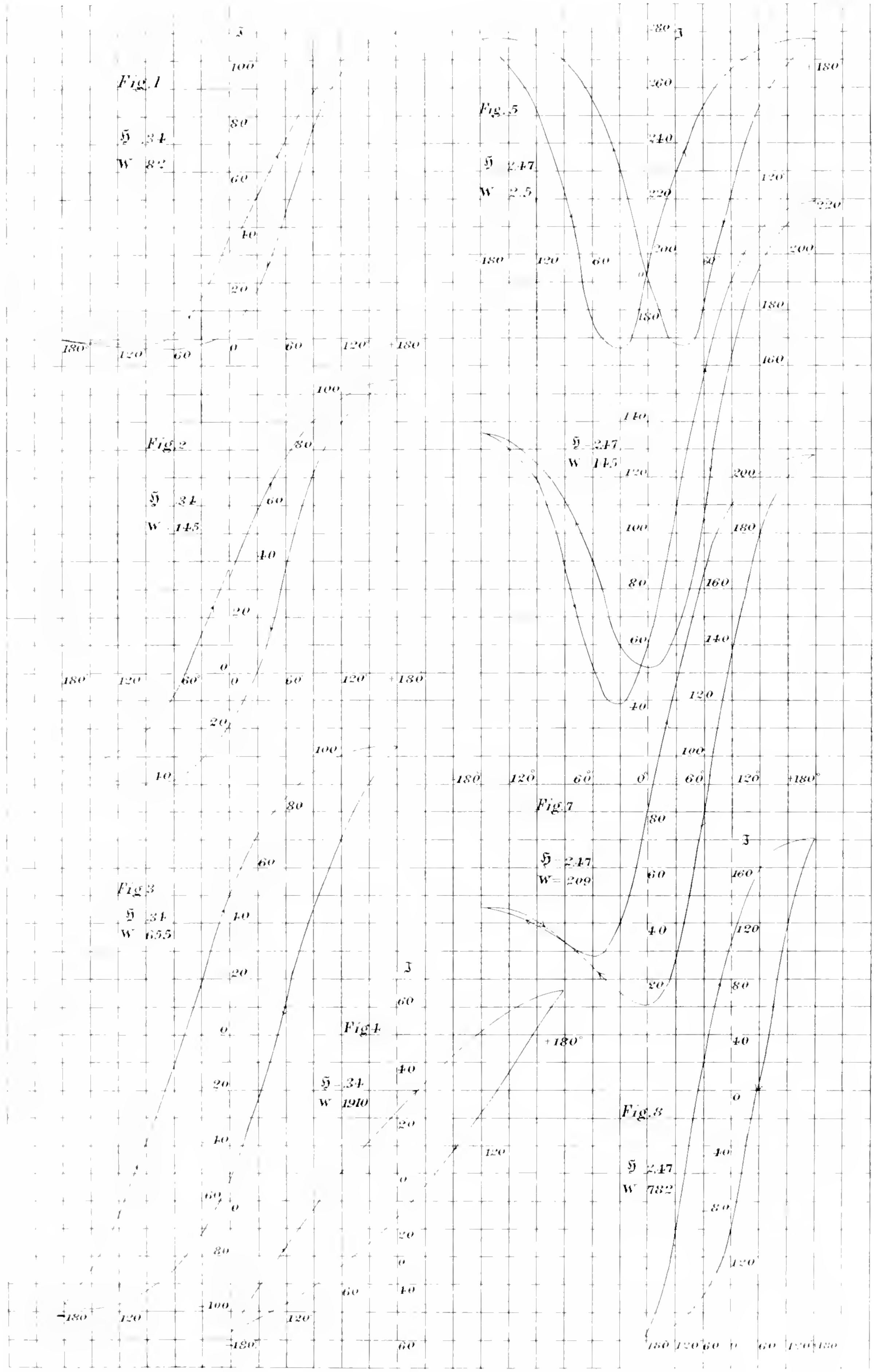


Fig. 1
ℌ 34
W 82
Fig. 2
ℌ 34
W 145
Fig. 3
ℌ 34
W 655
Fig. 4
ℌ 34
W 1910
Fig. 5
ℌ 247
W 2.5
ℌ 247
W 145
Fig. 7
ℌ 247
W = 209
Fig. 8
ℌ 247
W 782

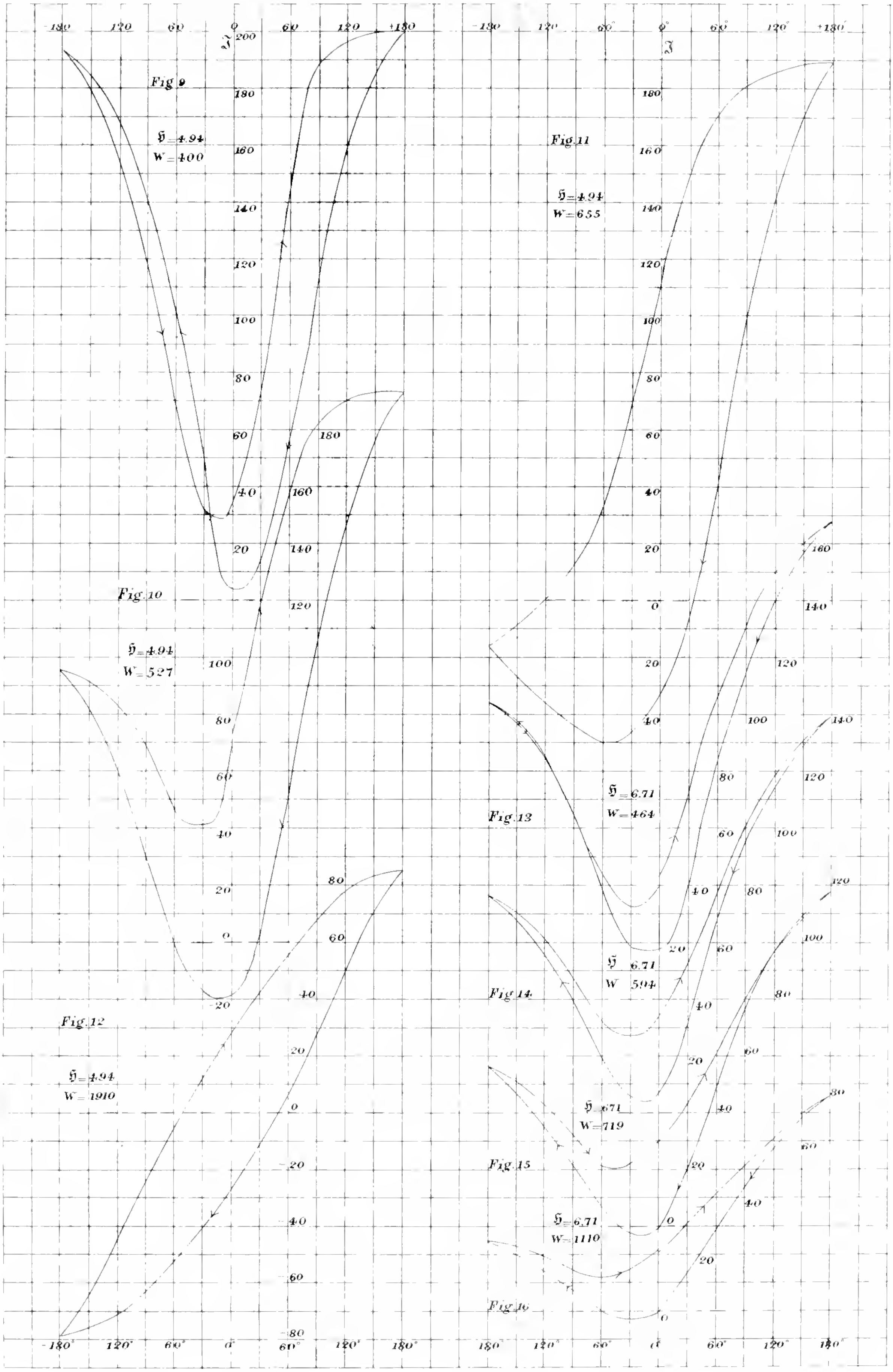


Fig. 9
$\mathfrak{H} = 4.94$
W = 400
Fig. 10
$\mathfrak{H} = 4.94$
W = 527
Fig. 11
$\mathfrak{H} = 4.94$
W = 655
Fig. 12
$\mathfrak{H} = 4.94$
W = 1910
Fig. 13
$\mathfrak{H} = 6.71$
W = 464
Fig. 14
$\mathfrak{H}$ 6.71
W 504
Fig. 15
$\mathfrak{H}$ 6.71
W = 719
Fig. 16
$\mathfrak{H} = 6.71$
W = 1110

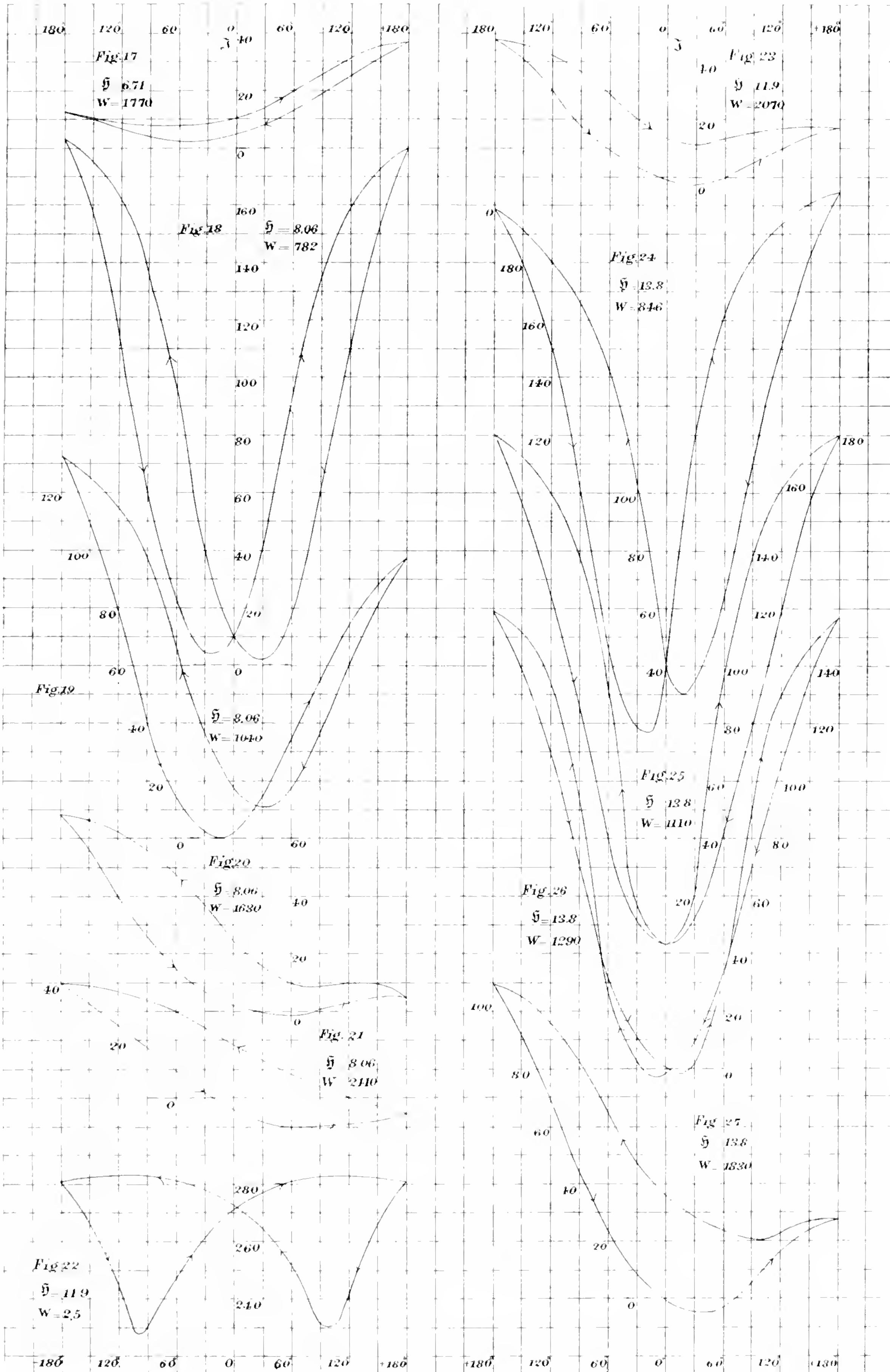


Fig. 17
𝔥 = 6.71
W = 1770
Fig. 18
𝔥 = 8.06
W = 782
Fig. 19
𝔥 = 8.06
W = 1040
Fig. 20
𝔥 = 8.06
W = 1630
Fig. 21
𝔥 = 8.06
W = 2110
Fig. 22
𝔥 = 11.9
W = 2.5
Fig. 23
𝔥 = 11.9
W = 2070
Fig. 24
𝔥 = 13.8
W = 846
Fig. 25
𝔥 = 13.8
W = 1110
Fig. 26
𝔥 = 13.8
W = 1290
Fig. 27
𝔥 = 13.8
W = 1830

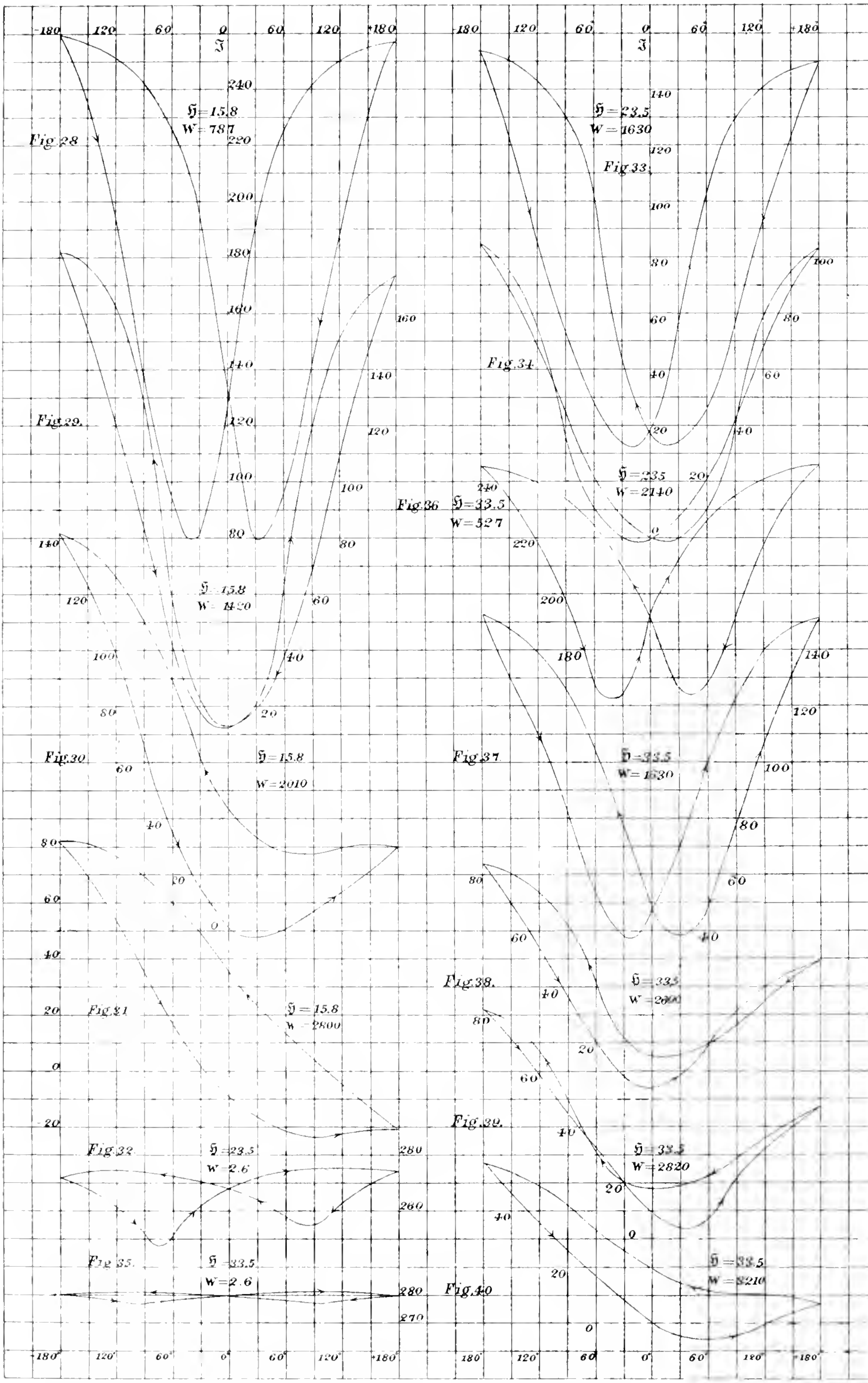


Fig. 28
$\mathfrak{H}$=15.8
W=787
Fig. 29.
$\mathfrak{H}$=15.8
W=1420
Fig. 30
$\mathfrak{H}$=15.8
W=2010
Fig. 31
$\mathfrak{H}$=15.8
W=2800
Fig. 32
$\mathfrak{H}$=23.5
W=2.6
Fig. 35.
$\mathfrak{H}$=33.5
W=2.6
Fig. 33.
$\mathfrak{H}$=23.5
W=1630
Fig. 34.
$\mathfrak{H}$=23.5
W=2140
Fig. 36
$\mathfrak{H}$=33.5
W=527
Fig. 37
$\mathfrak{H}$=33.5
W=1630
Fig. 38.
$\mathfrak{H}$=33.5
Fig. 39.
$\mathfrak{H}$=33.5
W=2820
Fig. 40
$\mathfrak{H}$=33.5
W=3210
$\mathfrak{I}$
180° 120° 60° 0° 60° 120° 180°

Appendix.

	$\mathfrak{H} = .34$				$\mathfrak{H} = 2.47$				$\mathfrak{H} = 4.94$			
τ	$\mathfrak{J}$ $W=82$ (*Fig.* 1)	$\mathfrak{J}$ $W=273.$	$\mathfrak{J}$ $W=655$ (*Fig.* 3)	$\mathfrak{J}$ $W=1280.$	$\mathfrak{J}$ $W=2.5$ (*Fig.* 5)	$\mathfrak{J}$ $W=145$ (*Fig.* 6.)	$\mathfrak{J}$ $W=273.$	$\mathfrak{J}$ $W=782$ (*Fig.* 8.)	$\mathfrak{J}$ $W=82.$	$\mathfrak{J}$ $W=400$ (*Fig.* 9.)	$\mathfrak{J}$ $W=655$ (*Fig.* 11.)	$\mathfrak{J}$ $W=1270.$
+0	32	54	51	33	195	59	112	109	207	36	115	19
+60	73	99	87	61	253	155	196	159	250	143	171	90
+120	100	112	100	77	275	209	215	178	274	197	186	113
+180	110	115	103	83	276	220	220	182	280	200	189	119
+120	96	93	71	50	252	194	185	121	259	160	142	65
+60	47	24	8.4	7.1	179	107	53	9.4	171	57	43	−1
−0	4.5	−52	−51	−29	195	51	−29	−82	208	77	−33	−51
−60	−3.1	−81	−85	−53	277	88	−69	−130	257	103	−50	−87
−120	−2.4	−87	−95	−68	275	125	−79	−150	280	169	−38	−108
−180	00	−86	−97	−73	256	136	−77	−154	282	193	−16	−113
−120	−.4	−66	−65	−41	253	118	−64	−95	251	155	−0.1	−63
−60	2.5	−15	−8.6	−1.3	180	55	−16	11	176	68	35	−1.3
−0	38	54	51	34	194	59	110	107	209	35	110	49
	$W=145$ (*Fig.* 2.)	$W=400.$	$W=910.$	$W=1910$ (*Fig.* 4.)	$W=82.$	$W=209$ (*Fig.* 7.)	$W=527.$		$W=273.$	$W=527$ (*Fig.* 10.)	$W=910.$	$W=1910$ (*Fig.* 12.)
+0	34	57	41	23	107	81	126		92	73	79	26
+60	78	97	75	43	191	168	181		173	159	121	55
+120	99	110	92	59	240	203	200		210	190	150	78
+180	105	114	97	65	251	209	204		221	193	152	86
+120	91	85	61	38	224	176	159		184	149	69	49
+60	39	16	4	7.6	139	77	44		85	51	−15	6.8
−0	21	−60	−46	−16	110	11	−78		87	−19	−85	−27
−60	−37	−90	−73	−35	180	23	−140		186	−2.8	−134	−53
−120	−37	−98	−84	−49	222	40	−163		236	61	−150	−71
−180	−36	−99	−86	−56	235	48	−167		257	96	−151	−77
−120	−29	−62	−51	−30	210	42	−125		216	83	−98	−44
−60	−7.0	3.0	−3.7	−1.1	128	26	−17		112	47	−7.2	4.2
−0	38	61	42	24	108	81	−122		95	64	81	28

	$\mathfrak{H}=6.71$			$\mathfrak{H}=8.06$			$\mathfrak{H}=11.92$			$\mathfrak{H}=13.85$	
τ	$\mathfrak{J}$ $W=464$ (*Fig.* 13.)	$\mathfrak{J}$ $W=719$ (*Fig.* 15.)	$\mathfrak{J}$ $W=1770$ (*Fig.* 17.)	$\mathfrak{J}$ $W=655.$	$\mathfrak{J}$ $W=1010$ (*Fig.* 19.)	$\mathfrak{J}$ $W=2410$ (*Fig.* 21.)	$\mathfrak{J}$ $W=2.5$ (*Fig.* 22.)	$\mathfrak{J}$ $W=1110.$	$\mathfrak{J}$ $W=2070$ (*Fig.* 13.)	$\mathfrak{J}$ $W=846$ (*Fig.* 24.)	$\mathfrak{J}$ $W=1290$ (*Fig.* 26.)
$+\ 0°$	15	30	12	58	0	$-\ 1.8$	271	26	$-\ 1.8$	40	0
$+\ 60$	101	59	21	155	32	$-\ 7.9$	281	134	0.0	163	33
$+120$	148	95	32	195	76	$-\ 8.1$	283	176	9.6	196	131
$+180$	166	116	39	204	97	$-\ 7.4$	281	185	18	203	156
$+120$	140	95	28	126	62	3.3	243	115	17	152	103
$+\ 60$	79	49	13	31	15	7.3	249	27	12	65	36
0	17	1.7	4	54	18	17	273	15	15	40	0
$-\ 60$	12	7.6	3	152	66	30	281.7	110	28	142	31
-120	86	38	8	194	112	37	282	143	42	182	131
-180	104	56	11	205	131	41	281	151	49	199	157
-120	85	44	8	126	78	24	244	90	28	149	107
$-\ 60$	15	24	7	32	17	8.4	247	19	7.6	50	38
$-\ 0$	10	31	11	55	5	$-\ 1.5$	271	24	$-\ 1.7$	39	.5
	$W=591$ (*Fig.* 14.)	$W=1110$ (*Fig.* 16.)		$W=782$ (*Fig.* 18.)	$W=1630$ (*Fig.* 20.)		$W=971.$	$W=1170.$		$W=1110$ (*Fig.* 25.)	$W=1830$ (*Fig.* 27.)
$+\ 0°$	33	22		10	4.5		83	$-\ 1.7$		0.0	$-\ 1.0$
$+\ 60$	74	41		89	$-\ 1.3$		168	93		93	$-\ 2.1$
$+120$	118	61		159	2.6		192	159		163	14
$+180$	137	74		179	6.4		197	175		179	27
$+120$	116	58		107	8.9		117	111		125	24
$+\ 60$	63	27		19	10		12	22		43	21
0	7	0		11	25		81	$-\ 12$		2	32
$-\ 60$	19	0		95	48		144	73		87	66
-120	58	14		163	63		162	127		161	97
-180	76	25		182	70		166	139		179	109
-120	61	19		112	41		95	83		125	67
$-\ 60$	32.5	14		23	15		24	14		42	26
$-\ 0$	33.3	22		9	3		86	$-\ 2$		9	$-\ .5$

	$\mathfrak{H} = 15.78$				$\mathfrak{H} = 23.5$			$\mathfrak{H} = 33.54$			
τ	$\mathfrak{J}$ $W=527$	$\mathfrak{J}$ $W=1040$	$\mathfrak{J}$ $W=2010$ (*Fig.* 30.)	$\mathfrak{J}$ $W=2800$ (*Fig.* 31.)	$\mathfrak{J}$ $W=2.5$ (*Fig.* 32.)	$\mathfrak{J}$ $W=1070$	$\mathfrak{J}$ $W=2140$ (*Fig.* 34.)	$\mathfrak{J}$ $W=2.5$ (*Fig.* 35.)	$\mathfrak{J}$ $W=1070$	$\mathfrak{J}$ $W=2140$	$\mathfrak{J}$ $W=2820$ (*Fig.* 39.)
+ 0°	200	35	3,5	− 8,2	268	97	.4	279,8	88	19	7,9
+ 60	214	153	1,4	− 22	274	160	12	281,1	147	60	5,1
+120	259	200	16	− 24	276	186	74	280,2	176	95	32
+180	262	212	30	− 22	275	195	102	279,8	188	107	46
+120	197	111	30	4	263	131	66	276,7	148	70	36
+ 60	115	51	29	14	255	53	23	277,7	84	30	23
− 0	199	31	46	36	269	99	− .4	280,5	90	18	17
− 60	244	132	189	60	273	161	12	281,4	149	67	28
−120	261	204	127	[illegible]	275	187	73	281,1	177	107	66
−180	266	216	141	83	273	196	106	279,8	188	120	82
−120	204	118	99	52	261	131	67	277,0	148	79	57
− 60	111	55	40	18	252	54	24	276,7	104	31	21
− 0	203	32	4,9	− 7,7	268	97	.5	279,8	88	16	7,6
	$W=782$ (*Fig.* 28.)	$W=1420$ (*Fig.* 29.)	$W=2260$		$W=527$	$W=1630$ (*Fig.* 33.)		$W=527$ (*Fig.* 36.)	$W=1630$ (*Fig.* 37.)	$W=2690$ (*Fig.* 38.)	$W=3210$ (*Fig.* 40.)
+ 0	123	11	− 3,5		174	18		194	47	5,6	1,1
+ 60	221	56	− 18		209	102		228	103	18	− 5,3
+120	249	152	− 17		229	141		242	139	10	− .4
+180	256	172	− 7,9		236	150		247	152	49	5,3
+120	187	107	.7		194	86		218	106	34	8,8
+ 60	81	38	11		124	26		171	49	19	12
− 0	131	14	25		175	18		188	18	16	19
− 60	225	63	52		210	99		225	104	12	34
−120	253	164	102		230	143		241	142	73	19
−180	260	184	122		236	153		246	154	83	57
−120	194	121	79		177	90		218	109	53	37
− 60	97	44	31		127	29		169	51	23	16
− 0	128	13	3,3		174	18		193	47	3,9	.8

On the Magnetization and Retentiveness of Nickel Wire under combined Torsional and Longitudinal Stresses.

By

H. Nagaoka, *Rigakushi,*

of the

Imperial University.

With Plates XX—XXIV.

In the experiment described in the preceding paper, the magnetizing field around the nickel wire was kept constant, while the wire itself was subjected to varying twists. The quite unexpected results thus obtained suggested to me the advisability of examining carefully the magnetization of nickel wire at constant twists under the influence of gradually increasing and decreasing magnetizing fields. The distinct lines of research presented themselves; and the experiments naturally fall to be discussed under two heads.

(1.) There is the determination of the induced magnetism due to various magnetizing forces in a twisted nickel wire subjected at the same time to longitudinal stress.

(2.) There is the investigation of the relation between the induced and residual magnetisms under the conditions just specified.

I. Relation between Magnetization and Magnetizing Force.

The experiments to be described are concerned with the peculiarities of magnetization of nickel wire subject to combined torsional and longitudinal stresses. The wire under examination was first demagnetized by successive reversals of a current of gradually diminishing strength. A definite load was then applied, and the wire

twisted through a definite angle by means of the twisting apparatus described in the preceding paper. The wire was then subjected to magnetizing currents of gradually increasing strengths, and the amount of induced magnetism measured by means of the deflection of the contiguous magnetometer mirror. The details of the arrangement and method are as follows.

The magnetizing current was obtained from 14 large Daniell cells, and the strength was varied continuously by means of a liquid slide. A commutator, consisting of rocker and mercury pools, was included in the circuit, so as to facilitate the reversing of the diminishing current which was necessary for demagnetizing the wire in position. The reversals were continued till the wire became magnetically neutral.

There was also an arrangement for compensating the direct electromagnetic action of the magnetizing coil—a real necessity in such experiments with nickel, because of the comparatively small susceptibility of this metal. The arrangement consisted of a ring with a few coils of wire wound round it. This ring mounted at the proper height on a wooden stand, which could be moved along a groove cut in the plank on which the magnetometer rested, was set in front of the magnetometer. The magnetizing current was led through this small coil, whose distance from the magnetometer was adjusted, so as to compensate the effect due to the solenoid. It was very difficult to set the ring exactly at the desired position, so that it was generally necessary to apply small corrections to the readings obtained.

In the first experiment a nickel wire 1 mm. in diameter and 40 cms. long was previously heated red hot. This was placed in the magnetizing solenoid and demagnetized without at first any application of longitudinal stress. The strength of the magnetizing field

was then gradually increased till it attained the value of 29 C. G. S units, magnetometer readings being taken at intervals. The residual magnetism was determined as the magnetizing current was gradually reduced from its maximum value to zero. The normal curve of magnetization thus obtained is represented in Fig. 1.*(a)*. After this, the wire was demagnetized by reversals, and twisted through an angle of 40°, so as to give a twist of 1° per cm. Under these slightly altered conditions, the experiment was repeated. Up to the strength of field $\mathfrak{H} = 4.2$, things were sensibly the same as in the former case. But as $\mathfrak{H}$ was raised to 5.2, a very sudden change occurred in the magnetization. The magnetometer reading rose abruptly from 8 to 97; and then as $\mathfrak{H}$ was still further increased to 6.1, the reading rose to 144. But after $\mathfrak{H}$ attained the value of 9.1, the rate of increase of induced magnetism per unit of the increase of the magnetizing field suddenly diminished, almost immediately falling off to a value which remained nearly constant for higher fields. That is, the magnetization curve at these higher fields becomes almost a straight line. Thus it appears that twisted nickel has an enormous *differential susceptibility* $(d\mathfrak{J}/d\mathfrak{H})$* within a certain quite limited range of field, and that for fields outside the lower and higher limits the differential susceptibility is comparatively small, and at high fields practically constant. Curve Fig. 1 *(b)* represents the case just discussed. No peculiar character as compared with the normal curve *(a)* is seen at a glance. The "wendepunkt" or point of maximum susceptibility occurs in a smaller field for the twisted nickel than for the untwisted. The value of the maximum susceptibility is also greater for the twisted nickel. But when once the "wendepunkt" for the twisted nickel is passed, the differential susceptibility rapidly diminishes in such a

* This term is suggested by Dr. C. G. Knott. Susceptibility always means the ratio of the magnetization to the magnetizing force that is producing it; and it is convenient, especially in discussing the peculiarities of nickel, to have a simple but suggestive phrase for "the rate of change of magnetization per unit increase of field."

way that above a certain field (27.6) the twisted nickel has a smaller susceptibility than the untwisted. This is shown by the curve *(b)* crossing the curve *(a)* at this particular field.

The wire was now demagnetized, and twisted through an angle of 80°, that is 2° per cm. The magnetization curve obtained in this state is shown graphically in Fig. I *(c)*. The chief characteristics remain the same as for *(b)*; but the "wendepunkt" occurs sooner, and the curve rises more abruptly to this point than for the case of smaller twist. Another thing to be remarked is the increase of residual magnetism. For the untwisted nickel, the ratio of the residual to the induced magnetism is .75 for $\mathfrak{H} = 29$; for the wire twisted through an angle of 1° per cm. it is .95; and for the twist of 2° per cm., it is .96. For a twist of 120° or 3° per cm. these characteristics acquire the maximum condition; the curves of magnetization rising less abruptly, and the maximum susceptibility as shown by the "wendepunkt" becoming smaller, when the twist is increased to 4°.5 per cm. A twist of 9° per cm. still further diminishes this abruptness; but the recovery of the wire towards the original magnetic condition in the untwisted state is very slow. Indeed at all twists the magnetization curve presents much the same characteristics. The general results of these experiments are as follows. Nickel wire twisted under no load acquires its maximum susceptibility in a particular field of force, which is a function of the twist, and is a minimum for a twist of 3° or 4° per cm. In other words, if we draw a line from the origin to the "wendepunkt," the line (the tangent of whose inclination gives the maximum susceptibility) is, so to speak, gradually deflected to higher inclinations as the wire is twisted up to 3° or 4° per cm. It then begins to move back again in the direction of its original position, but very slowly, so that there is not very much difference in the curves for highly twisted wires and a moderately twisted

wire. The maximum value of the differential susceptibility is very great for twisted wire, and is greatest for the twisted wire which has the greatest maximum susceptibility. After passing the "wendepunkt," the magnetization curve becomes almost straight, the differential susceptibility becoming very small and of such a nature as to cause the nickel always to cut the curve for untwisted nickel. This point of intersection occurs sooner for higher twists. And finally the amount of residual magnetism is enormously increased by twisting the wire even through a small angle.

In the experiments now to be described, the wire was subjected to extra loads. It was necessary, however, to take a new wire cut of course from the same specimen, and treated preliminarily as the first piece was.

The first extra load used was nearly 1 kg.; and with this load exactly the same series of experiments was gone through as in the previous case. In Fig. 2, the curves of magnetization are shown graphically. They are very similar to the curves of Fig. 1 ; only in this case it was not possible with the strength of field used to get the curves for the twisted nickels to intersect the curve for the untwisted nickel. In all probability, however, the crossing will occur for high enough fields.

It was natural to enquire, here, as to the after effect of the twist. It is known that, because of the elastic after-strain, the wire does not return exactly to its former position when the twisting stress is relieved. It seemed likely that something analogous would occur in reference to the magnetic characteristics of the wire. Accordingly the wire, after it had been twisted, was allowed to hang freely under the action of the load ; and under these conditions, the magnetization curve was obtained. Comparing it with the first curve, we see that there is distinct evidence of a change in the magnetic properties of

the wire. The susceptibility of the wire that has been twisted is in general diminished to a slight extent, although in one or two places, it is increased.

Further experiments made by subjecting the wire to longitudinal stresses of 5.14 and 10.14 kgs. gave the same general results. The curves are shown in Fig. 3 and Fig. 4 respectively. All the curious characteristics are the same as before. Comparing the curves for untwisted with the curves for twisted wire, we see how marked is the difference between these two conditions, and especially when the wire is subjected to considerable longitudinal stresses. The curves show very readily that the magnetic susceptibility for a field of 27 units of the untwisted wire with a load of 5 kgs. is only one half the susceptibility for the same with a twist of 1° per cm. But judging from the march of the curve, we may safely conclude that the susceptibility of the untwisted wire will become greater than that of the twisted wire, when the strength of the field is sufficiently increased. With a load of 10 kgs., the susceptibility of the wire which has suffered a torsion of 3° per cm. is 3 times as great as that of the untwisted wire even at the field of 27 units.

Another thing to be noticed is the receding of the "wendepunkt" for the twisted wire under heavy load. This point tends to move gradually towards the right as the load is increased at constant twist. This is not so well marked in the first two experiments, but is quite apparent when the longitudinal stress amounts to 5 or 10 kgs. Thus for the unloaded wire with a twist of 1° per cm., the "wendepunkt" lies at $\mathfrak{H} = 6.3$, while for the wire under the longitudinal stress of 5 kgs., it occurs at $\mathfrak{H} = 12$. Similarly for a wire with a twist of 3° per cm., the "wendepunkt" occurs at $\mathfrak{H} = 4.3$ for the unloaded, and at $\mathfrak{H} = 11$ for the loaded wire. This is still more marked when the wire is loaded with 10 kgs.

The following tables contain the observed numbers :—

I. W = 2.5 kgs. cm^2.

$\mathfrak{H}$	$\mathfrak{J}$ $\tau=0$.	$\mathfrak{J}$ $\tau=1^\circ$ per cm.	$\mathfrak{J}$ $\tau=2^\circ$ per cm.	$\mathfrak{J}$ $\tau=3^\circ$ per cm.	$\mathfrak{J}$ $\tau=4.5$ per cm.	$\mathfrak{J}$ $\tau=9^\circ$ per cm
2.2	10.1	4.3	1.8	2.0	2.0	4.0
3.2	...	...	...	47.4	9.7	11.2
3.4	...	...	69.5	...	...	...
4.2	22.6	13.2	238.2	243.6	135.1	164.8
5.2	36.5	160.2	282.3	285.9	279.0	260.5
6.1	70.3	238.3	289.7	293.0	288.9	276.4
7.1	109.4	269.6	295.2	295.8	...	...
8.1	131.7	277.7	297.5	297.7	293.5	285.1
10.1	170.9	286.3	301.1	300.3	...	...
11.9	200.6	291.4	303.3	301.8	298.0	290.4
13.9	220.8	295.7	304.8	...	...	...
15.8	236.8	299.3	305.9	304.4	300.8	293.2
19.6	272.9	304.3	307.7	305.9	302.3	291.4
23.5	295.4	308.9	309.4	...	303.4	295.7
29.2	316.1	313.0	311.7	307.6	304.6	297.3
23.5	310.7	310.4	309.5	306.4	304.1	296.8
15.8	298.2	306.1	307.7	305.7	303.9	296.5
8.1	278.0	299.6	306.4	305.5	301.6	294.9
0	237.9	290.2	300.0	298.0	293.2	286.1

II. W = 145 kgs. cm^2.

$\mathfrak{H}$	$\mathfrak{J}$ $\tau=0^\circ$ per cm	$\mathfrak{J}$ $\tau=1^\circ$ per cm.	$\mathfrak{J}$ $\tau=3^\circ$ per cm.	$\mathfrak{J}$ $\tau=9^\circ$ per cm.	$\mathfrak{J}$ $\tau=15^\circ$ per cm.	$\mathfrak{J}$ (Untwisted)
2.2	6.3	7.3	4.6	3.0	2.3	3.3
3.2	8.7	16.2	9.9	8.6	3.6	5.3
4.2	12.4	32.5	196.4	24.8	17.3	7.9
5.2	23.1	86.0	229.4	55.4	49.2	19.3
6.1	40.3	192.1	247.2	121.1	109.4	35.6
7.1	58.9	209.7	256.4	229.0	206.3	50.3
8.1	77.1	216.5	264.0	249.0	242.6	67.7
10.0	112.2	226.5	270.4	256.6	255.1	101.0
11.9	133.7	233.3	273.9	261.4	260.2	135.6
15.8	165.0	239.3	277.2	267.3	264.5	178.5
19.6	196.2	252.9	278.9	270.4	267.3	201.6
23.5	219.5	262.1	280.0	272.3	269.4	218.5
27.3	236.0	269.3	280.8	273.9	270.7	229.0
23.5	231.0	267.3	...		...	224.7
15.8	216.8	262.0	...		...	214.2
8.1	196.7	255.8	278.9	270.1	267.3	201.8
0	167.5	245.9	272.7	263.7	260.7	181.5

III. $W = 655$ kgs. cm^2.

$\mathfrak{H}$	$\mathfrak{I}$ $\tau = 0$.	$\mathfrak{I}$ $\tau = 1°$ per cm.	$\mathfrak{I}$ $\tau = 3°$ per cm.	$\mathfrak{I}$ $\tau = 5°.5$ per cm.	$\mathfrak{I}$ $\tau = 9°$ per cm.	$\mathfrak{I}$ (Untwisted.)
2.2	3.8	3.0	0.5	1.7	1.0	4.0
3.1	6.1	1.8	0.8	1.5	2.6	6.1
4.2	9.2	5.9	2.5	6.3	1.8	8.3
5.2	11.6	7.6	3.8	7.4	6.4	10.6
6.1	15.3	9.6	4.8	9.2	7.6	11.7
7.1	18.3	16.0	6.8	12.0	8.7	13.0
8.1	23.4	30.5	11.4	14.5	11.9	15.2
9.0	27.9	56.9	29.7	19.1	13.7	17.0
10.0	31.0	95.2	150.1	50.0	18.8	20.1
11.0	...	125.2	184.6	127.2	25.1	22.9
11.9	34.3	140.3	199.0	196.0	44.2	25.1
12.9	...	149.0	204.1	203.0	135.8	28.1
13.9	40.6	156.1	208.9	206.7	184.1	35.0
15.8	47.4	165.2	213.5	211.4	198.7	41.6
17.7	57.3	171.9	217.8	214.5	204.1	48.3
19.6	67.3	178.2	219.5	218.0	207.2	57.3
23.5	80.4	185.8	222.4	221.3	212.2	70.3
27.3	92.7	190.2	224.9	224.2	217.5	81.2
23.5	85.5	188.8	224.4	222.8	215.2	77.7
15.8	74.4	185.3	222.6	220.1	212.9	66.0
8.1	58.6	180.2	221.4	218.1	210.7	53.9
4.2	51.2	177.2	220.8	216.3	208.9	47.9
0	42.2	173.6	217.8	214.2	205.9	39.6

IV. $W = 1290$ kgs. cm^2.

$\mathfrak{H}$	$\mathfrak{I}$ $\tau = 0$.	$\mathfrak{I}$ $\tau = 1°$ per cm.	$\mathfrak{I}$ $\tau = 3°$ per cm.	$\mathfrak{I}$ $\tau = 5°.5$ per cm.	$\mathfrak{I}$ $\tau = 9°$ per cm.	$\mathfrak{I}$ (Untwisted.)
2.2	2.0	3.3	0.8	0.8	0.5	3.5
4.2	6.1	7.4	3.6	3.3	1.8	6.6
6.1	8.7	11.2	5.9	5.0	4.5	9.6
8.1	11.7	15.2	8.6	8.3	7.1	12.2
10.0	14.9	19.6	12.9	12.5	9.1	14.9
11.9	18.2	24.3	19.8	21.5	11.2	17.7
13.9	21.5	29.7	162.5	74.3	13.0	20.5
15.8	25.1	34.7	170.4	163.4	22.1	24.8
17.7	28.4	41.3	173.3	169.0	131.0	27.2
19.6	31.7	48.5	175.4	173.3	151.0	30.0
21.5	35.1	58.1	177.2	176.1	157.2	32.7
23.5	38.1	65.5	179.0	178.0	161.7	35.3
27.3	45.0	82.8	181.8	182.7	168.3	41.6
23.5	39.6	78.9	..	181.3	165.8	36.5
15.8	28.1	69.3	176.7	176.9	161.7	25.9
8.1	16.3	59.4	172.1	...	155.8	14.9
4.2	9.2	54.1	170.0	171.6	152.1	8.3
0	3.3	48.7	167.1	165.3	148.5	3.3

Reviewing the results of these experiments we see that twisting produces many singular effects on the magnetic properties of nickel. In the first place, maximum susceptibility occurs, for the twisted wire, at lower values of the magnetizing force than for the untwisted wire. The amount of twist which must be applied to obtain the maximum susceptibility in the weakest field is about 3° per cm. For higher twists, the "wendepunkt" occurs at the higher values of the field. The critical value of twist is nearly constant for all values of longitudinal stress. Also, the wire which, twisted to this critical amount, gives the greatest maximum susceptibility, gives at the same twist the greatest maximum differential susceptibility—in other words the curve rises most abruptly to its turning point. The field for maximum differential susceptibility is very slightly smaller than the field for maximum susceptibility. As the latter is passed, the differential susceptibility diminishes markedly in value and remains pretty constant in the higher fields. Ultimately the magnetic susceptibility of the twisted nickel becomes less than that of the normal wire, the curves crossing each other in high magnetic fields. Again the field for maximum susceptibility increases with load; but the maximum susceptibility itself diminishes. Also the susceptibility in fields of moderate strength is more sensitive to twisting for the greater loads. This effect of load is more marked with regard to the residual magnetism. Thus with a load of 10 kgs. in a field of 27, a wire with a twist of 3° per cm. shows 50 times as much residual magnetism as it did in the untwisted condition. Finally the wire released from torsion behaves in a different way from the normal one, the magnetic effect of stress outliving the removal of it.

II. The effect of combined longitudinal and torsional stresses on the retentiveness of nickel wire.

After the preceding experiments were finished, I determined to

examine the residual magnetism of the nickel wire under the action of two stresses, especially at the point where the susceptibility suddenly increases. The experiments were conducted in very much the same way as before. A nickel wire 40 cms. long and 1 mm. thick was placed in a double solenoid, the second solenoid serving to neutralize the earth's field. After the wire was demagnetized, magnetizing forces of gradually increasing magnitudes were applied and removed. The residual magnetism was then measured. All the applications and removal of the magnetizing force were conducted very gradually and readings were taken at each step.

The wire was treated exactly as before as regards both the longitudinal and the torsional stresses. The observations for each of the combinations of longitudinal and torsional stresses are given in the following tables, which also contain the ratios of the temporary and residual magnetisms.

I. $W = 2.5$ kgs. cm².

	$\tau=0$			$\tau=0.5°$ per cm.			$\tau=1°.5$ cm.		
$\mathfrak{H}$	Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$
1.9	8.3	1.5	...	3.3	.3	...	3.3	1.7	...
3.9	20.6	5.9	.29	16.5	7.3	.44	154.1	150.2	.97
4.8	...	...	...	28.1	15.8	.56	219.5	214.5	.98
5.8	57.8	34.7	.60	67.3	52.8	.78	246.8	239.9	.97
7.7	116.3	83.2	.72	193.1	172.1	.89	260.7	252.5	.97
9.7	150.2	110.9	.74	221.1	196.4	.89	269.4	258.1	.96
11.6	181.5	133.0	.74	234.3	205.3	.88	275.6	261.2	.95
13.5	206.3	150.6	.73	247.5	215.5	.87	...	...	...
15.4	229.4	167.6	.73	259.1	220.6	.85	284.0	266.0	.94
17.4	219.5	181.5	.73	267.6	226.1	.84	...	...	...
19.3	261.7	188.9	.73	276.8	230.8	.83	291.7	268.0	.92
23.2	290.4	209.9	.72	290.7	237.6	.82	297.0	269.6	.91
27.0	307.6	219.5	.71	300.8	240.9	.80	303.3	271.4	.90

$\mathfrak{H}$	$\tau=4°.5$ per cm. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	$\tau=9°.0$ per cm. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	Untwisted. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$
1.9	5.0	2.6	.53	3.3	1.2	...	8.3	1.5	...
3.9	84.8	80.9	.95	52.0	48.2	.93	21.5	13.5	.62
4.8	200.8	194.7	.97	146.2	140.7	.96	...	...	...
5.8	233.1	226.5	.97	222.8	217.8	.98	45.5	34.2	.75
7.7	255.3	249.8	.97	247.5	240.7	.96	104.6	87.9	.84
9.7	267.3	256.6	.96	255.8	247.5	.96	150.2	128.7	.86
11.6	272.3	260.4	.96	260.7	250.8	.96	169.2	143.9	.85
15.4	282.2	262.7	.93	267.3	253.4	.95	198.0	163.4	.83
19.3	287.4	264.2	.92	272.9	255.8	.94	216.2	175.9	.81
23.2	292.1	266.1	.91	277.2	257.4	.93	231.0	183.5	.79
27.0	296.1	267.0	.90	280.5	257.9	.92	243.7	189.8	.78

II. W = 145 kgs. cm².

$\mathfrak{H}$	$\tau=0$. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	$\tau=0°.5$ per cm. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$	$\tau=1°.5$ per cm. Temp. Mag.	Resid. Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$
1.9	4.0	...	...	4.0	0	...	2.3	0	...
3.9	9.7	...	...	9.1	.7	...	4.5	.8	...
5.8	18.3	4.6	.25	26.1	14.1	.54	16.2	10.2	.63
6.8	...	...	...	38.1	24.1	.63	51.5	44.6	.86
7.7	30.4	12.4	.41	54.5	39.3	.72	177.2	170.8	.96
9.7	54.3	30.5	.56	98.8	78.0	.79	229.4	221.3	.96
11.6	88.1	58.1	.66	152.0	127.9	.84	242.1	232.3	.96
13.5	120·1	85.0	.71	173.6	147.5	.85	250.0	238.6	.95
15.4	143.2	102.8	.72	187.8	158.2	.84	254.6	242.1	.95
17.4	159.4	113.2	.71	200.1	166.2	.83	...	...	...
19.3	174.2	124.1	.71	209.9	172.9	.82	262.8	246.5	.94
23.2	...	...	...	224.9	182.2	.80	...	...	...
27.0	212.9	146.2	.68	237.6	189.1	.79	272.3	251.6	.92

$\tau = 3°.0$ per cm. $\tau = 4°.5$ per cm. $\tau = 9°$ per cm.

$\mathfrak{H}$	Temp. Mag	Resid. Mag.	Resid / Temp.	Temp Mag	Resid. Mag.	Resid / Temp	Temp Mag.	Resid. Mag.	Resid. / Temp.
1.9	1.2	0	...	1.2	0	...	1.2	...	...
3.9	2.5	0	...	2.5	0	..	2.8	.3	...
5.8	3.8		.	5.9	2.1	.36	5.1	1.7	.32
6.8	184.8	181.5	.98	46.5	42.6	.92	21.1	17.0	·80
7.7	219.6	216.2	.98	239.6	236.0	.985	93.9	90.8	.98
9.7	259.9	255.8	.98	258.2	254.1	.98	242.2	238.6	.98
11.6	266.1	261.5	.98	264.2	259.7	.98	253.9	249.0	.98
13.5		.	..	238.1	232.8	.98	259.1	253.9	.98
15.4	273.1	266.3	.98	270.8	264.8	.98	262.0	255.8	.97
17.4			.	..	.	..	..	.	...
19.3	277.2	268.3	.97	274.9	265.0	.97	266.8	258.4	.97
23.2	279.5	259.3	.93	..	..	..	...	...	...
27.0	282.2	269.6	.96	279.2	256.8	.93	272.3	259.5	.95

III. W = 1040 kgs. cm^2

$\tau = 0°$ $\tau = 1°.5$ per cm.

$\mathfrak{H}$	Temp Mag	Resid. Mag	Resid. / Temp	Temp. Mag.	Resid. Mag	Resid. / Temp
1.9	1.5	..	...	3.3	.7	...
3.9	4.1	.3	...	8.7	4.3	...
5.8	8.1	.5	...	15.3	8.7	.57
7.7	11.4	.8	.07	29.0	19.5	.67
9.7	15.2	1.7 ?	.11 ?	63.5	51.0	.80
11.6	19.5	2.0	.10	95.7	82.2	.85
13.5	22.4	2.3	.11	111.4	94.7	.85
15.4	25.2	3.3	.13	122.4	104.0	.85
19.3	33.7	4.1	.12	135.0	115.5	.85
23.2	41.4	5.0	.12	148.0	121.1	.82
27.0	47.9	5.9	.12	155.9	124.1	.79

$\mathfrak{H}$	$\tau=4^\circ.5$ per cm. Temp Mag.	Resid Mag	$\frac{\text{Resid.}}{\text{Temp}}$	$\tau=9^\circ$ per cm. Temp Mag.	Resid Mag.	$\frac{\text{Resid.}}{\text{Temp.}}$
1.9	1.2	0	...	2.6	0	...
3.9	5.3	1.0	...	5.4	1.0	.18
5.8	11.1	4.6	.41	...		...
7.7	22.8	13.7	.60	20.0	11.9	.59
9.7	70.1	58.7	.84	48.2	36.0	.75
11.6	129.1	114.7	.88	98.0	83.5	.85
13.5	139.1	124.9	.90	128.4	114.3	.89
15.4	146.0	128.4	.88	138.8	120.8	.87
19.3	156.1	134.0	.86	150.8	128.7	.85
23.2	166.2	137.9	.84	159.7	133.0	.83
27.0	173.0	141.9	.82	168.3	138.6	.80

The curves showing the temporary and residual magnetisms are given in Figs. 5_a, 6_a, 7_a, and Figs. 5_b, 6_b, 7_b; and the curves showing the ratio of the residual to the induced magnetism in Figs. 5_c, 6_c, and 7_c.

The inspection of these figures will show how curious a change is produced in the residual magnetism of twisted nickel wire, when the magnetizing force is that which corresponds to maximum susceptibility. In some cases, the amount of residual magnetism is indeed enormous, reaching to .98 of the induced magnetism.

Examining the curves obtained for the wire which was only subject to the action of its own weight and the twisting rod, we see that for the untwisted wire, the ratio of residual to temporary magnetism increases to a maximum at about field 10. Its value is then .74. At higher fields, this ratio gradually diminishes. The curve representing these ratios is shown in Fig. 5_c. But when the wire is twisted only through an angle of $0^\circ.5$ per cm., the curves of induced and residual magnetisms are quite different from those of the untwisted wire. The ratio of residual to temporary magnetisms increases up to

.89 instead of .74. However, this large retentiveness occurs just at the field where the susceptibility is at its maximum. The rate at which this ratio grows with the field is almost uniform till the maximum is reached, if we leave out of account the observations at very low fields which cannot in the circumstances be regarded as at all accurate. The curves in Fig. 5_c are therefore straight up to the maximum. For the twisted wire the maximum is passed very abruptly; and for the rest of their course all the curves are approximately straight lines.

With a twist of 1°.5 per cm, the amount of residual magnetism increases still more; and at the maximum point which again occurs near the wendepunkt, the ratio of the residual to the temporary attains the value of .98. It is indeed wonderful what a great effect simple twisting of nickel produces upon its retentiveness. The approach to the maximum ratio takes place more suddenly in this than in the previous case, the curve sloping more steeply. There is nothing singular after the maximum is passed, but the rate of fall for higher fields becomes distinctly less for the larger twist. When the twist is increased to 4°.5 or 9° per cm, there is a distinct tendency in the curve to return to its former state. Thus the slope gradually becomes less steep, and the maximum occurs at higher fields. But the ratio of the residual to the temporary magnetism still keeps to its late value. Again the rate of fall of the ratio after the maximum is reached gradually lessens as the twist is increased.

From these experiments, we gather that twist applied to nickel wire increases its residual magnetism, which attains a maximum in the field corresponding to the maximum susceptibility. For moderate twists this maximum residual falls short of the temporary magnetism by only 2 per cent. The sooner the "wendepunkt" occurs, the more rapid is the rate of increase of the residual magnetism; and

consequently, there exists a twist for which the maximum slope of the percentage curve attains a greatest value. The rate at which the residual magnetism falls off in higher fields is diminished as the twist is increased.

The next series of experiments related to the combined effect of torsional and longitudinal stresses. For this purpose, a new wire properly prepared was loaded with 1.14 kgs. The normal curve (Fig. 6) show a slight decrease of retentiveness; and the curve obtained for the twist of 0°.5 per cm. does not show those curious characteristics of the twisted nickel. The retentiveness, however, is greatly increased, and at its maximum, the residual magnetism is .85 of the induced. But when the twist is increased to 1°.5, all the curious properties of twisted nickel already mentioned become apparent and confirm the results already obtained for the unloaded wire. In this case the maximum ratio of the residual to the induced magnetism attains the enormous value of .985. The greatest maximum slope in the ratio curves again occurs for the value of twist for which the "wendepunkt" occurs sooner. This is for a twist of nearly 3° per cm. As may be seen from the experiments on the relation of $\mathfrak{J}$ and $\mathfrak{H}$, this is the twist which gives the maximum differential susceptibility. Hence it would appear that the twist of about 3° per cm. has a certain critical significance in the relations of twist to magnetization for the specimens of nickel wire used.

When the load is increased to 3.14 kgs., the maximum ratio of residual to temporary magnetism diminishes to .97, but the general characteristics still remain the same.

The experiments made on the wire which was loaded with a weight of 8.14 kgs. also brings out clearly the effect of twist on the retentiveness of nickel. The residual magnetism for no twist is indeed very small with such a large load; but merely twisting the wire

through 1°.5 per cm. is sufficient to increase the maximum retentiveness more than 6 times for a field of 27, although the absolute value of the residual magnetism falls far short of the values obtained for small longitudinal stresses. The maximum value of the ratio of the residual to induced magnetism is .90 for a twist of 4°.5 per cen. These phenomena are represented in Fig. 7_a, 7_b, 7_c.

All these curious properties of the twisted nickel must be accounted for by the change of molecular structure caused by the torsional stresses. But as we know nothing regarding molecular arrangements, we can make no definite assertion how the change takes place. We may however conceive the wire in the normal state as consisting of an assemblage of rows of molecules arranged in a straight line along the length of the wire. On twisting the wire, these rows or filaments of molecules will no longer be straight, but will become a spiral. Since all such molecular filaments in the wire suffer similar distortion, the axes of the molecules in the unmagnetized nickel will be indifferently placed in all directions either in the normal or the twisted wire. Now if the wire is magnetized longitudinally, the magnetizing force will be parallel to the molecular filaments in the untwisted wire, but in the twisted wire the corresponding molecular filaments will no longer be parallel to the magnetizing force. Such considerations would lead us to expect distinct differences in the curves of magnetization for the two states of nickel.

If, according to Weber's theory, the application of a magnetizing force tends to turn the axes of the molecules in one direction, the results of my experiments show that the twist applied to the wire makes the molecules turn more easily. There also exists a critical value of twist, beyond or below which the molecules turn with less ease. This twist is about 3° per cm. in the wire experimented. But in spite of the ease of the molecules in turning, they seem to be un-

able to turn beyond a certain angle, so that when once the sudden rotation of molecules takes place, the rate of turning becomes very small. It also appears that when the sudden turning of the molecules takes place, the tendency of the molecules to revert to their former positions is very small, the retentiveness being large

All this is a mere hypothesis, since we know nothing of the original arrangement of the molecules, and far less of how they are changed. If however we assume Weber's theory, we may regard the curious phenomena presented by twisted nickel as being accompanied by such motions of the magnetic molecules as have just been described.

Specific Volume of Camphor and of Borneol determined with proximate accuracy.

By

Mitsuru Kuhara, Ph. D.

The materials employed in the experiments were ordinary commercial camphor and borneol, purified by repeated sublimation. Their melting and boiling points* were found to be as follows :

	Melt. point.	Boil. point.
Camphor	177.7°	205.3°
Borneol	—	209.7°

In these determinations a number of experiments were repeated and in each necessary corrections were made.

The determination of the specific gravity of liquid camphor and of liquid borneol was made with the use of a cylindrical specific gravity bottle of a small size whose capacity had carefully been ascertained by filling it with boiled distilled water. Either the camphor or the borneol was fused in a long cylindrical vessel over the paraffin bath, and the specific gravity bottle and its stopper, tied separately with platinum wire, were both introduced into this cylindrical vessel and heated together with the melted camphor or borneol. As the boiling point of the liquid camphor and of the liquid borneol is always found a few degrees higher than that of the vapour, the

* In most text-books the melting and boiling points of camphor are given to be 174° and 204° respectively, and the boiling point of borneol to be 212°.

moment the temperature of the liquid reached 205.3° or 207.9° (boiling points of their vapours), the stopper was instantly closed and the bottle filled with the liquid was taken out, cooled and weighed. These experiments were conducted under a higher barometric pressure than the normal, all the measurements being made by means of the corrected thermometer. The specific gravity was calculated according to the following formula referring to the water of 4°.

$$\text{Specific gravity} = \frac{W}{V(1+3\beta\,(T-t))}$$

W = Weight of the liquid camphor or borneol at T.

T = Boiling point of camphor or borneol.

V = Volume of water at t (or the capacity of the bottle at t).

t = Temperature when the bottle filled with water was weighed.

3β = Coefficient of the cubical expansion of glass = 0.0000254.

I. Experiments on Camphor $C_{10}H_{16}O$

The following three experiments were performed with a bottle of the capacity of 6.28114 c.c. at 30.5°.

(1) Camphor in the bottle weighed 5.1304 grms at 205.3°.

(2) ,, ,, ,, ,, ,, 5.1089 ,, ,, ,,

(3) ,, ,, ,, ,, ,, 5.1189 ,, ,, ,,

The fourth experiment was performed with a bottle of the capacity of 2.98632 c.c. at 40.5°.

(4) Camphor in the bottle weighed 2.4252 grms at 205.3°.

Seven more experiments were performed with a bottle of the capacity of 2.74518 c.c. at 40.5°.

(5) Camphor in the bottle weighed 2.2400 grms at 205.3°.

(6)	Camphor	in	the	bottle	weighed	2.2470	grms	at	205.3°.
(7)	,,	,,	,,	,,	,,	2.2405	,,	,,	,,
(8)	,,	,,	,,	,,	,,	2.2330	,,	,,	,,
(9)	,,	,,	,,	,,	,,	2.2355	,,	,,	,,
(10)	,,	,,	,,	,,	,,	2.2290	,,	,,	,,
(11)	,,	,,	,,	,,	,,	2.2315	,,	,,	,,

Specific gravity at 205.3° deduced according to the formula given above:

(1)	0.8131	(7)	0.8127
(2)	0.8097	(8)	0.8100
(3)	0.8113	(9)	0.8109
(4)	0.8087	(10)	0.8085
(5)	0.8125	(11)	0.8094
(6)	0.8147	Mean	0.8110

$$\text{Specific volume} = \frac{\text{Molec. wt}}{\text{Sp. gravity}} = \frac{152}{0.811} = 187.42$$

II. Experiments on Borneol $C_{10}H_{18}O$

The experiments were performed with a bottle of the capacity of 2.74518 c.c. at 40.5°.

(1)	Borneol	in	the	bottle	weighed	2.2284	grms	at	209.7°.
(2)	,,	,,	,,	,,	,,	2.2284	,,	,,	,,
(3)	,,	,,	,,	,,	,,	2.2265	,,	,,	,,
(4)	,,	,,	,,	,,	,,	2.2245	,,	,,	,,
(5)	,,	,,	,,	,,	,,	2.2390	,,	,,	,,
(6)	,,	,,	,,	,,	,,	2.2258	,,	,,	,,

Specific gravity at 209.7°.

(1)	0.8082	(4)	0.8071
(2)	0.8082	(5)	0.8120
(3)	0.8075	(6)	0.8073

Mean.............. 0.8083

$$\text{Specific volume} = \frac{\text{Molec. wt}}{\text{Sp. gravity}} = \frac{154}{0.8083} = 190.5$$

Specific volume of camphor and of borneol calculated with Kopp's values of the atomic volumes of carbon, hydrogen and oxygen, supposing the former as a ketone and the latter as an alcohol, is found to be as follows:

Camphor.		Borneol.	
C_{10}	$10 \times 11 = 110$	C_{10}	$10 \times 11 = 110$
H_{16}	$16 \times 5.5 = 88$	H_{18}	$18 \times 5.5 = 99$
O	$1 \times 12.2 = 12.2$	O	$1 \times 7.8 = 7.8$
Specific volume	210.2	....	216.8

Thus it is seen that the calculated values are much greater than those found by experiments. Also the specific volume of benzene and of some of its derivatives calculated with Kopp's values is often found to be much greater than those obtained experimentally. In Watt's Dictionary of Chemistry, 3rd Supplement, page 2126, the following statements are given. "Lothar Meyer makes $H=3$ and Löschmidt $C=14$ and $H=3.5$; and by assuming that half the carbon atoms in benzene have the value 11 and the remainder the value 14, and that hydrogen has the constant value 3.5, we obtain a value for this hydrocarbon which is identical with the observed values." Thus:

Calculated.		Found. Kopp	Ramsay
C_3 .... $3 \times 14 = 42$			
C_3 .... $3 \times 11 = 33$	96	95.8	95.9
H_6 .... $6 \times 3.5 = 21$			

I tried to apply these values of carbon and hydrogen and Kopp's values of oxygen (12.2 and 17.8) to camphor and borneol, supposing each to consist of a closed chain of six carbon atoms like benzene, and I found that the calculated specific volumes are almost concordant with those observed.

Camphor.	Calculated	Found	Borneol.	Calculated	Found
$C_3 = 3 \times 14$			$C_3 = 3 \times 14$		
$C_7 = 7 \times 11$			$C_7 = 7 \times 11$		
$H_{16} = 16 \times 3.5$	187.2...	187.42	$H_{18} = 18 \times 3.5$	189.8...	190.5
$O = 1 \times 12.2$			$O = 1 \times 7.8$		

I tried farther to compute the specific volumes of some of the benzene derivatives whose experimental numbers are already known, by applying similar method of calculation to them, thus:

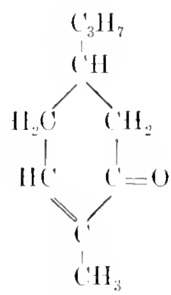
	Found. Kopp	Ramsay	Yoshida	Calculated. C=14 & 11; H=3.5; O=12.2 & 7.8	C=11; H=5.5; O=12.2 & 7.8
Benzene	95.8 ...	95.9 ...	— ...	96.0 ...	99.0
Phenol	103.6 ...	106.9 ...	— ...	103.8 ...	106.9
Benzylalcohol .	123.7 ...	— ...	— ...	121.8 ...	128.8
Benzaldehyde .	118.4 ...	— ...	— ...	119.2 ...	122.2
Ethyl benzoate	172.4–174.8	— ...	163.1*...	162.0 ...	174.0
Benzoic acid...	126.9 ...	— ...	— ...	127.0 ...	130.0
Naphthalene...	149.2 ...	— ...	— ...	150.0 ...	154.0

* This number was observed according to Ramsay's method by Mr. H. Yoshida, Science College, Imperial University of Tokio.

It thus appears that the numbers found by the new way of calculation agree better with the observed ones, in some cases, than with those calculated with Kopp's values.

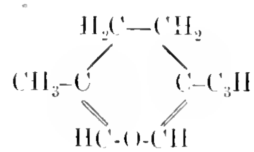
Now the principal formulæ for camphor suggested by different chemists are six in number, namely:

(1)



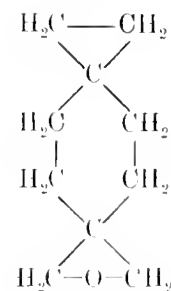
Kekulé.

(2)



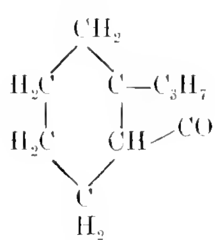
Victor Meyer.

(3)



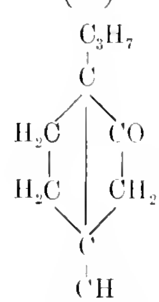
Hlasiwetz.

(4)



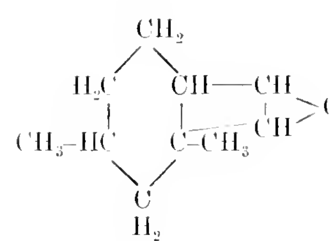
Kachler.

(5)



Kanonnikow.

(6)



Armstrong.

If we suppose the above method of calculation to be true there may exist in camphor, a closed chain of six carbon atoms and

(1) Ber. d. deut. chem. Gesell. VI - 929.
(2) „ „ „ „ „ III - 116.
(3) „ „ „ „ „ III - 540.
(4) Annalen d. Chem. CLXV - 185.
(5) Ber. d. deut. chem. Gesell. XVI - 3051.
(6) „ „ „ „ „ „ XI - 1698.

camphor itself may be a ketone, and then the formula 1, 4 or 5 may represent its constitution, as any one of the remaining formulæ would represent it as an oxide. Kanonnikow,* however, who determined the refraction equivalent of camphor and of its allied compounds by applying Bruhl's method, states that camphor has no double union between carbon atoms. Hence it may be concluded that one of the formulæ 4 and 5 represents the constitution of camphor, and borneol is its corresponding alcohol.

* Ber. d. deut. chem. Gesell. XVI - 3051.

Fig. 1.

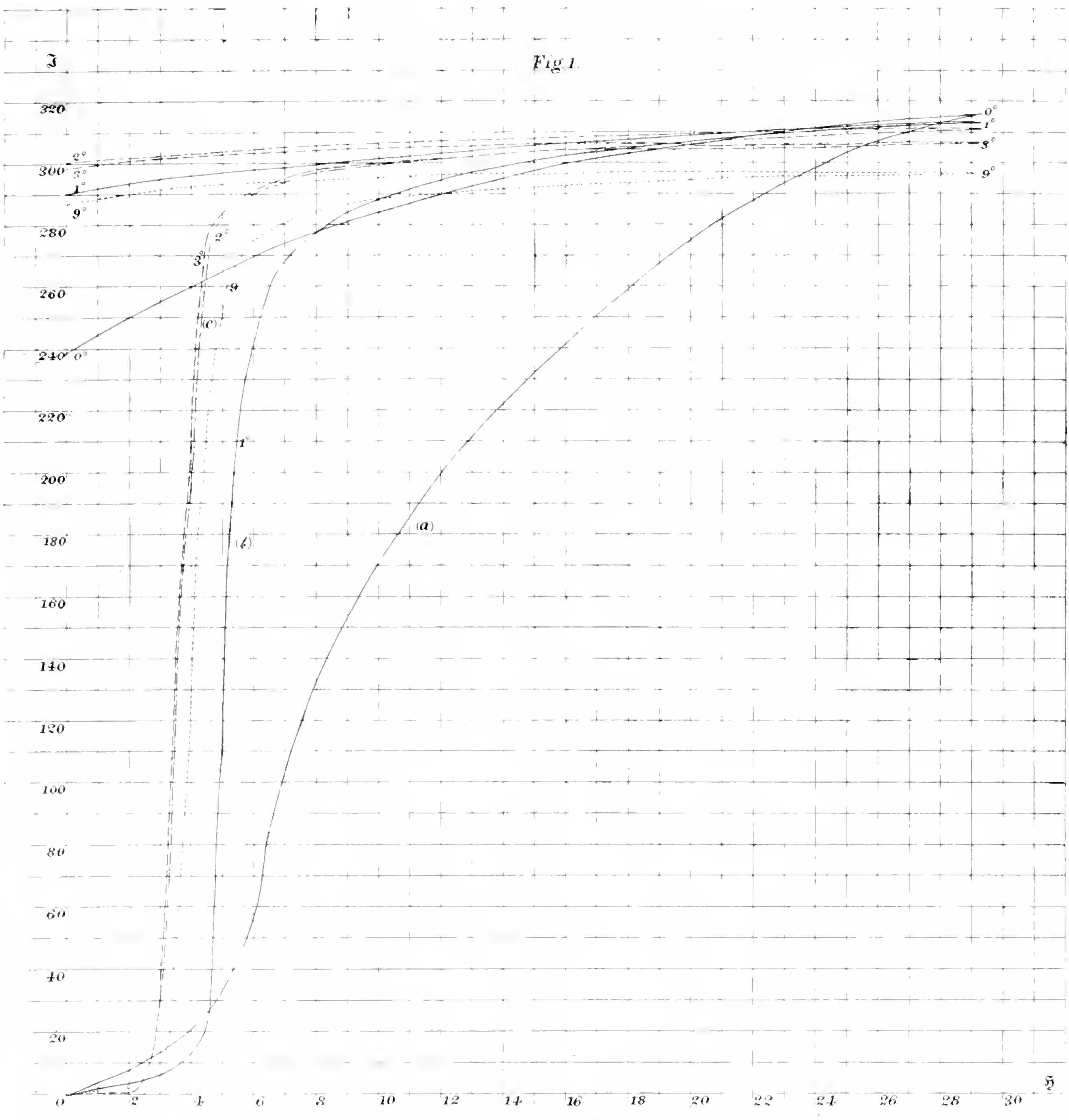

ℑ
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
ℌ
(a)
(b)
(c)

Fig. 2.

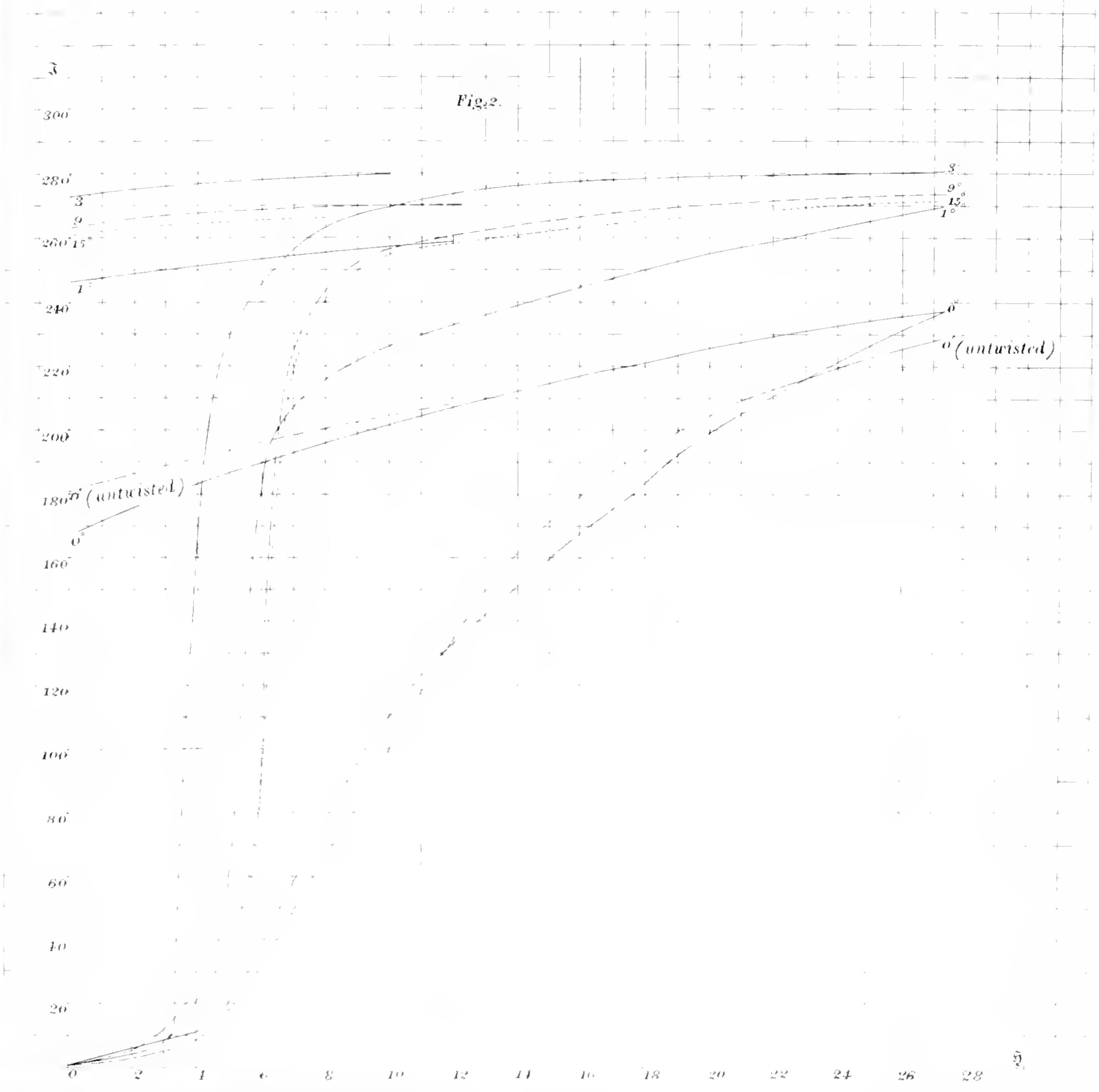
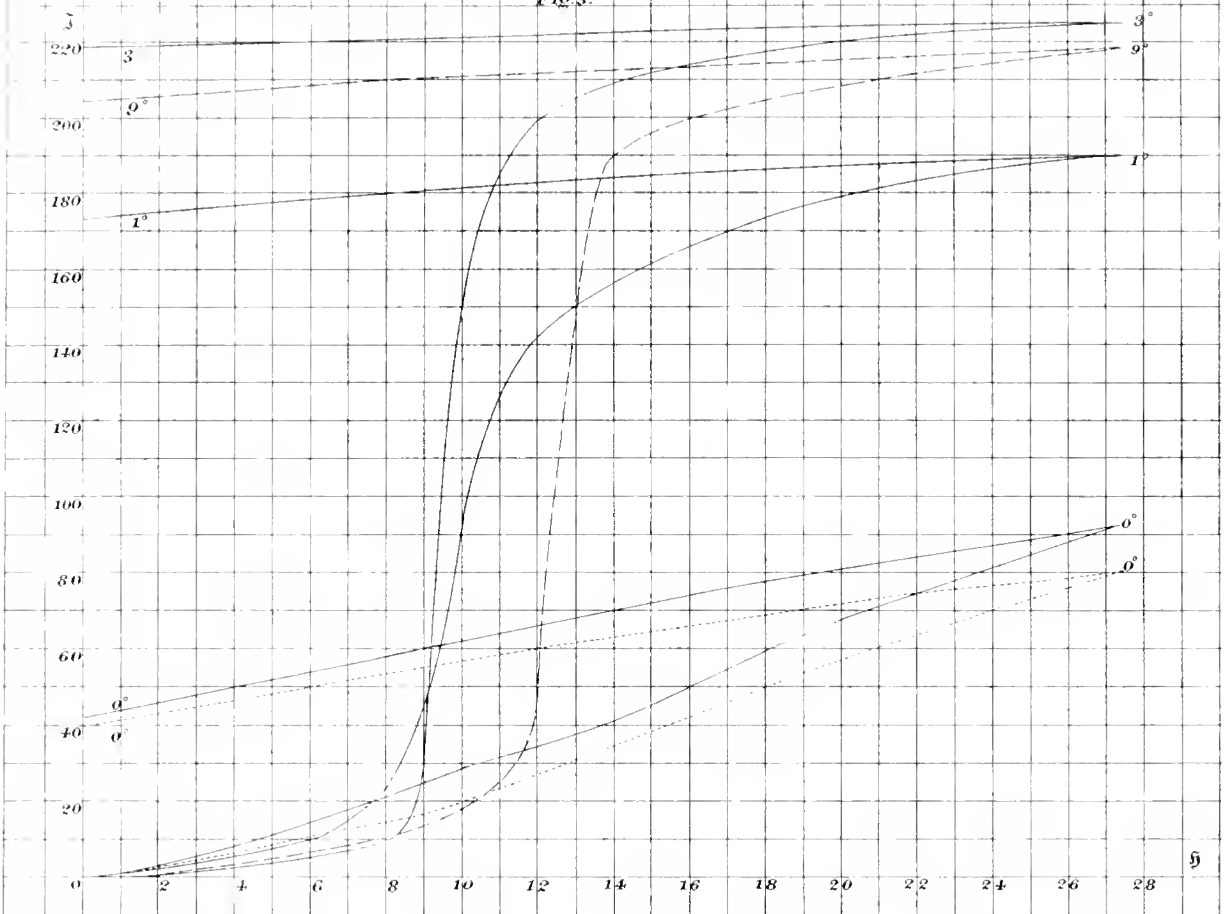
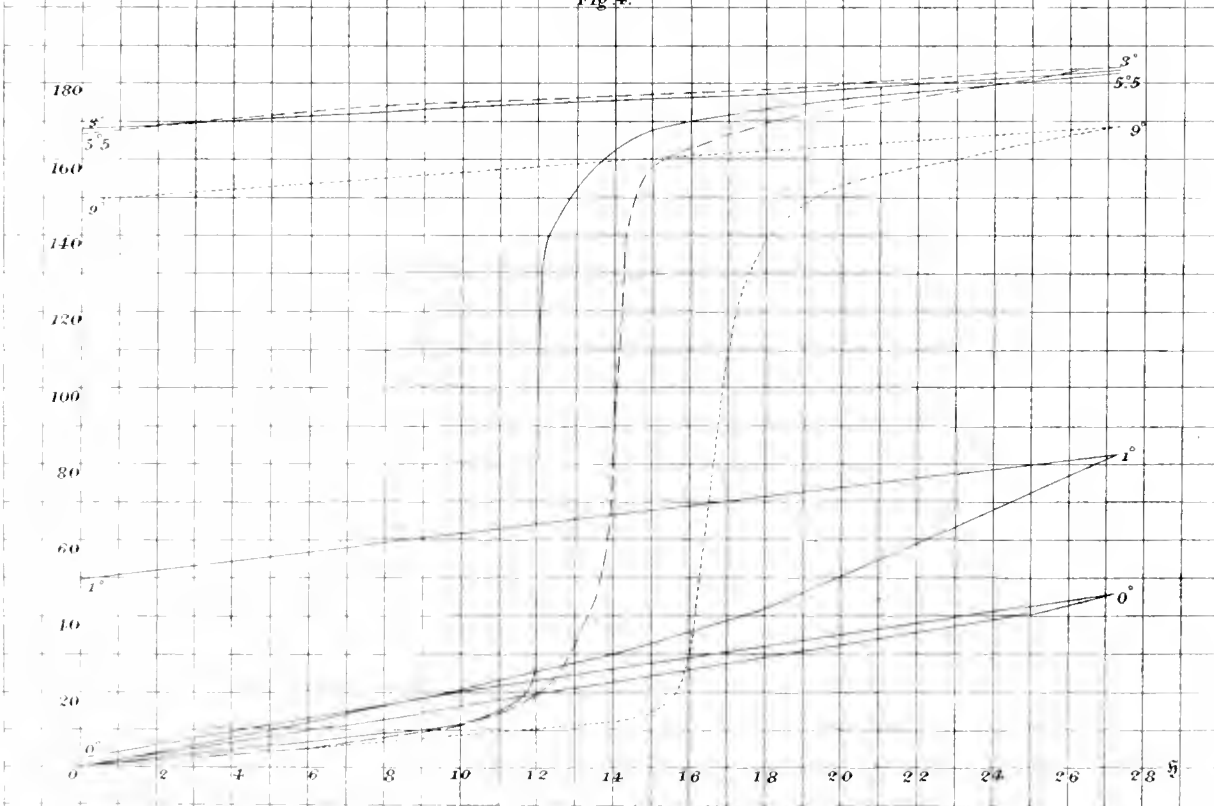

ℑ
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
ℌ
0° (untwisted)
0° (untwisted)

Fig. 3.

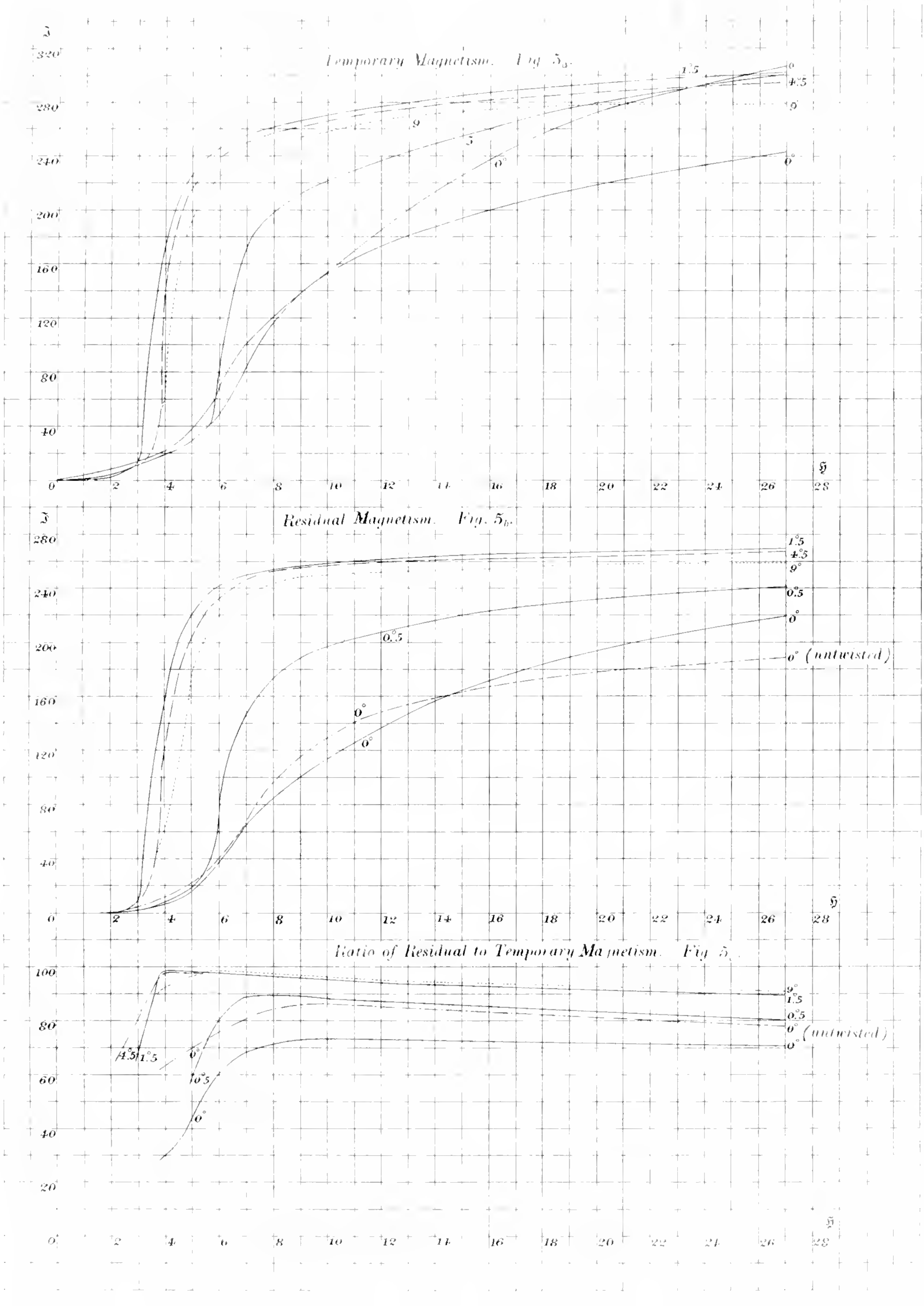


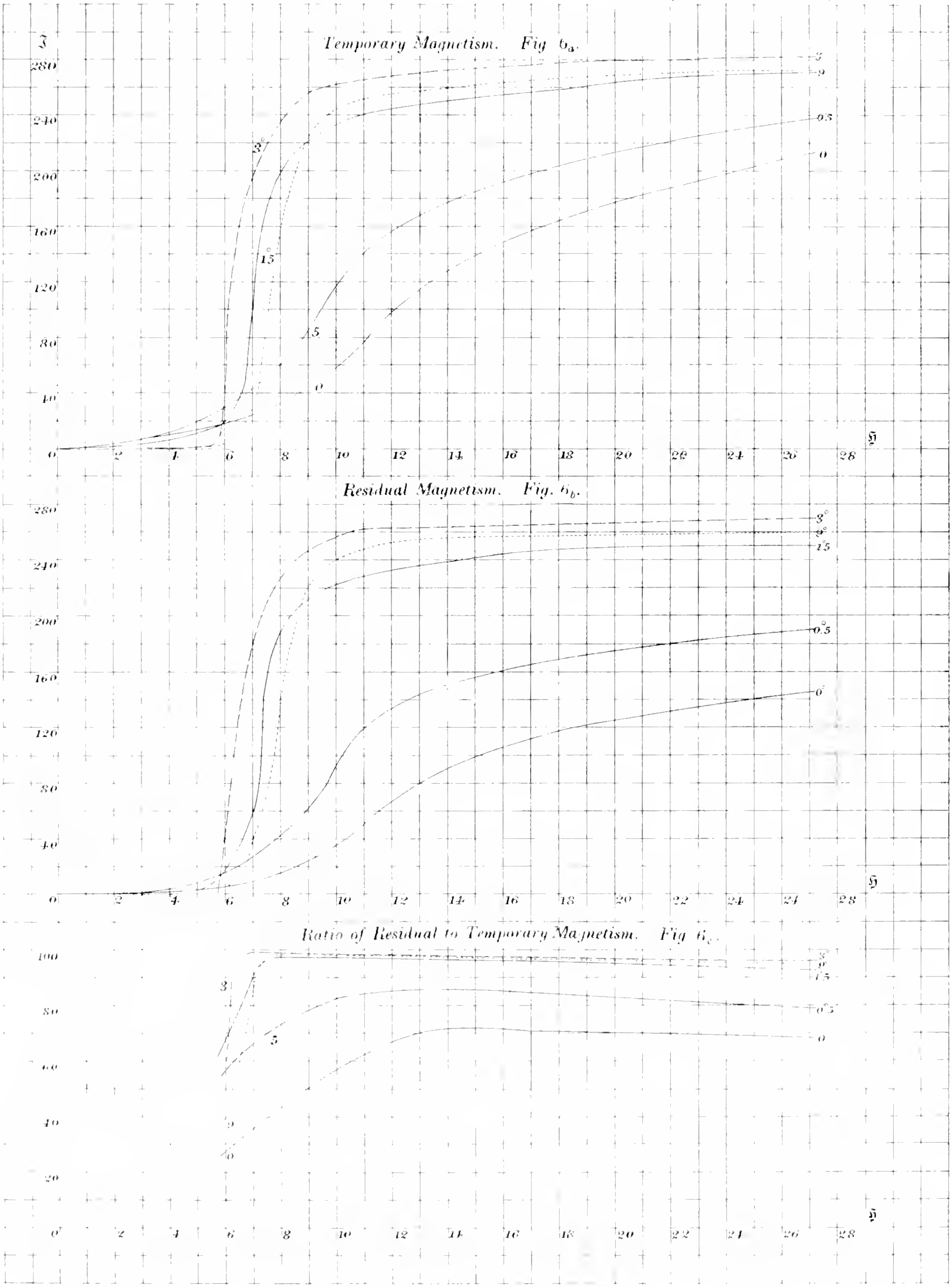
ℑ
220
200
180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
ℌ
3°
9°
1°
0°

Fig. 4.



180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
ℌ
3°
5°5
9°
1°
0°


Temporary Magnetism. Fig 5a.
Residual Magnetism. Fig. 5b.
Ratio of Residual to Temporary Magnetism. Fig 5c.
0° (untwisted)
0° (untwisted)


Temporary Magnetism. Fig 6a.
Residual Magnetism. Fig. 6b.
Ratio of Residual to Temporary Magnetism. Fig 6c.
3°
9°
15°
0°.5
0°

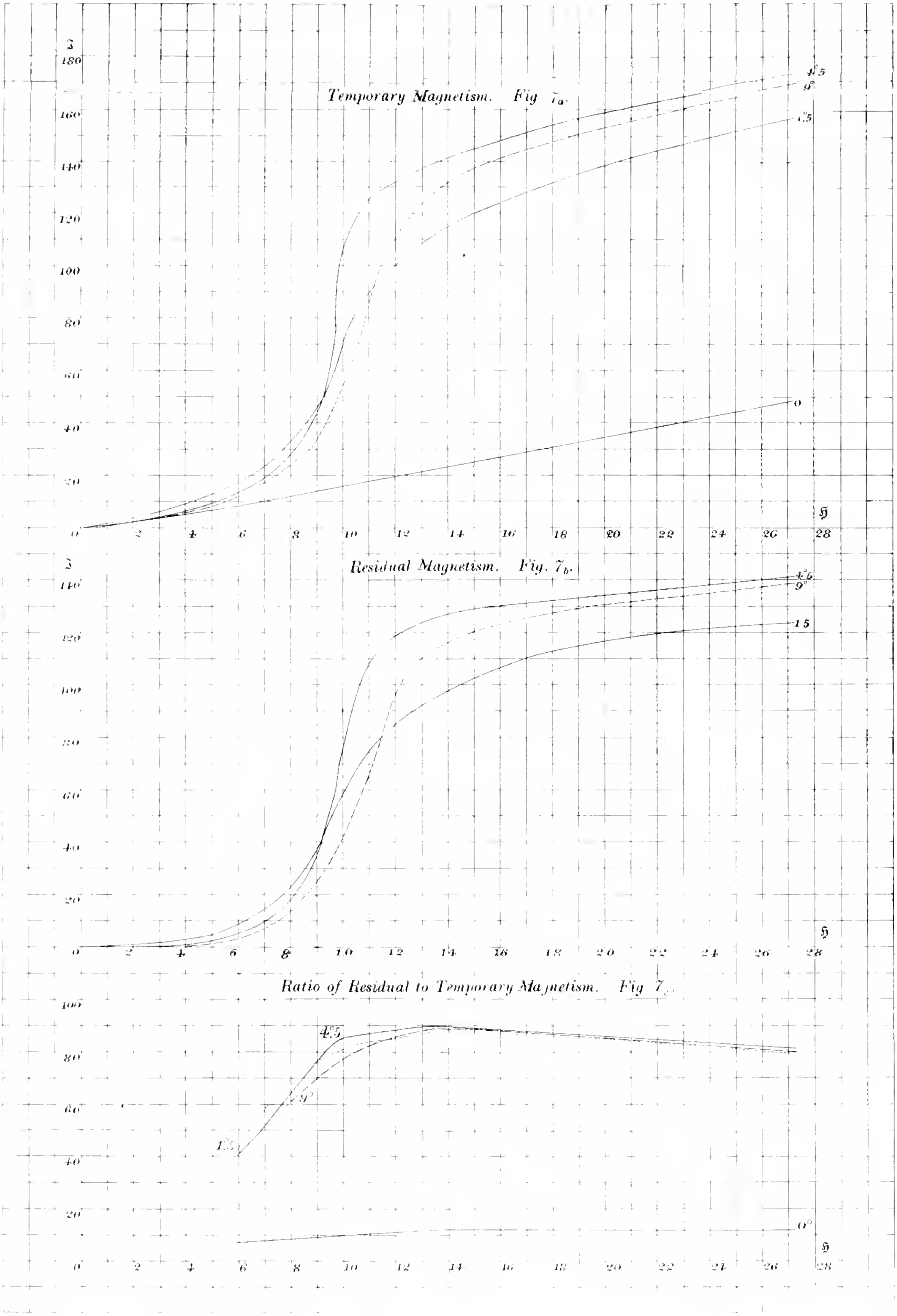

Temporary Magnetism. Fig 7a.
Residual Magnetism. Fig. 7b.
Ratio of Residual to Temporary Magnetism. Fig 7c.
4°5
9°
1°5
0
0°

Fig. 1

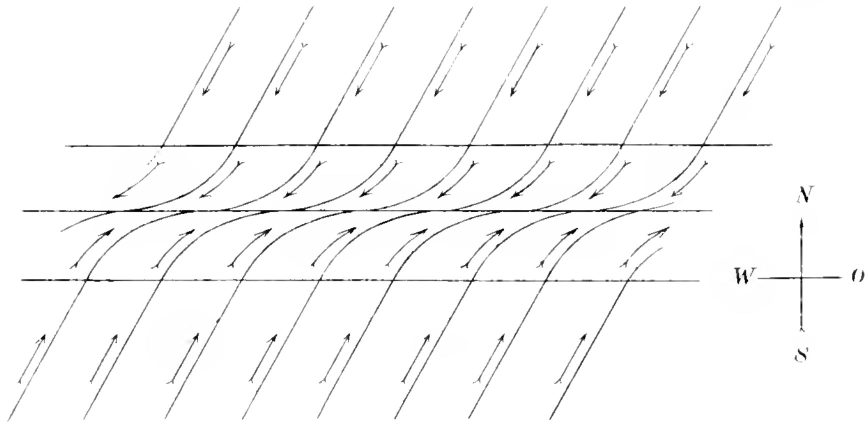


Fig. 2

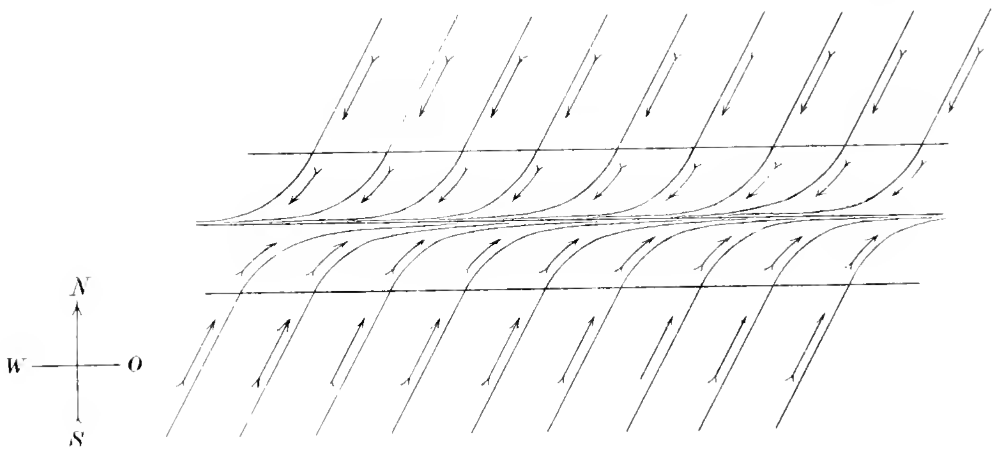
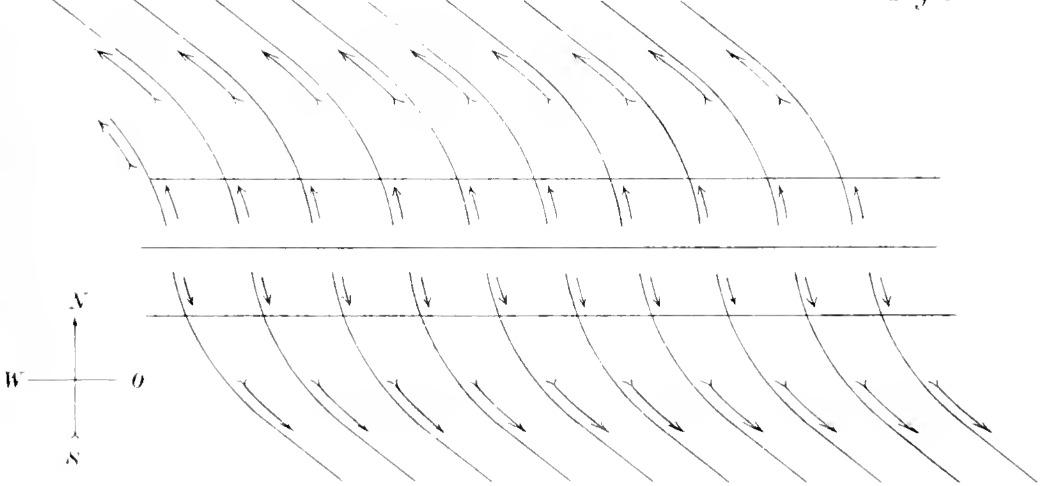
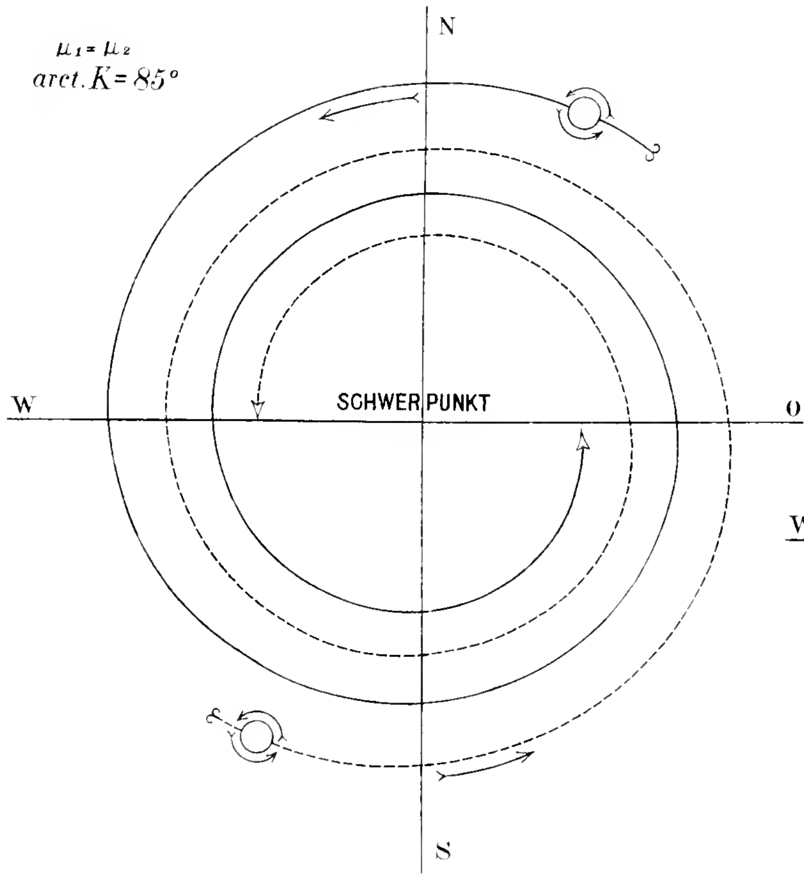
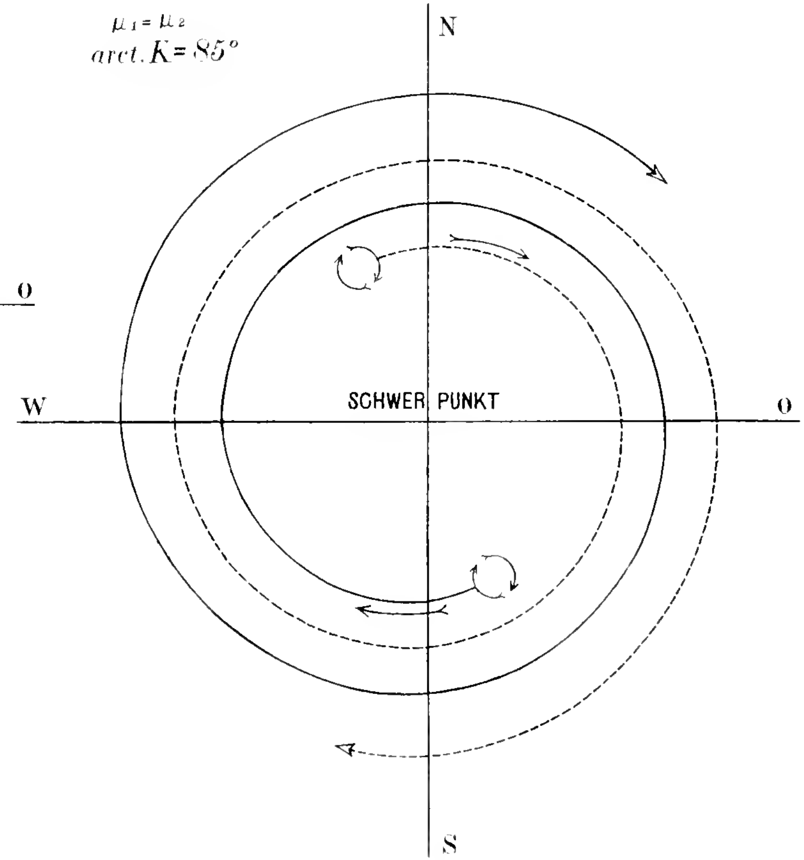


Fig. 3




$\mu_1 = \mu_2$
$arct. K = 85°$
N
W
SCHWER PUNKT
O
S


$\mu_1 = \mu_2$
$arct. K = 85°$
N
W
SCHWER PUNKT
O
S

Beitraege zur Theorie der Bewegung der Erdatmosphaere und der Wirbelstuerme

(Zweite Abhandlung)

von

Dr. Phil. Diro Kitao.

Professor für Physik und Mathematik an der Kaiserlichen Akademie für Forst-und Landwirthschaft zu Tôkyô.

---

§ *VIII. Wirbelgebiet der geradlinigen Isobaren.*

Ganz so vollständig wie der Fall eines kreisförmigen Gebietes der Verticalströmung* lässt sich nun derjenige Fall behandeln, wo das Gebiet der Verticalströmung durch zwei unendlich grosse Ebenen begrenzt ist.

Wir nehmen an: die Ebenen, welche das Gebiet der Verticalströmung begrenzen, seien parallel gerichtet der (yz) Ebene und ihr gegenseitiger Abstand sei δ. Wenn wir den Coordinatenanfang in *die* Ebene legen, welche diesen Abstand der beiden unendlichen Ebenen halbirt, welche wir die Mittelebene nennen wollen, so hat man bekanntlich als Potential† einer mit der Masse $\frac{\gamma}{4\pi}$ erfüllten unendlich grossen Schichte von der Dicke δ für einen äusseren Punkt

---

* Dr. Diro Kitao. Beiträge zur Theorie der Bewegung der Erdatmosphäre und der Wirbelstürme. Dieses Journal. Vol. I pag. 183—209.

† Thomson und Tait. Handbuch der theoret. Physik. übersetzt von H. Helmholtz und G. Wertheim. 1874 Band I zweiter Theil pag. 41.

$$\varphi_a = -\frac{\gamma\delta}{2} x + Const$$

und für einen inneren Punkt

$$\varphi_i = -\frac{\gamma}{2} x^2 + Const$$

so dass also die Bedingungen, denen φ genügen muss

$$\varDelta \varphi_a = 0 \qquad \varDelta \varphi_i = -\gamma$$

und

$$\frac{\partial \varphi_a}{\partial x} = \frac{\partial \varphi_i}{\partial x} \qquad \text{für} \quad x = \frac{\delta}{2}$$

erfüllt sind, da die Normale der unendlichen Ebene überall mit der Richtung der x Achse zusammenfällt. Für die negative Hälfte der (xy) Ebene hat man dabei überall δ negativ zu setzen.

Wir fassen zunächst den Fall in's Auge, wo die Verticalströmung eine aufsteigende ist und setzen wieder

$$u = \frac{\partial W}{\partial y} + \frac{\partial \varphi}{\partial x} \qquad v = -\frac{\partial W}{\partial x} + \frac{\partial \varphi}{\partial y}$$

Es ist sehr leicht die Bewegung in äusseren wirbelfreien Gebiete zu finden. Da, wie wir allgemein gefunden haben

$$W_a = \frac{2\lambda \sin\theta}{\kappa} \cdot \varphi_a$$

und in dem vorliegenden Fall W_a von y unabhängig ist, so folgt augenblicklich

$$u = \frac{\partial \varphi_a}{\partial x} = -\frac{\gamma\delta}{2}$$

$$v = -\frac{\partial W}{\partial x} = \frac{2\lambda \sin\theta}{\kappa}\frac{\gamma\delta}{2} \qquad w = 0$$

Die Geschwindigkeit der Luft im äusseren wirbelfreien Gebiet ist demnach constant und jedes Lufttheilchen strömt auf der positiven (südlichen) Hälfte der unendlichen (xy) Ebene von SW her, und auf der negativen (nördlichen) Hälfte aber von NO her, und zwar

geradlinig, da die Gleichung für Windbahn $\frac{dy}{dx} = \frac{v}{u}$ ergiebt

$$y = -\frac{2\lambda \sin\theta}{\kappa} \cdot x + Const$$

Als Isodynamen erhält man ferner aus (50) (pag. 172. vol. I d. Journals)

$$\Phi = \frac{\gamma\delta}{2}\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right) x + Const$$

Hieraus findet man für den Druck im äusseren wirbelfreien Gebiet

$$p_a = Const + \mu G + \frac{\mu\gamma\delta}{2}\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right) x - \frac{\mu\gamma^2\delta^2}{8}\left(1 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right)$$

Die Isobaren sind demnach der y Achse parallele Geraden, und ihre Gradienten dabei constant, $= \frac{\mu\gamma\delta}{\kappa}\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right)$.

Um die Bewegung im inneren wirbelerfüllten Gebiete zu bestimmen brauchen wir nur W für einen inneren Punkt aufzustellen. Zu dem Ende setzen wir den Werth für φ_i in die Gleichung (42) (pag. 169, Vol. I) so dass wir unter der Voraussetzung, dass die Bewegung eine stationäre sei, erhalten

$$-\gamma x \frac{d\Delta W_i}{dx} + (\kappa - \gamma)\,\Delta W_i + 2\lambda \sin\theta\,\gamma = 0 \qquad (84)$$

Wenn wir hierbei zunächst annehmen, dass $\kappa \gtrless \gamma$ sei, und $\frac{\kappa}{\gamma} = m$ setzen, so folgt durch Integration

$$\Delta W_i = Ax^{(m-1)} - \frac{2\lambda \sin\theta}{(m-1)} = \frac{d^2 W_i}{dx^2}$$

wo A eine willkürliche Constant ist. Eine nochmalige Integration ergiebt hieraus

$$\frac{dW_i}{dx} = \frac{Ax^m}{m} - \frac{2\lambda \sin\theta . x}{(m-1)}.$$

Die zweite Integrationsconstante ist $= 0$ zu setzen, wenn die Gesch-

windigkeit in der Mittelebene ($x=0$) verschwinden soll, während die erste Integrationsconstante A gemäss der Grenzbedingung

$$\frac{\partial W_i}{\partial x}=\frac{\partial W_a}{\partial x} \qquad \text{für } x=\frac{\delta}{2}$$

bestimmt werden muss. Dies ergiebt

$$A=+\frac{\lambda \sin\theta.\delta}{(m-1)}\left(\frac{2}{\delta}\right)^{m}=\frac{2\lambda\sin\theta}{(m-1)}\left(\frac{2}{\delta}\right)^{(m-1)}$$

mithin folgt

$$\frac{dW_i}{dx}=\frac{2\lambda\sin\theta.x}{(m-1)}\left[\frac{1}{m}\left(\frac{2x}{\delta}\right)^{m-1}-1\right]$$

Weiter folgt aus $\Delta W_i=-\zeta+2\lambda\sin\theta$ als Rotationsgeschwindigkeit eines Lufttheilchens

$$\zeta=\frac{2\lambda\sin\theta}{(m-1)}\left[m-\left(\frac{2x}{\delta}\right)^{m-1}\right]$$

und folglich als Deviationswinkel

$$tag\ i=\frac{2\lambda\sin\theta}{\kappa(m-1)}\left[m-\left(\frac{2x}{\delta}\right)^{m-1}\right]$$

Der Deviationswinkel nimmt demnach stetig nach der Mittelebene zu, wo entweder $tag\ i=\frac{2\lambda\sin\theta\, m}{\kappa(m-1)}$ oder $=\infty$, je nachdem $m>1$ oder $m<1$ ist. Als Componenten der Geschwindigkeit im inneren wirbelerfüllten Gebiete erhalten wir ferner

$$\begin{aligned}
u&=-\gamma x\\
v&=-\frac{2\lambda\sin\theta.x}{(m-1)}\left[\frac{1}{m}\left(\frac{2x}{\delta}\right)^{m-1}-1\right] \qquad (84_a)\\
w&=\gamma.z.
\end{aligned}$$

Die Componenten v ist positiv für die nördliche Hemisphäre auf der positiven Hälfte der $(x\,y)$ Ebene, gleichgiltig, ob $m > 1$ oder $m < 1$ ist. Ist nämlich $m > 1$, so ist, da $\frac{2x}{\delta}$ immer ein echter Bruch ist, die eingeklammerte Grösse demnach negativ. Ist dagegen $m < 1$, so ist $\frac{1}{m}\left(\frac{2x}{\delta}\right)^{m-1} > 1$, da $\left(\frac{2x}{\delta}\right)$ ein echter Bruch ist, und die eingeklammerte Grösse daher positiv. Da aber $(m-1)$ zugleich negativ ist, so folgt, dass v auch in diesem Fall positiv sein muss. Als Windbahn findet man

$$y = -\frac{2\,\lambda \sin\theta\, x}{\gamma(m-1)} \cdot \left[1 - \frac{1}{m^2}\left(\frac{2x}{\delta}\right)^{m-1}\right] + const. \qquad (84_b)$$

also im Allgemeinen eine Parabel höheren Grades, welche die Normale der Mittelebene unter einem Winkel schneidet, dessen trigonometrische Tangente $= \frac{2\,\lambda \sin\theta.\, m}{(m-1)}$ ist, wenn $m > 1$. Sie geht aber parallel zur Mittelebene, wenn $m < 1$ ist.

Die Bewegung der Luft im wirbelerfüllten Gebiete lässt sich etwa, wie folgt, charakterisiren. Auf der südlichen Hälfte der (xy) Ebene strömt die Luft in der Richtung N z. O, und nährt sich, sich allmälig umbiegend in der Nähe der Mittelebene, um so rascher dem reinen Ost je grösser γ gegen κ ist, d. i. je grösser die Geschwindigkeit der aufsteigenden Strömung gegen Reibungswiderstand an der Erdoberfläche ausfällt. Die Luftbewegung geschieht auf der nördlichen Hälfte in diametral entgegengesetzter Richtung; sie ist anfangs S z. W und geht in der Nähe der Mittelebene allmälig zum reinen West über, und zwar um so rascher, je grösser γ gegen κ ausfällt. Die Figur (1) Taf. XXV. veranschaulicht den ungefähren Verlauf der Windbahnen im innern und äusseren Gebiete für die nördliche Hemisphäre

In Folge der vertical aufsteigenden Strömung beschreibt dabei jedes Lufttheilchen eine Curve doppelter Krümmung, deren horizontale Projection jene Parabel höheren Grades ist. Aus der ersten und dritten Gleichung in (84_a) ergiebt sich durch Integration

$$x\,z = const.$$

Die Projection der Strömungslinien auf die $(x\,z)$ Ebene ist eine Hyperbel. Diejenige auf die $(z\,y)$ Ebene ist, wenn wir die Coordinaten eines Theilchens zur Zeit 0 mit x_0, y_0, z_0 bezeichnen

$$y = -\frac{2\,\lambda\,sin\,\theta}{(m-1)}\left(\frac{z_0 x_0}{z}\right)\left[1-\frac{1}{m^2}\left(\frac{2x_0 z_0}{z.\delta}\right)^{m-1}\right] + const.$$

Diese Gleichung stellt im Allgemeinen eine hyperbelartige Curve dar, welche die Axen der Coordinaten z und y zu ihren Asymptoten hat, wenn $m > 1$ ist. Indem ein Lufttheilchen also auf solchen Linien hinaufströmt, erreicht es nie die Mittelebene, wenn es einmal um ein Endliches von derselben entfernt war; denn die Zeit, welche dazu nöthig ist, erweist sich als unendlich gross. Aus der ersten, und dritten Gleichung in (84_a) ergiebt sich durch Integration

$$x = x_0\,e^{-\gamma t} \qquad\qquad z = z_0\,e^{+\gamma t}.$$

und hieraus weiter durch Elimination von x aus (84_b)

$$y = -\frac{2\,\lambda\,sin\,\theta}{(m-1)}\frac{x_0}{\gamma}\left[e^{-\gamma t} - \frac{1}{m^2}\left(\frac{2x_0}{\delta}\right)^{(m-1)} e^{-\gamma t}\right] + const.$$

Um die Mittelebene muss daher thatsächlich eine Windstille herrschen, da die Luft nur vertical aufwärts strömt.

Von nicht geringerem Interess ist der Ausdruck für den Druck. Die Gleichung für die Isodynamen (44) (pag. 170 Vol. I) verwandelt sich in unserem besonderen Fall in

$$\frac{d^2\phi}{dx^2} = \frac{d^2 W_i}{dx^2}\frac{d W_i}{dx} + \left(\frac{d^2 W_i}{dx^2} - 2\,\lambda\,sin\,\theta\right)\frac{d^2 W_i}{dx^2} + \kappa\,\gamma$$

Indem man für $\frac{d^3 W_i}{dx^3}$ $\frac{d^2 W_i}{dx^2}$ und $\frac{dW_i}{dx}$ ihren bereits bekannten Werth einsetzt, und zweimal integrirt, erhält man

$$\Phi_i = \frac{\delta^2 \lambda^2 \sin^2\theta}{2m^2(m-1)^2}\left(\frac{2x}{\delta}\right)^{2m} - \frac{\lambda^2 \sin^2\theta\, \delta^2}{(m-1)^2\,(m+1)}\left(\frac{2x}{\delta}\right)^{m+1}$$
$$+ \frac{x^2}{2}\left(\frac{4\lambda^2 \sin^2\theta\, m}{(m-1)^2} + \kappa\gamma\right) + Cx + C'$$

wo C und C' zwei willkürliche Constanten sind, die gemäss der Bedingungen, die an der Grenzebene zu erfüllen sind, bestimmt werden müssen; d. i. gemäss der Bedingungen

$$\Phi_i = \Phi_a \qquad \frac{d\Phi_i}{dx} = \frac{d\Phi_a}{dx} \qquad \text{für } x = \frac{\delta}{2}$$

woraus die beiden Gleichungen hervorgehen,

$$C' + C\frac{\delta}{2} + \left(\frac{\lambda \sin\theta . \delta}{m-1}\right)^2\left(\frac{1}{2m^2} - \frac{1}{m+1}\right) + \frac{\delta^2}{8}\left(m\,\frac{4\lambda^2 \sin\theta}{(m-1)^2} + \kappa\gamma\right)$$
$$= \frac{\gamma\delta^2}{2}\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right) + Const$$

$$C + \frac{2}{\delta}\left(\frac{\lambda \sin\theta . \delta}{m-1}\right)^2\left(\frac{1}{m} + 1\right) + \frac{2}{\delta}\left(m\,\frac{4\lambda^2 \sin^2\theta}{(m-1)^2} + \kappa\gamma\right)$$
$$= \frac{\gamma\delta}{2}\left(\kappa + \frac{2\lambda^2 \sin^2\theta}{\kappa}\right)$$

Da in der ersten Gleichung rechter Hand eine dritte willkürliche Constante auftritt, so können wir $C' = 0$ setzen, und erhalten somit

$$C = \frac{\gamma\delta}{2}\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right) - \frac{2}{\delta}\left[\left(\frac{\delta\lambda \sin\theta}{m-1}\right)^2\left(\frac{1}{m} + 1\right) + m\,\frac{4\lambda^2 \sin^2\theta}{(m-1)^2} + \kappa\gamma\right]$$

Das Quadrat der resultirenden Geschwindigkeit ist

$$= \gamma^2(x^2 + z^2) + \left(\frac{2}{\delta}\right)^{2m}\left(\frac{\delta\lambda \sin\theta}{m-1}\right)^2\frac{x^{2m}}{m^2}$$
$$- \frac{4}{m}\left(\frac{2}{\delta}\right)^m\frac{\delta\lambda^2 \sin^2\theta}{(m-1)^2}\,x^{m+1} + \frac{4\lambda^2 \sin^2\theta}{(m-1)^2}\,x^2$$

$$= \gamma^2 (x^2 + z^2) + \frac{1}{m^2}\left(\frac{\delta \lambda \sin \theta}{m-1}\right)^2 \left(\frac{2x}{\delta}\right)^{2m}$$

$$- \frac{2}{m} \frac{\lambda^2 \sin^2\theta . \delta^2}{(m-1)^2}\left(\frac{2x}{\delta}\right)^{m+1} + \frac{4\lambda^2 \sin^2\theta}{(m-1)^2} x^2$$

da

$$\Phi = \frac{1}{2} (u^2 + v^2 + w^2) + \frac{p}{\mu} - G$$

ist, so folgt schliesslich als Gleichung für Druck

$$p_i = \mu G - \gamma^2 \frac{\mu}{2} (x^2 + z^2) + \frac{\mu}{m(m+1)}\left(\frac{\delta \lambda \sin \theta}{m-1}\right)^2 \left(\frac{2x}{\delta}\right)^{m+1}$$

$$+ \frac{\mu x^2}{2}\left(\frac{4\lambda^2 \sin^2\theta}{(m-1)} + \kappa\gamma\right) + C\mu x$$

oder etwas anderes geordnet

$$p_i = \mu G - \frac{\mu\gamma^2}{2} z^2 + \frac{\mu}{m(m+1)}\left(\frac{\delta \lambda \sin \theta}{m-1}\right)^2 \left(\frac{2x}{\delta}\right)^{m+1}$$

$$+ \frac{\mu x^2}{2}\left[\frac{4\lambda^2 \sin^2\theta}{(m-1)} + \gamma^2 (m-1)\right] + C\mu x.$$

Der Druck ist, wis es zu erwarten stand, um die Mittelebene ein Minimum, und wächst nach Aussen zu stetig, so lange $m > 1$, d. h. $\kappa > \gamma$ ist. Wird $m < 1$ d. h. $\kappa < \gamma$, so wird der Factor von x^2 negativ, so könnte es innerhalb des Wirbelgebietes ein Maximum geben, weil der zweite Differentialquotient von p_i auch innerhalb der Wirbelgebietes einmal verschwinden kann, was auch der Fall ist, wenn γ grösser als $2\lambda \sin \theta$ ist. Es ist nämlich

$$\frac{d^2 p_i}{dx^2} = \mu \left[\left(\frac{\delta \lambda \sin \theta}{1-m}\right)^2 \left(\frac{2}{\delta}\right)^{m+1} x^{m-1} - \frac{4\lambda^2 \sin^2\theta}{(1-m)} - \gamma^2 (1-m)\right]$$

Setzt man dies $= 0$, so erhält man

$$\left(\frac{2x}{\delta}\right)^{(m-1)} = \left(\frac{2x}{\delta}\right)^{-(1-m)} = (1-m) + \frac{\gamma^2 (1-m)^3}{4 \lambda \sin^2 \theta}.$$

woraus das oben Gesagte hervorgeht. Ob aber das Maximum in der That einem inneren Punkte angehört, das hängt von einem sehr verwickelten Verhältniss zwischen $\lambda \sin\theta$, γ und κ ab.

Wir erledigen jetzt den Fall, wo $\kappa = \gamma$ wird, und gehen von der Gleichung (84) aus, welche sich in diesem besonderen Fall verwandelt in

$$-\gamma x \frac{d\Delta W_i}{dx} + 2\lambda \sin\theta . \gamma = 0$$

Das Integral hiervon ist

$$\Delta W_i = 2\lambda \sin\theta \log(x) + C$$

wo C eine willkürliche Constante ist. Die Rotationsgeschwindigkeit der Lufttheilchen in der Mittelebene wird demnach unendlich gross. Die nochmalige Integration ergiebt

$$\frac{dW_i}{dx} = 2\lambda \sin\theta . x . [\log(x) - 1] + Cx$$

Die zweite willkürliche Constante ist $= 0$ zu setzen, wenn die Geschwindigkeit in der Mittelebene verschwinden soll. Der Grenzbedingung wird es gegnügt, wenn wir setzen

$$C = -2\lambda \sin\theta \log\left(\frac{\delta}{2}\right)$$

so dass wir erhalten

$$\frac{dW_i}{dx} = -2\lambda \sin\theta \left(1 - \log\frac{2x}{\delta}\right)$$

Als Geschwindigkeitscomponenten findet man somit

$$u = -\gamma x$$
$$v = 2\lambda \sin\theta x . \left(1 - \log\frac{2x}{\delta}\right)$$
$$w = \gamma . z .$$

als Rotationsgeschwindigkeit

$$\zeta = 2\lambda \sin\theta - \Delta W_i = 2\lambda \sin\theta \left[1 - \log\left(\frac{2x}{\delta}\right)\right]$$

und hieraus als Deviationswinkel

$$tag\, i = \frac{2\lambda \sin\theta}{\kappa} \left[1 - \log\left(\frac{2x}{\delta}\right)\right]$$

Da $tag\, i = \infty$ wird für $x = 0$, so müssen die Lufttheilchen in der Nähe der Mittelebene parallel zu derselben strömen. Demgemäss ist die Gleichung der Windbahn

$$y = -\frac{2\lambda \sin\theta}{\kappa} \cdot x \left[2 - \log\left(\frac{2x}{\rho}\right)\right] + const$$

eine transcendente Curve, welche sich der y Achse fortwährend nährt, ohne sie je zu erreichen.

Die Figur (2) Taf. XXV stellt den Verlauf der Windbahnen in diesem Fall dar.

Die Lufttheilchen, welche einmal nicht auf der Erdoberfläche waren, werden dabei durch die Verticalströmung emporgerissen, und steigen hinauf auf Linien doppelter Krümmung, deren horizontale Projection jene transcendente Linie ist, deren verticale Projectionen auf die $(x\, z)$ Ebene und auf die $(z\, y)$ Ebene aber durch folgende Gleichungen

$$x\, z = const.$$

$$y = -\frac{2\lambda \sin\theta}{\kappa} \left(\frac{z_0 x_0}{z}\right) \left[2 - \log\left(\frac{2z_0 x_0}{\delta^2}\right)\right] + const.$$

bestimmt sind. Die erste Curve ist wieder eine Hyperbel, und die zweite eine transcendente Linie, bestehnd aus zwei Zweigen, die sich sowohl y Achse als der z Achse asymptotisch nähren.

Sowohl die Isodynamen, als die Isobaren bestehn selbstredend aus zur y Achse parallelen Geraden, und ihre Ausdrücke können auch ohne mindeste Schwierigkeit gefunden werden.

Ganz eben so leicht und vollständig, wie der vorhergehende Fall, lässt sich auch der Fall behandeln, wo die Luft in dem durch zwei unendliche parallele Ebenen begrenzten Gebiete niederströmt, mit der Geschwindigkeit $w = -\gamma . z.$, ein Fall, in welchem wieder, wie in dem Fall des kreisförmigen Gebietes der Verticalströmung, alle für den Fall der vertical aufsteigenden Strömung aufgestellten Ausdrücke ungiltig werden, da sie, wenn wir statt γ überall $-\gamma$ einführen, bei jedem Werthe von γ, in der Mittelebene unendlich gross werden. Wir setzen

$$\varphi_a = \frac{\gamma}{2}\,\delta . x + const \qquad \varphi_i = \frac{\gamma}{2} x^2$$

so dass die Bedingungen

$$\Delta \varphi_a = 0 \qquad \Delta \varphi_i = \gamma \qquad \frac{\partial \varphi_i}{\partial x} = \frac{\partial \varphi_a}{\partial x} \qquad \text{für } x = \frac{\delta}{2}$$

erfüllt werden.

Wenn wir die Hülfsfunction W für das innere Gebiet bilden, so begegnen wir auch hier derselben Erscheinung, wie in dem Fall eines kreisförmigen Gebietes der Verticalströmung, dass im inneren Gebiete nur Wirbel mit constanter Rotationsgeschwindigkeit vorhanden sein können. Es ist daher

$$\Delta W_i = const.$$

Aus der Gleichung (84) fliesst ohne Weiteres, indem wir $-\gamma$ für γ setzen

$$\Delta W_i = \frac{2 \lambda \sin \theta . \gamma}{\kappa + \gamma}$$

Mithin folgt

$$\frac{dW_i}{dx} = \frac{2 \lambda \sin \theta . \gamma}{\kappa + \gamma} . x$$

wo die Integrationsconstante $= 0$ zu setzen ist, da $\frac{dW_i}{dx}$ in der Mittelebene verschwinden soll.

Die für das äussere Gebiet giltige Lösung, $W_a = \frac{2\lambda \sin \theta}{\kappa} \varphi_a$ genügt auch hier der Grenzbedingung nicht. Es ist hieraus zu schliessen, dass auch in dem vorliegenden Fall Wirbel mit variabler Rotationsgeschwindigkeit ausserhalb des Gebietes der Verticalströmung existiren müssen. Die Gleichung (43) (pag. 169. Vol. I.) wird in dem vorliegenden Fall

$$\frac{\partial \varphi_a}{\partial x} \frac{\partial \Delta W_a}{\partial x} + \kappa \Delta W_a = 0$$

d. h.

$$\frac{\gamma\delta}{2} \frac{d\Delta W_a}{dx} + \kappa \Delta W_a = 0$$

Das Integral hiervon ist

$$\Delta W_a = C\, e^{-\frac{2\kappa}{\gamma\delta} \cdot x}$$

wo C eine Constante ist. Hieraus folgt weiter

$$\frac{dW_a}{dx} = -\frac{\gamma\delta}{2\kappa} C e^{-\frac{2\kappa}{\delta\gamma} x} + C'$$

Die zweite Integrationsconstante C' bestimmt sich durch die Bemerkung, dass $\frac{dW_a}{dx}$ für unendlich grosses x, $= \frac{\gamma\delta}{2}\left(\frac{2\lambda \sin\theta}{\kappa}\right)$ werden muss. Mithin folgt

$$\frac{dW_a}{dx} = -\frac{\gamma\delta}{2\kappa} C e^{-\frac{2\kappa}{\gamma\delta} x} + \frac{\gamma\delta}{2} \cdot \frac{2\lambda \sin\theta}{\kappa}$$

Der Grenzbedingung

$$\frac{dW_a}{dx} = \frac{dW_i}{dx} \qquad \text{für } x = \frac{\delta}{2}$$

genügt man, indem man setzt

$$C = \frac{2\lambda \sin\theta . \gamma}{(\gamma + \kappa)} \quad e^{\frac{\kappa}{\gamma}}$$

so dass wir schliesslich erhalten

$$\frac{dW_a}{dx} = \frac{\gamma\delta\lambda\sin\theta}{\kappa}\left[1 - \frac{\gamma}{\gamma+\kappa}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right]$$

Die Componenten der Geschwindigkeit im inneren Gebiete sind

$$u = \gamma x$$

$$v = -\frac{2\lambda\sin\theta.\gamma}{(\kappa+\gamma)}.\, x$$

$$w = -\gamma . z.$$

und die Rotationsgeschwindigkeit eines Lufttheilchens

$$\zeta = \frac{2\lambda\sin\theta}{(\kappa+\gamma)}\,\kappa.$$

und hieraus folgt als Deviationswinkel

$$tag\, i = \frac{2\lambda\sin\theta}{(\kappa+\gamma)}$$

also constant, und wird um so kleiner, je grösser γ gegen κ ausfällt. Als Windbahn findet man

$$y = -\frac{2\lambda\sin\theta}{\kappa+\gamma}.\, x + const$$

also eine Gerade, welche mit der positiven y Achse einen Winkel einschliesst, dessen trigonometrische Tangente $= -\frac{2\lambda\sin\theta.}{(\kappa+\gamma)}$ ist.

Indem die Lufttheilchen niederwärts strömen, fahren sie aus der Grenzebene heraus auf Curven, deren Projectionen auf die $(z\,x)$ und $(y\,z)$ Ebene durch die Gleichungen dargestellt werden.

$$x z = const. \qquad y z = const.$$

d. h. ein System gleichseitiger Hyperbeln.

Als Ausdruck der Isodynamen findet man ferner für einen inneren Punkt

$$\Phi_i = -\gamma\kappa\left(1 + \frac{4\lambda^2\sin^2\theta}{(\kappa+\gamma)^2}\right)\frac{x^2}{2} + const$$

Da das Quadrat der resultirenden Geschwindigkeit

$$= \gamma^2 z^2 + \gamma^2 x^2 \left(1 + \frac{4\lambda^2 \sin^2\theta}{(\kappa+\gamma)^2}\right)$$

ist, so kommt hieraus, als Ausdruck für den Druck des inneren Gebietes

$$p_i = Const + \mu G - \frac{\gamma^2 z^2}{2} - \gamma(\kappa+\gamma)\left(1 + \frac{4\lambda \sin^2\theta}{(\kappa+\gamma)^2}\right)\frac{x^2}{2}$$

Wir wenden uns zu, zu der Betrachtung der Bewegung im äusseren Gebiete. Die Componenten der Geschwindigkeit sind gegeben in

$$u = \frac{\gamma\delta}{2}$$

$$v = -\frac{\gamma\delta \sin\theta}{\kappa}\left(1 - \frac{\gamma}{\gamma+\kappa}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right)$$

Die Rotationsgeschwindigkeit eines Theilchens ist

$$\zeta = 2\lambda \sin\theta\left(1 - \frac{\gamma}{\kappa+\gamma}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right)$$

Hieraus folgt als Deviationswinkel

$$tag\, i = \frac{2\lambda \sin\theta}{\kappa}\left(1 - \frac{\gamma}{\kappa+\gamma}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right)$$

tag i wird sonach an der Grenzebene, $= \frac{2\lambda \sin\theta}{\kappa+\gamma}$, und wächst von da ab allmälig bis zum Maximum $\frac{2\lambda \sin\theta}{\kappa}$.

Als Windbahn findet man

$$y = -\frac{2\lambda \sin\theta}{\kappa}\left(x + \frac{\gamma^2\delta}{2\kappa(\gamma+\kappa)}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right) + const.$$

Die ist demnach eine Exponentialcurve, welche sich der Gerade $y = -\frac{2\lambda \sin\theta}{\kappa}\cdot x + const.$ asymptotisch nährt. Die Figur (3) Taf. XXV. veranschaulicht den ungefähren Verlauf der Windbahnen in den beiden Gebieten

Die Isobaren sind zur x Achse senkrechte parallele Geraden wie die Isodynamen, deren Gleichung

$$\Phi_a = -\frac{2\lambda \sin^2\theta\, \gamma^3 \delta^2}{\kappa^2(\gamma+\kappa)}\left(1 - \frac{\gamma}{4(\gamma+\kappa)}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right) e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)} + Const.$$

ist. Da das Quadrat der resultirenden Geschwindigkeit

$$= \frac{\gamma^2\delta^2}{4} + \frac{\lambda^2 \sin^2\theta}{\kappa^2}\, \gamma^2\, \delta^2 \left(1 - \frac{\gamma}{\gamma+\kappa}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}\right)^2$$

ist, so folgt als Ausdruck für den Druck im äusseren Gebiete

$$p_a = Const. + \mu\, G - \frac{\gamma^2\delta^2\mu}{8}\left(1 + \frac{4\lambda^2 \sin^2\theta}{\kappa^2}\right) - \frac{3\mu\lambda^2 \sin^2\theta\, \gamma^3\delta^2}{\kappa^2(\gamma+\kappa)}\, e^{-\frac{\kappa}{\gamma}\left(\frac{2x}{\delta}-1\right)}$$

Wie es zu erwarten war, herrscht um die Mittelebene ein Maximum des Drucks, welcher dann nach Aussen zu stetig abnimmt, um schliesslich in hinreichend grosser Entfernung vom Gebiete der Verticalströmung zu einem neuen Maximum asymptotisch aufzusteigen. Was die Bewegung der Lufttheilchen unter diesem Druckverhältniss anbelangt, so strömen sie durchaus im anticyklonalen Sinne; sie gehen auf der nördlichen Hälfte geradlinig in der Richtung S z.W und auf der südlichen Hälfte des inneren Gebietes in der Richtung N z.O. und indem sie aus der Grenzebene heraustreten, wird ihre Strömungsrichtung noch mehr nach W respect. O abgelenkt., und die Bahnen welche sie beschreiben, krümmen sich zuerst rasch nach W. resp. O., um in hinreichender Ferne vom Gebiete der Verticalströmung wieder geradlinig zu werden.

Die bisher entwickelten Ausdrück gelten für die nördliche Hemisphäre. Offenbar erhält man hieraus solche für die südliche Hemisphäre, wenn man überall statt θ, $-\theta$ einführt. Die Ausdrücke für

den Druck, wie für die Geschwindigkeitscomponenten u und w änderen sich nicht, wohl aber für die Componente der Geschwindigkeit parallel zum Parallelkreis; sie wechselt ihr Vorzeichen, wenn man für $+\theta$, $-\theta$ setzt. Es folgt hieraus, dass die Strömungsrichtung der Luft unter denselben Umständen auf der südlichen Hemisphäre um einen rechten Winkel im Sinne der Erdrotation gedreht erscheinen muss.

Haben die beiden unendlichen Ebenen welche das Gebiet der Verticalströmung begrenzen, eine andere Lage, als *die* parallel zur y Achse (zum Parallelkreis), so ist der Fall leicht auf den oben gefundenen zurückführbar. Es sei ψ der Winkel, welchen die unendlichen Ebenen mit der positiven y Achse, d. h. mit der Richtung von W. nach O. einschliessen. Wir führen ein anderes Coordinatensystem ξ, η ein, welche so definirt sind, dass

$$\xi = x \cos\psi + y \sin\psi$$

$$\eta = y \cos\psi - x \sin\psi$$

Dann giebt $\xi = \frac{\delta}{2}$ die eine unendliche Ebene, und $\xi = -\frac{\delta}{2}$ die andere. Da ferner die Linien $\xi = Const.$, $\eta = Const.$ Geraden sind, und sich senkrecht schneiden, so ist

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{\partial^2\varphi}{\partial \xi^2} + \frac{\partial^2\varphi}{\partial \eta^2} = 0 \qquad oder = \pm\gamma$$

erfüllt, so wie

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = \frac{\partial^2 W}{\partial \xi^2} + \frac{\partial^2 W}{\partial \eta^2} = -\xi + 2\lambda \sin\theta$$

Wenn wir in die allgemeine Gleichung ((42) pag. 169 Vol. I) statt x, y, ξ, η einführen, so finden wir

$$\left(\frac{\partial \Delta W}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \Delta W}{\partial \eta}\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial \varphi}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial W}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial W}{\partial \eta}\frac{\partial \eta}{\partial y}\right)$$

$$+\left(\frac{\partial \Delta W}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial \Delta W}{\partial \eta}\frac{\partial \eta}{\partial y}\right)\left(\frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial \varphi}{\partial \eta}\frac{\partial \eta}{\partial y}-\frac{\partial W}{\partial \xi}\frac{\partial \xi}{\partial x}-\frac{\partial W}{\partial \eta}\frac{\partial \eta}{\partial x}\right)$$

$$+(\kappa-\gamma)\Delta W+2\lambda \sin\theta.\gamma=0$$

Wenn wir hierin setzen

$$\frac{\partial \varphi}{\partial \eta}=0 \qquad \frac{\partial \Delta W}{\partial \eta}=0 \qquad \frac{\partial W}{\partial \eta}=0$$

so dass wir und φ, W und ΔW als Functionen von ξ allein vorzustellen haben, so folgt

$$\frac{\partial \Delta W}{\partial \xi}\frac{\partial \xi}{\partial x}\left(\frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial W}{\partial \xi}\frac{\partial \xi}{\partial y}\right)+\frac{\partial \Delta W}{\partial \xi}\frac{\partial \xi}{\partial y}\left(\frac{\partial \varphi}{\partial \xi}\frac{\partial \xi}{\partial y}-\frac{\partial W}{\partial \xi}\frac{\partial \xi}{\partial x}\right)$$

$$+(\kappa-\gamma)\Delta W+2\lambda \sin\theta.\gamma=0$$

d. i.

$$\frac{\partial \Delta W}{\partial \xi}\frac{\partial \varphi}{\partial \xi}\left[\left(\frac{\partial \xi}{\partial x}\right)^2+\left(\frac{\partial \xi}{\partial y}\right)^2\right]+(\kappa-\gamma)\Delta W+2\lambda \sin\theta.\gamma=0$$

Da aber

$$\left(\frac{\partial \xi}{\partial x}\right)^2+\left(\frac{\partial \xi}{\partial y}\right)^2=\cos^2\psi+\sin^2\psi=1$$

so folgt

$$\frac{d\varphi}{d\xi}\frac{d\Delta W}{d\xi}+(\kappa-\gamma)\Delta W+2\lambda \sin\theta.\gamma=0$$

ganz wie die Gleichung (84), wenn wir hierin

$$\varphi=\pm\frac{\gamma\xi^2}{2}+const.$$

setzen.

Sind daher die beiden unendlichen Ebenen gegen den Parallelkreis um den Winkel ψ geneigt, so findet man die für diesen Fall giltige Lösung, wenn man in alle bisher aufgestellte Ausdrücke statt x, y, $x\cos\psi+y\sin\psi$, respect. $y\cos\psi-x\sin\psi$ einsetzt.

§ IX. *Mehrfache Wirbelbildungen in der Erdatmosphäre.*

Existiren in der Atmosphäre mehr als ein wirbelerfüllter cylindrischer Raum der Verticalströmung, so wird die Bewegung der Luft so verwickelt, und die dadurch verursachte analytische Schwierigkeit auch in den einfachsten Fällen so bedeutend, dass es bisher gelungen ist, die Aufgabe bis zum gewissen Grade zu lösen, und zwar nicht einmal strenge, sondern nur unter der Voraussetzung, dass die gegenseitigen Abstände der Wirbelgebiete gegen die Querschnitte derselben unendlich gross seien, oder was dasselbe ist, dass die Querschnitte der Wirbelgebiete gegen ihren Abstand, der dann endlich sein darf, unendlich klein seien.

Es seien in der Atmosphäre n wirbelerfüllte cylindrische Räume von gegebenem Querschnitte vorhanden, gebildet theils von vertical aufsteigender, theils von vertical niedersteigender Strömung, deren Geschwindigkeiten durch die Gleichungen

$$w_1 = \pm\gamma_1 z \qquad w_2 = \pm\gamma_2 z \qquad w_3 = \pm\gamma_3 z \qquad u.\ s.\ w.$$

bestimmt sind, wo das obere Zeichen im Fall der aufsteigenden Luftströmung, und das untere im Fall der niedersteigenden Luftströmung zu nehmen ist. Dabei wollen wir annehmen, dass das Gebiet der vertical aufsteigenden Strömung immer mit dem Wirbelgebiete zusammenfalle, und dass, wenn gleich das Gebiet der niedersteigenden Strömung möglicherweise mit dem Wirbelgebiet *nicht* zusammenfällt, und dieses gegen jenes unendlich gross ist, der Querschnitt des anticyklonalen Wirbelgebietes doch noch gegen die Entfernungen je zweier Wirbelgebiete unendlich klein sei.

Wir betrachten zunächst die Gleichung

$$\frac{\partial \Delta \bar{w}'}{\partial t} + \frac{\partial \Delta \bar{w}'}{\partial x}\left(\frac{\partial \varphi}{\partial x} + \frac{\partial \bar{w}'}{\partial y}\right) + \frac{\partial \Delta \bar{w}'}{\partial y}\left(\frac{\partial \varphi}{\partial y} - \frac{\partial \bar{w}'}{\partial x}\right) + (\kappa - \gamma)\Delta \bar{w}' + 2\lambda \sin\theta . \gamma = 0$$

wo W, φ Functionen von x, y, t sind, welche innerhalb des Wirbelgebietes, dem x, y gehören, die Bedingungen

$$\Delta W = -\zeta + 2\lambda \sin\theta \qquad \Delta\varphi = \mp\gamma$$

und ausserhalb desselben aber die Bedingungen

$$\Delta W = 0 \qquad \Delta\varphi = 0$$

erfüllen müssen.

Es seien $x_1\, y_1$, $x_2\, y_2$, x_3, y_3 *u. s. w.* die Coordinaten eines Punktes innerhalb des Wirbelgebietes 1, 2, 3, *u, s.* w., und, W_1, W_2, W_3, *u. s, w* und φ_1, φ_2, φ_3 *u, s, w.* die Lösungen der Gleichungen

$$\Delta W_1 = -\zeta_1 + 2\lambda \sin\theta \qquad \Delta W_2 = -\zeta_2 + 2\lambda \sin\theta$$

$$\Delta W_3 = -\zeta_3 + 2\lambda \sin\theta$$

oder allgemein

$$\Delta W_m = -\zeta_m + 2\lambda \sin\theta \tag{85}$$

und $\qquad \Delta\varphi_1 = \mp\gamma_1 \qquad \Delta\varphi_2 = \mp\gamma_2, \qquad \Delta\varphi_3 \mp \lambda_3$

oder allgemein

$$\Delta\varphi_m = \mp\gamma_m \tag{86}$$

Wenn wir setzen

$$W = \sum_1^n W_m \qquad \varphi = \sum_1^n \varphi_m$$

so befriedigen diese Functionen für jeden Punkt ausserhalb der Wirbelgebiete die Bedingungen

$$\Delta W = 0 \qquad \Delta\varphi = 0$$

da diese Gleichungen linear und homogen sind. Für einen Punkt innerhalb eines Wirbelgebietes verschwinden in

$$\Delta W = \sum_1^n \Delta W_m \qquad \Delta\varphi = \sum_1^n \Delta\varphi_m$$

alle Glieder, bis auf ein Glied für das Wirbelgebiet, innerhalb dessen der Punkt x, y liegt, so dass für das Wirbelgebiet m

$$\Delta W = \Delta W_m = -\zeta_m + 2\lambda \sin \theta$$

$$\Delta \varphi = \Delta \varphi_m = \mp \gamma_m$$

Wir erhalten somit als Bestimmungsgleichung für W_m, wenn n Wirbelgebiete vorhanden sind,

$$\frac{\partial \Delta W_m}{\partial t} + \frac{\partial \Delta W_m}{\partial x}\left(\frac{\partial \sum_1^n \varphi_m}{\partial x} + \frac{\partial \sum_1^n W_m}{\partial y}\right) + \frac{\partial \Delta W_m}{\partial y}\left(\frac{\partial \sum_1^n \varphi_m}{\partial y} - \frac{\partial \sum_1^n W_m}{\partial y}\right)$$

$$+ (\kappa - \gamma)\, \Delta W_m \pm 2\lambda \sin \theta . \gamma_m = 0 \tag{89}$$

Die Anzahl solcher Gleichungen ist n und reicht daher hin, für n Gebiete die Hilfsfunction W_n zu bestimmen.

Die Ausdrücke für die Geschwindigkeitscomponenten sind dann für das m'te Gebiet

$$u_m = \frac{\partial \sum_1^n W_m}{\partial y} + \frac{\partial \sum_1^n \varphi_m}{\partial x} \qquad v_m = -\frac{\partial \sum_1^n W_m}{\partial x} + \frac{\partial \sum_1^n \varphi_m}{\partial y}$$

$$w_m = \pm \gamma_m . z$$

während für das äussere wirbelfreie Gebiet im Allgemeinen

$$u = \frac{\partial \sum_1^n W_m}{\partial y} + \frac{\partial \sum_1^n \varphi_m}{\partial x} \qquad v = -\frac{\partial \sum_1^n \varphi_m}{\partial x} + \frac{\partial \sum_1^n \varphi_m}{\partial x}$$

$$w = 0$$

vorausgesetzt, dass die Grenzbedingungen erfüllt sind

$$-\frac{\partial \sum_1^n W_m}{\partial n_a} = \frac{\partial \sum_1^n W_m}{\partial n_i} \qquad -\frac{\partial \sum_1^n \varphi_m}{\partial n_a} = \frac{\partial \sum_1^n \varphi_m}{\partial n_i} \tag{88}$$

wo n_a die nach Aussen und n_i die nach Innen des Wirbelgebietes m gezogene Normale bedeutet.

Wir setzen

$$W_m = \frac{1}{2\pi}\iint (\zeta_m - 2\,\lambda \sin\theta) \log \rho_m \, dx_m \, dy_m$$

$$\varphi_m = \mp \frac{\gamma_m}{2\pi}\iint \log \rho_m \, dx_m \, dy_m$$

wo

$$\rho_m = \sqrt{(x-x_m)^2 + (y-y_m)^2}$$

ist, und die Integration in dem ersteren Ausdruck über das Wirbelgebiet, und diejenige in dem letzteren Ausdruck über das Gebiet der Verticalströmung auszudehnen ist. Die Functionen

$$W = \frac{1}{2\pi}\sum_1^n \iint (\zeta_m - 2\lambda \sin\theta) \log \rho_m \, dx_m \, dy_m$$

$$\varphi = \frac{1}{2\pi}\sum_1^n \mp \gamma_m \iint \log \rho_m \, dx_m \, dy_m$$

genügen den Gleichungen (85) (86), so wie der Grenzbedingung (88) und wenn W_n gemäss der Gleichungen (87) für jedes vorhandene Wirbelgebiet bestimmt ist, so ist die Bewegung der Luft sowohl im Inneren als Äusseren der Wirbelgebiete vollständig bestimmt.

Nun vereinfachen sich die Gleichungen (87) wesentlich durch die Einführung der Annahme, dass die Querdimension der Wirbelgebiete gegen ihre Entfernungen zu einander unendlich klein sei. Wenn wir den ersten Differentialquotienten

$$\frac{\partial \sum_1^n W_m}{\partial x} = \frac{1}{2\pi}\sum_1^n \iint (\zeta_m - 2\lambda \sin\theta)\, \frac{(x-x_m)}{\rho_m^2}\, dx_m \, dy_m$$

betrachten, so sieht man, da ρ_n in jedem Summengliede, dessen Index nicht m ist, unendlich gross ist, dass dann $\frac{x-x_n}{\rho_n^2}$ unendlich klein ist, wie $\frac{1}{\rho_n}$. Jedes Glied, dessen Index nicht m ist, daher unendlich klein gegen dasjenige, dessen Index m ist.

Bezeichnen wir die Summe der Gleider, welche den Index m nicht haben, mit R_m, so dass für das Wirbelgebiet m

$$R_m = \frac{1}{2\pi}\sum_1^{n-1}\iint(\zeta_m - 2\lambda \sin\theta)\log\left(\sqrt{(x-x_m)^2+(y-y_m)^2}\right)dx_m\,dy_m$$

Die Coordinaten x_1 y_1, x_2, y_2 *u. s. w.* sind unendlich gross, da die Entfernungen der anderen $(n-1)$ Wirbelgebiete unendlich gross sind, und da der Spielraum der Coordinaten x, y, insofern sie einen Punkt innerhalb des Wirbelgebietes m bestimmen, dagegen unendlich klein ist, so können wir mit Vernachlässigung von unendlich kleiner Grösse, wie $\frac{1}{\rho_1}$ etc., in R_m x, y, mit den Coordinaten eines gewissen mitteleren Punktes in dem Wirbelgebiete (m) vertauchen, so dass wir schreiben können

$$R_m = \frac{1}{2\pi}\sum_1^{(n-1)}\log\left(\sqrt{(x_n-x_m)^2+(y_n-y_m)^2}\right)\iint(\zeta_m^{\prime\prime} - 2\lambda \sin\theta)\,dx_m\,dy_m$$

Diese Grösse hängt nun von x, y gar nicht ab. Es folgt daher, dass innerhalb des Wirbelgebietes m überall gesetzt werden können, mit Vernachlässigung unendlich kleiner Grösse, wie $\frac{1}{\rho_1^2}$ etc

$$\frac{\partial R_m}{\partial x} = \frac{\partial R_m}{\partial x_m} = A_m \qquad\qquad \frac{\partial R_m}{\partial y} = \frac{\partial R_m}{\partial y_m} = B_m$$

Bezeichnet man ferner die Summe der Glieder in $\frac{\partial\varphi}{\partial x}$, $\frac{\partial\varphi}{\partial y}$, deren Index nicht m ist, mit R'_m, so folgt durch dieselbe Betrachtung, wie oben, dass man überall setzen kann

$$\frac{\partial R'_m}{\partial x} = \frac{\partial R'_m}{\partial x_m} = A'_m \qquad\qquad \frac{\partial R'_m}{\partial y_m} = \frac{\partial R'_m}{\partial x_m} = B'_m$$

Diese Grössen A_m, B_m, A'_m, B'_m hängen von x, y nicht ab, sind aber im allgemeinen Functionen von der Zeit, welche zu ermitteln sind, gemäss der Differentialgleichungen, welche wir noch aufzustellen haben.

Die Gleichung (89) wird dann

$$\frac{\partial \Delta W'_m}{\partial t} + \frac{\partial \Delta W'_m}{\partial x}\left(\frac{\partial W'_m}{\partial y} + \frac{\partial \varphi_m}{\partial x} + B_m + A'_m\right) + \frac{\partial \Delta W'_m}{\partial y}$$
$$\left(\frac{\partial \varphi_m}{\partial y} - \frac{\partial W'_m}{\partial x} + B'_m - A_m\right) + (\kappa - \gamma)\Delta W'_m + 2\lambda \sin\theta . \gamma_m = 0. \qquad (89\,a)$$

Macht man hierin

$$W'_m = W''_m - (B'_m - A_m)\,x - (A'_m + B_m)\,y. \qquad (89\,b)$$

was die Bedingungsgleichung, welcher W'_m genügt, ebenfalls erfüllt, wenn W''_m einer solchen genügt, da

$$\Delta W'_m = \Delta W''_m - \Delta\{(B'_m - A_m)\,x + (A'_m + B_m)\,y\} = \Delta W''_m.$$

ist, so folgt aus (89 a) als Bestimmungsgleichung der neuen Function W''_m augenblicklich

$$\frac{\partial \Delta W'_m}{\partial t} + \frac{\partial \Delta W''_m}{\partial x}\left(\frac{\partial W''_m}{\partial y} + \frac{\partial \varphi_m}{\partial x}\right) + \frac{\partial \Delta W''_m}{\partial y}\left(\frac{\partial \varphi_m}{\partial y} - \frac{\partial W''_m}{\partial x}\right)$$
$$+ (\kappa + \gamma)\Delta W''_m + 2\lambda \sin\theta . \gamma_m = 0$$

Es erhellt hieraus, dass der Einfluss, welchen die Rotationsgeschwindigkeit eines Lufttheilchens in einem Wirbelgebiete von $(n-1)$ anderen Wirbelgebieten erleidet, unter diesen Umständen hinreichend berücksichtigt ist, wenn man W'_m für einen Punkt innerhalb des Wirbelgebietes so bildet, als wäre das Wirbelgebiet allein vorhanden und der so gebildeten Function W'_m das Glied $-(B'_m - A_m)\,x - (A'_m + B_m)\,y$ hinzufügt.

§ X. *Luftbewegung im äusseren wirbelfreien Gebiete bei mehrfachen Wirbelbildungen.*

Für das Gebiet der Erdatmosphäre, wo die Lufttheilchen keine andere Rotationsgeschwindigkeit haben, als die durch Erdrotation veranlasste, lassen sich die Bewegungen der Luft auch hier ganz allgemein verfolgen, wenn sie stationär sind. Die allgemeine Lösung für ein solches äussere Gebiet ist wieder

$$W' = \frac{2\lambda \sin\theta}{\kappa} \cdot \varphi$$

Mithin

$$W = \frac{2\lambda \sin\theta}{\kappa} \sum_1^n \varphi_m$$

Hieraus erhalten wir als Componenten der Geschwindigkeit

$$u = \frac{2\lambda \sin\theta}{\kappa} \sum_1^n \frac{\partial \varphi_m}{\partial y} + \sum_1^n \frac{\partial \varphi_m}{\partial x}$$

$$v = -\frac{2\lambda \sin\theta}{\kappa} \sum_1^n \frac{\partial \varphi_m}{\partial x} + \sum_1^n \frac{\partial \varphi_m}{\partial y}$$

Die Differentialgleichung für die Windbahn ist, wenn wir setzen

$$K = \frac{2\lambda \sin\theta}{\kappa}$$

$$\frac{dy}{dx} = \frac{\sum_1^n \frac{\partial \varphi_m}{\partial y} - K \sum_1^n \frac{\partial \varphi_m}{\partial x}}{\sum_1^n \frac{\partial \varphi_m}{\partial x} + K \sum_1^n \frac{\partial \varphi_m}{\partial y}}$$

woraus folgt

$$K\left(\sum_1^n \frac{\partial \varphi_m}{\partial y} dy + \sum_1^n \frac{\partial \varphi_m}{\partial x} dx\right) = \sum_1^n \frac{\partial \varphi_m}{\partial y} dx - \sum_1^n \frac{\partial \varphi_m}{\partial x} dy \qquad (89\ c)$$

Behufs der Integration dieser Gleichung betrachten wir ein Integral von der Form

$$\psi = \frac{1}{2}\left(\int \sum_1^n \frac{\partial \varphi_m}{\partial y} dx - \int \sum_1^n \frac{\partial \varphi_m}{\partial x} dy\right)$$

Es ist

$$\frac{\partial \psi}{\partial x} = \frac{1}{2}\left(\sum_1^n \frac{\partial \varphi_m}{\partial y} - \int \sum_1^n \frac{\partial^2 \varphi_m}{\partial x_2} dy.\right)$$

da aber vermöge $\Delta \varphi_m = 0$, $\frac{\partial^2 \varphi_m}{\partial x^2} = \quad \frac{\partial^2 \varphi_m}{\partial y^2}$ ist, so folgt,

$$\frac{\partial \psi}{\partial x} = \sum_1^n \frac{\partial \varphi_m}{\partial y} \qquad (90)$$

Bildet man ferner $\frac{\partial \psi}{\partial y}$, so findet man,

$$\frac{\partial \varphi}{\partial y} = -\sum_1^n \frac{\partial \varphi_m}{\partial x} \qquad (91)$$

die Substitution der Ausdrücke (90) (91) in (89 c) ergiebt

$$K\left(\sum_1^n \frac{\partial \varphi_m}{\partial x}\,dx + \sum_1^n \frac{\partial \varphi_m}{\partial y}\,dy\right) = -\frac{\partial \psi}{\partial y}dy - \frac{\partial \psi}{\partial x}\,dx = -\,d\psi$$

Mithin folgt durch Integration

$$K\sum_1^n \varphi_m + \psi = const.$$

d. h.

$$K\sum_1^n \varphi_m + \frac{1}{2}\left[\int \sum_1^n \frac{\partial \varphi_m}{\partial y}\,dx - \int \sum_1^n \frac{\partial \varphi_m}{\partial x}\,dy\right] = const.$$

Wenn nun φ_m lauter symmetrische Functionen von $x-x_m$, $y-y_m$ sind, wo x_m, y_m die Coordinaten eines gewissen mitteleren Punkt in dem Wirbelgebiete (m) bedeuten, so lässt sich diese Gleichung der Windbahn in eine andere Form bringen. Wir setzen

$$\sum_1^n \psi_m = \frac{1}{2}\sum_1^n\left(\int \frac{\partial \varphi_m}{\partial x_m}\,dy_m - \int \frac{\partial \varphi_m}{\partial y_m}\,dx_m\right)$$

Da φ_n symmetrisch in Bezug auf $(x-x_m)$, $(y-y_m)$ gebaut sein soll so ist

$$\frac{\partial \varphi_m}{\partial x_m} = -\frac{\partial \varphi_m}{\partial x} \qquad \frac{\partial \varphi_m}{\partial y_m} = -\frac{\partial \varphi_m}{\partial y}$$

Mithin

$$\sum_1^n \psi_m = -\frac{1}{2}\sum_1^n \int \frac{\partial \varphi_m}{\partial x}\,dy_m + \frac{1}{2}\sum_1^n \int \frac{\partial \varphi_m}{\partial y}\,dx_m = -\frac{1}{2}\frac{\partial}{\partial x}\int \sum_1^n \varphi_m\,dy_m$$
$$+\frac{1}{2}\frac{\partial}{\partial y}\int \sum_1^n \varphi_m\,dx_m$$

Es ist andererseits

$$\sum_1^n \int \varphi_m\,dy_m = \sum_1^n \iint \frac{\partial \varphi_m}{\partial y}\,dy_m\,dy = -\sum_1^n \iint \frac{\partial \varphi_m}{\partial y_m}\,dy_m\,dy = -\sum_1^n \int \varphi_m\,dy$$

$$\sum_1^n \int \varphi_m\,dx_m = \sum_1^n \iint \frac{\partial \varphi_m}{\partial x}\,dx_m\,dx = -\sum_1^n \iint \frac{\partial \varphi_m}{\partial x_m}\,dx_n\,dx = -\sum_1^n \int \varphi_m\,dx$$

so folgt

$$\sum_1^n \psi_m = \frac{1}{2}\int \sum_1^n \frac{\partial \varphi_m}{\partial x}\,dy - \frac{1}{2}\int \sum_1^n \frac{\partial \varphi_m}{\partial y}\,dx \text{ d. h.} = -\psi$$

Es folgt hieraus als Gleichung der Windbahn

$$\sum_1^n \left[K\varphi_m + \frac{1}{2}\int\left(\frac{\partial\varphi_m}{\partial y_m}dx_m - \frac{\partial\varphi_m}{\partial x_m}dy_m\right)\right] = const.$$

Drückt man die Variabeln y_m, x_n in Polarcoordinaten aus, so dass

$$y - y_m = \rho_m \sin \chi_m \qquad (x - x_m) = \rho_m \cos \chi_m$$

so erhält man leicht

$$\sum_1^n \left(\kappa\varphi_m + \frac{1}{2}\int\frac{\partial\varphi_m}{\partial \log \rho_m}d\chi_m - \frac{1}{2}\int\frac{\partial\varphi_m}{\partial\chi_m}d\log\rho_m\right) = const.$$

Eine particuläre Lösung der Gleichung $\Delta\varphi_m = 0$, ist

$$\varphi_m = \mu_m \log \rho_m$$

wo μ_m eine gewisse Constante ist, welche noch näher bestimmt werden muss. Es folgt für diesen Fall unmittelbar

$$\sum_1^n \mu_m \left(K \log \rho_m + \frac{1}{2}\chi_m\right) = Const.$$

oder indem man wieder zum rechtwinkeligen Coordinatensystem zurückkehrt

$$\sum_1^n \mu_m\left(K\log\sqrt{(x-x_m)^2+(y-y_m)^2} + \frac{1}{2}\, arctag\,\frac{(y-y_m)}{(x-x_m)}\right) = const. \quad (92)$$

also eine transcendente Spirale von ausserordentlich verwickelter Natur.

Der Deviationswinkel ist auch hier constant, so dass

$$tag\, i = \frac{2\lambda \sin\theta}{\kappa}$$

Die Windbahnen können hiernach gezeichnet werden, wenn die Isodynamen bekannt sind. Als Gleichung dieser erhalten wir für unseren Fall

$$\psi = -\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right)\sum_1^n \varphi_m$$

woraus für unseren besonderen Fall hervorgeht

$$\Phi = -\left(\kappa + \frac{4\lambda^2 sin^2\,\theta}{\kappa}\right)\sum_1^n \mu_m\, log\sqrt{(x-x_m)^2+(y-y_m)^2}$$

Als Ausdruck für den Druck findet man hieraus, da die Resultirende der Geschwindigkeit

$$=\sqrt{\left(1+\frac{4\lambda^2 sin^2\,\theta}{\kappa^2}\right)\left[\left(\frac{\partial\sum_1^n\varphi_m}{\partial x}\right)^2+\left(\frac{\partial\sum_1^n\varphi_m}{\partial y}\right)^2\right]}$$

ist,

$$p = Const. - \mu\left(\kappa + \frac{4\lambda^2 sin^2\,\theta}{\kappa}\right)\sum_1^n\varphi_m$$
$$-\frac{\mu}{2}\left(1+\frac{4\lambda^2 sin^2\,\theta}{\kappa^2}\right)\left[\left(\frac{\partial\sum_1^n\varphi_m}{\partial x}\right)^2+\left(\frac{\partial\sum_1^n\varphi_m}{\partial y}\right)^2\right]+\mu G$$

woraus für unseren speciellen Fall hervorgeht

$$p = Const. - \mu\left(\kappa + \frac{4\lambda^2 sin^2\,\theta}{\kappa}\right)\sum_1^n\mu_m\, log.\sqrt{(x-x_m)^2+(y-y_m)^2}$$
$$-\frac{\mu}{2}\left(1+\frac{4\lambda^2 sin^2\,\theta}{\kappa^2}\right)\left[\left(\sum_1^n\frac{\mu_m(x-x_m)}{(x-x_m)^2+(y-y_m)^2}\right)^2\right.$$
$$\left.+\left(\sum_1^n\frac{\mu_m(y-y_m)}{(x-x_m)^2+(y-y_m)^2}\right)^2\right]+\mu G$$

die Isobaren fallen demnach in diesem Fall mit Isodynamen nicht zusammen.

§ X. *Eigenbewegungen der Wirbelgebiete.*

Die also bestimmten Windbahnen wie Isodynamen, und Isobaren verändern indessen ihre Lage gegen die Axen der Coordinaten, wie ihre Gestalten fortwährend., denn ein jedes Wirbelgebiet verharrt nicht an seiner Stelle, sondern es bewegt sich um einen gewissen ruhenden Punkt in Folge der Luftströmungen, welche die anderen Wirbelgebiete in der Atmosphäre hervorbringen. Da nun $x_1\, y_1$, $x_2\, y_2$

u, s, w. wie festgesetzt wurde, einen gewissen mittleren Punkt je eines Wirbelgebietes bestimmen, so ist nöthig x_1 y_1 x_2 y_2, u. s. w. als Functionen von der Zeit t darzustellen.

Behufs dieses wollen wir die Coordinaten x_1 y_1 etc. so definiren, dass für jedes Wirbelgebiet

$$x_m \iint (\zeta_m - 2\lambda \sin\theta)\, d\omega_m = \iint (\zeta_m - 2\lambda \sin\theta)\, x\, d\omega_m$$

$$y_m \iint (\zeta_m - 2\lambda \sin\theta)\, d\omega_m = \iint (\zeta_m - 2\lambda \sin\theta)\, y\, d\omega_m$$

wo $d\omega_m$ ein Querschnittelement des Wirbelgebietes bedeutet, und nennen diesen Punkt den Schwerpunkt des entsprechenden Wirbelgebietes. Da nun die Bewegung stationär ist, und die Grössen, ζ_m, $d\omega_m$ von t nicht abhängen, so folgt durch Differentiation

$$\frac{dx_m}{dt} \iint (\zeta_m - 2\lambda \sin\theta)\, dx_m = \iint (\zeta_m - 2\lambda \sin\theta)\, \frac{dx}{dt}\, d\omega_m$$

$$\frac{dy_m}{dt} \iint (\zeta_m - 2\lambda \sin\theta)\, d\omega_m = \iint (\zeta_m - 2\lambda \sin\theta)\, \frac{dy}{dt}\, d\omega_m$$

Setzt man hierin

$$\frac{dx}{dt} = u = \sum_1^n \frac{\partial W_m}{\partial y} + \sum_1^n \frac{\partial \varphi_m}{\partial x} \qquad W_m = \frac{1}{2\pi} \int (\zeta_m' - 2\lambda \sin\theta) \log \rho_m\, d\omega_m'$$

$$\frac{dy}{dt} = v = -\sum_1^n \frac{\partial W_m}{\partial x} + \sum_1^n \frac{\partial \varphi_m}{\partial y} \qquad \varphi_m = \mp \frac{\gamma_m}{2\pi} \int \log \rho_m\, d\omega_m'$$

so kommt

$$\frac{dx_n}{dt} \iint (\zeta_n - 2\lambda \sin\theta)\, d\omega_n$$
$$= \frac{1}{2\pi} \sum_1^n \iint (\zeta_m' - 2\lambda \sin\theta)\, \frac{(y - y_m')}{\rho_m^2}\, (\zeta_n - 2\lambda \sin\theta)\, d\omega_n\, d\omega_m'$$
$$+ \frac{1}{2\pi} \sum_1^n \mp \gamma_m \iint \frac{(x - x_m')}{\rho_m^2} \cdot (\zeta_m' - 2\lambda \sin\theta)\, d\omega_n\, d\omega_m'$$

$$\frac{dy_n}{dt} \iint (\zeta_n - 2\lambda \sin\theta)\, d\omega_n$$
$$= \frac{1}{2\pi} \sum_1^n \iint (\zeta_m' - 2\lambda \sin\theta)(\zeta_n - 2\lambda \sin\theta)\, \frac{(x - x_m)}{\rho_m^2}\, d\omega\, d\omega_m'$$
$$+ \frac{1}{2\pi} \sum_1^n \mp \gamma_m \iint \frac{(y - y_m')}{\rho_m^2}\, (\zeta_m' - 2\lambda \sin\theta)\, d\omega_n\, d\omega_m'$$

Das m'te Glied der ersten Reihe in der ersten Gleichung verschwindet, denn: vertauscht man in dem m'ten Gliede x' mit x, was offenbar auf das schliessliche Resultat der Integration gleichgiltig ist, so erhält man

$$-\iint(\zeta_m - 2\lambda \sin \theta)(\zeta'_m - 2\lambda \sin \theta)\,\frac{(x'_m - x)}{\rho_m^2}\,d\omega'_m\,d\omega_m$$
$$=\iint(\zeta'_m - 2\lambda \sin \theta)(\zeta_m - 2\lambda \sin \theta)\,\frac{(x - x'_m)}{\rho_m^2}\,d\omega_m\,d\omega'_m$$

woraus nothwendig vervorgeht, dass

$$\iint(\zeta_m - 2\lambda \sin \theta)(\zeta'_m\ 2\lambda \sin \theta)\,\frac{(x - x'_m)}{\rho'_m}\,d\omega_m\,d\omega'_m = 0$$

Was das m'te Glied in der zweiten Reihe anbetrifft, so verschwindet es im allgemeinen nicht. Um zu untersuchen, ob es überhaupt verschwinden kann, setzen wir

$$\Omega = \int \log \rho_m\,d\omega_m$$

wo die Integration über das Gebiet der Verticalströmung auszudehnen ist, so dass wir erhalten

$$\iint(\zeta'_m - 2\lambda \sin \theta)\left(\frac{x - x'_m}{\rho_m^2}\right)d\omega'_m\,d\omega'_m = \int(\zeta'_m - 2\lambda \sin \theta)\,\frac{\partial \Omega}{\partial x}\,d\omega'_m$$

Da nun aber

$$(\zeta'_m - 2\lambda \sin \theta)\,\frac{\partial \Omega}{\partial x} = \frac{\partial[(\zeta'_m - 2\lambda \sin \theta)\Omega]}{\partial x} - \Omega\,\frac{\partial \zeta'_m}{\partial x}$$

so folgt

$$\iint(\zeta'_m - 2\lambda \sin \theta)\,\frac{(x - x_m)}{\rho_m^2}\,d\omega_m\,d\omega'_m = \int \frac{\partial[\zeta'_m - 2\lambda \sin \theta)\,\Omega]}{\partial x}\,d\omega'_m - \int \Omega\,\frac{\partial \zeta'_m}{\partial x}\,d\omega'_m$$

Die Function Ω hängt nur von der Gestalt der cylindrischen Gebietes der Verticalströmung ab; sie erleidet keinerlei Sprünge, wenn der Punkt x, y durch die Grenzfläche der Verticalströmung hindurchgeht, und änderte nirgends ihr Vorzeichen. Bezeichnet

man ein Umfangselement des Querschnittes des Wirbelgebietes mit ds, die nach Innen gezogene Normale des Umfangs mit n, so lässt sich das erste Glied in dem vorstehenden Ausdrück auch so schreiben

$$-\int \overline{\Omega}\,(\overline{\zeta}_m - 2\lambda \sin\theta) \cos(nx)\, ds$$

wo $\overline{\Omega}$, $\overline{\zeta}_m$ den Werth von Ω und ζ_n an der Grenzfläche des Wirbelgebietes bedeutet. Nun haben die Lufttheilchen an der Grenzfläche keine andere Rotationsgeschwindigkeit als durch die Erdrotation. hervorgerufene, so dass demnach $\overline{\zeta}_n = 2\lambda \sin\theta$ ist; das Integral über den Umfang des Querschnittes verschwindet.

Wir erhalten somit

$$\iint (\zeta'_m - 2\lambda \sin\theta) \frac{(x - x'_m)}{\rho^2}\, d\omega'_m = -\int \Omega \frac{\partial \zeta'_m}{\partial x}\, d\omega'_m$$

Das rechter Hand stehende Integral ist offenbar eine Grösse wesentlich abhängig von der Gestalt des Wirbelgebietes und von der Art und Weise, wie ζ_m in dem Wirbelgebiete vertheilt ist. Wenn nun die Grenzlinie des Wirbelgebiets so beschaffen ist, dass durch passende Verlegung der Coordinatenaxen. das Wirbelgebiet durch x—, respect. y—Achse in je zwei vollkommen symmetrische Flächenstücke getheilt werden kann. so dass Ω eine positive symmetrische Function von x. y. wird. und wenn ζ_m, welches sein Vorzeichen innerhalb des Wirbelgebietes nicht änderen soll. so in Bezug auf die Coordinatenaxen vertheilt ist, dass auch ζ_m eine positive symmetrische Function von x. y und daher $\frac{\partial \zeta_m}{\partial x}$ wie $\frac{\partial \zeta_m}{\partial y}$. für positives x. respect. y positiv und für negatives x, respect. y, negativ wird, und dem absoluten Werthe nach gleich wird für das dem absoluten Werthe nach nämliche x und y, so verschwindet das Integral $\int \Omega \frac{\partial \zeta'_m}{\partial x} d\omega'_m$. Da die Gleider, die unter diesen Umständen verschwinden

$$\iint(\zeta'_m - 2\lambda \sin\theta)(\zeta_m - 2\lambda \sin\theta)\left(\frac{y - y'_m}{\rho_m^2}\right)d\omega_m d\omega'_m$$

$$\mp \gamma_m \iint(\zeta'_m - 2\lambda \sin\theta)\left(\frac{x - x'_m}{\rho_m^2}\right)d\omega_m d\omega'_m$$

$$-\iint(\zeta'_m - 2\lambda \sin\theta)(\zeta_m - 2\lambda \sin\theta)\left(\frac{x - x'_m}{\rho_m^2}\right)d\omega_m d\omega'_m$$

$$\mp \gamma_m \iint(\zeta'_m - 2\lambda \sin\theta)\left(\frac{y - y'_m}{\rho_m^2}\right)d\omega_m d\omega'_m$$

die Componenten derjenigen Geschwindigkeit bedeuten, welche das Wirbelgebiet (m) sich selbst ertheilt haben würde, wenn es allein in der Erdatmosphäre existirte, so folgt, dass ein jedes Wirbelgebiet sich selbst keine fortschreitende Bewegung zu ertheilen mag, wenn es symmetrisch in Bezug auf einen inneren Punkt gebildet und die Rotationsgeschwindigkeit eines jeden Lufttheilchens gleichfalls symmetrisch in Bezug auf denselben Punkt vertheilt ist. Wenn nicht, dann wohl. Dann sind $-\int \Omega \frac{\partial \zeta'_m}{\partial x} d\omega'_m$ und $-\int \Omega \frac{\partial \zeta'_m}{\partial y} d\omega_m$ für jedes Wirbelgebiet individuelle Constanten, welche wir mit α_m und β_m bezeichnen wollen, unabhängig von x_m und y_m, also auch von der Zeit. Denn setzt man

$$x = x_m + \alpha\,\xi + \beta\,\eta \qquad \alpha = \sin\psi \qquad \beta = \cos.\psi$$

$$y = y_m + \beta\,\xi - \alpha\,\eta$$

wo ζ, η die neuen Coordinaten sind, und ψ der Winkel ist, welchen die ξ Achse mit der x Achse einschliesst. Die Function Ω ändert sich gar nicht, da ρ_m^2 sich nicht ändert, und so auch ζ_m. Es ist nämlich

$$\frac{dx}{dt} = \frac{dx_m}{dt} + \alpha\frac{d\xi}{dt} + \beta\frac{d\eta}{dt} \qquad \frac{dy}{dt} = \frac{dy_m}{dt} + \beta\frac{d\xi}{dt} - \alpha\frac{d\eta}{dt}$$

oder

$$u_m = u_m + \alpha u' + \beta v' \qquad v = v_m + \beta u' - \alpha v'$$

Hieraus folgt durch Differentiation, da u_m und v_m von x, y nicht abhängt

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{\partial u'}{\partial \xi}(\alpha\beta-\alpha\beta)+\frac{\partial v'}{\partial y}(-\alpha\beta+\alpha\beta)$$
$$+\frac{\partial v'}{\partial \xi}(\alpha^2+\beta^2)-\frac{\partial u'}{\partial y}(\alpha^2+\beta^2)$$

d. h, da $\alpha^2+\beta^2=1$ ist

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\frac{\partial v'}{\partial \xi}-\frac{\partial u'}{\partial \eta}$$

Da ferner

$$\frac{\partial \zeta'}{\partial x}=\frac{\partial \zeta'}{\partial \xi}\frac{\partial \xi}{\partial x}+\frac{\partial \zeta'}{\partial \eta}\frac{\partial \eta}{\partial x}=\frac{\partial \zeta'}{\partial \xi}\beta-\frac{\partial \zeta'}{\partial \eta}\alpha.$$

$$\frac{\partial \zeta'}{\partial y}=\frac{\partial \zeta'}{\partial \xi}\frac{\partial \xi}{\partial y}+\frac{\partial \zeta'}{\partial \eta}\frac{\partial \eta}{\partial x}=\frac{\partial \zeta'}{\partial \xi}\alpha+\frac{\partial \zeta'}{\partial \eta}\cdot\beta.$$

so folgt

$$\int\Omega\frac{\partial \zeta'_m}{\partial x}d\omega'_m=\beta\int\Omega\frac{\partial \zeta'_m}{\partial \xi}d\omega'_m-\alpha\int\Omega\frac{\partial \zeta'_m}{\partial \eta}d\omega'_m=-\alpha_m$$

$$\int\Omega\frac{\partial \zeta'_m}{\partial y}d\omega'_m=\alpha\int\Omega\frac{\partial \zeta'_m}{\partial \xi}d\omega'_m+\beta.\int\Omega\frac{\partial \zeta'_m}{\partial \eta}d\omega'_m=-\beta_m$$

was das oben Gesagte enthält.

Als Bewegungsgleichungen für den Schwerpunkt des m'ten Wirbelgebietes erhalten wir somit

$$\frac{dx_m}{dt}\int(\zeta_m-2\lambda\sin\theta)\,d\omega_m=\mp\frac{\gamma_m\alpha_m}{2\pi}$$
$$+\sum_1^{(n-1)}\frac{1}{2\pi}\iint\frac{(\zeta_m-2\lambda\sin\theta)(\zeta'_m-2\lambda\sin\theta)(y-y'_m)}{r_m^2}d\omega_m d\omega'_m$$
$$+\sum_1^{(n-1)}\mp\frac{\gamma_n}{2\pi}\iint\frac{(\zeta'_m-2\lambda\sin\theta)(x-x'_m)}{r_m^2}d\omega_m d\omega'_m$$

$$\frac{dy_m}{dt}\int(\zeta_m-2\lambda\sin\theta)\,d\omega_m=\mp\frac{\gamma_m\beta_m}{2\pi}$$
$$-\sum_1^{(n-1)}\frac{1}{2\pi}\iint\frac{(\zeta'_m-2\lambda\sin\theta)(\zeta_m-2\lambda\sin\theta)(x-x'_m)}{r^2}d\omega_m d\omega'_m$$
$$+\sum_1^{(n-1)}\mp\frac{\gamma_m}{2\pi}\iint\frac{(\zeta'_m-2\lambda\sin\theta)(y-y'_m)}{r_m^2}d\omega_m d\omega'_m$$

Diejenigen für andere Wirbelgebiete sind nach diesem Muster aufzustellen.

Die unter dem Summenzeichen stehenden Ausdrücke sind als Functionen von den Coordinaten zu bestimmen, welche den Schwerpunkt eines jeden Wirbelgebietes bestimmen, was leicht bewerkstelligt werden kann, da ζ_m sich nicht ändert, wenn man die Coordinatenaxen, worauf die Integration bezogen ist, anderes verlegt.

Wesentlich einfacher gestaltet sich die Sache indessen, wenn wir annehmen, dass die Querschnitte eines jeden Wirbelgebietes unendlich klein sei gegen ihre gegenseitigen Abstände. Wir betrachten zunächst eine Function von der Form

$$W_m = \frac{1}{2\pi} \sum_1^{(n-1)} \int (\zeta_m - 2\lambda \sin\theta) \log(\rho_{n.m})\, d\omega_m$$

wo ρ_{nm} die Entfernung des Schwerpunktes x_m y_m im m'ten Wirbelgebiete von einem Punkt im n'ten Wirbelgebiete bedeutet, so dass

$$\rho_{nm} = \sqrt{(x_m - x_m)^2 + (y_n - y_m)^2}$$

ist. Differentirt man nun W_m nach x_m, so ist

$$\frac{\partial W_m}{\partial x_m} = -\frac{1}{2\pi} \sum_1^{n-1} \int (\zeta_m - 2\lambda \sin\theta)\, \frac{(x_n - x_m)}{\rho^2_{nm}}\, d\omega_m$$

Nun ist $\frac{x_n - x_m}{\rho_{nm}}$ der Cosinus des Winkels, welchen eine den Schwerpunkt des m'ten Gebietes mit einem Punkt des n'ten Gebietes verbindende Linie mit der x Achse einschliesst. Ist nun der Querschnitt des m'ten Gebietes unendlich klein gegen die Entfernung ρ_{nm}, so unterscheidet sich ρ_{nm} um unendliches Kleines von der Entfernung je zweier Schwerpunkte, und $\frac{x_n - x_m}{\rho_{nm}}$ auch um unendlich Kleines von dem Cosinus des Winkels, welchen die Verbindungslinien zweier Schwerpunkte im Gebiete n und m mit

x Achse einschliesst, so kann man mit Vernachlässigung unendlich kleiner Grösse höherer Ordnung als $\frac{1}{\rho_{mn}}$ setzen

$$\frac{\partial W'_m}{\partial x_m} = \frac{1}{2\pi}\left[\sum_1^{(n-1)} \frac{(x_m - x_n)}{\rho^2_{mn}} \int (\zeta_m - 2\lambda \sin \theta)\, d\omega_m\right]$$

wo x_n, y_n jetzt die Coordinaten des Schwerpunktes im n'ten Gebiete bedeuten. Oder indem man setzt

$$\int (\zeta_m - 2\lambda \sin \theta) = M_m.$$

erhält man

$$\frac{\partial W'_m}{\partial x_m} = \frac{1}{2\pi} \sum_1^{(n-1)} M_n \frac{\partial \log (\rho_{mn})}{\partial x_m}$$

Auf ganz dieselbe Weise erhält man

$$\frac{\partial W'_m}{\partial x_m} = \frac{1}{2\pi} \sum_1^{(n-1)} M_n \frac{\partial \log (\rho_{mn})}{\partial y_m}$$

Setzt man ferner

$$\varphi_m = \frac{1}{2\pi} \sum_1^{(n-1)} \mp \gamma_n \int \log \rho_{nm}\, d\omega_n$$

so erhält man durch dieselbe Betrachtung

$$\frac{\partial \varphi_m}{\partial x_m} = \frac{1}{2\pi} \sum_1^{(n-1)} \mp \lambda_n Q_n \frac{\partial \log \rho_{nm}}{\partial x_m} \qquad \frac{\partial \varphi_n}{\partial y_n} = \frac{1}{2\pi} \sum_1^{(n-1)} \mp \gamma_n Q_n \frac{\partial \log \rho_{mn}}{\partial y_m}$$

wo

$$\int d\omega_n = Q_n$$

ist, demnach den Querschnitt des n'ten Gebietes der Verticalströmung bedeutet

Betrachten wir andererseits ein Integral von der Form

$$\int (\zeta_m - 2\lambda \sin \theta) \frac{(x_n - x_m)}{\rho_m} d\omega_m.$$

wo $x_m\ y_m$ einen Punkt im m'ten Wirbelgebiet, und $x_n\ y_n$ den Schwerpunkt des n'ten Wirbelgebietes bestimmen. Ist nun wieder ρ_m gegen dem Querschnitt des m'ten Gebietes unendlich gross, so ist dieses Integral ersetzbar durch

$$\frac{x_n - x_m}{\rho_{nm}} \int (\zeta_m - 2\lambda \sin \theta)\, d\omega_m = \frac{\partial \log \rho_{nm}}{\partial x_n} M_m.$$

Es geht demnach aus dieser Betrachtung hervor: Ist der Querschnitt eines jeden Wirbelgebietes unendlich klein gegen die gegenseitigen Abstände, so ist für das n'te Wirbelgebiet

$$\sum_1^{n-1} \frac{1}{2\pi} \iint (\zeta_m - 2\lambda \sin \theta)(\zeta'_m - 2\lambda \sin \theta) \frac{(x - x_m)}{\rho_m} d\omega_n d\omega'_m$$
$$= \frac{M_n}{2\pi} \sum_1^{(n-1)} M_m \frac{\partial \log \rho_{nm}}{\partial x_n}$$

$$\frac{1}{2\pi} \sum_1^{(n-1)} \iint (\zeta_n - 2\lambda \sin \theta)(\zeta'_m - 2\lambda \sin \theta) \frac{(y - y_m)}{\rho_m^2} d\omega_n d\omega'_m$$
$$= \frac{M_n}{2\pi} \sum_1^{n-1} M_m \frac{\partial \log \rho_{nm}}{\partial y_n}$$

$$\frac{1}{2\pi} \sum_1^{n-1} \mp \gamma_m \iint (\zeta_n - 2\lambda \sin \theta) \frac{x - x_m}{\rho_m^2} d\omega_n d\omega'_m = \frac{M_n}{2\pi} \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{\partial \log \rho_{nm}}{\partial x_n}$$

$$\frac{1}{2\pi} \sum_1^{n-1} \mp \gamma_m \iint (\zeta_n - 2\lambda \sin \theta) \frac{(y - y_m)}{\rho_m^2} d\omega_n d\omega'_m = \frac{M_n}{2\pi} \sum_1^{n-1} \mp \gamma_m Q_m \frac{\partial \log \rho_{nm}}{\partial y_n}$$

Mithin erhalten wir als Bewegungsgleichungen des Schwerpunktes des n'ten Wirbelgebietes

$$\frac{dx_n}{dt} = \frac{1}{2\pi} \sum_1^{n-1} M_n \frac{\partial \log \rho_{mn}}{\partial y_n} + \frac{1}{2\pi} \sum_1^{n-1} \mp \gamma_m Q_m \frac{\partial \log \rho_{nm}}{\partial x_n} \mp \frac{\gamma_n \alpha_n}{2\pi M_n}$$

$$\frac{dy_n}{dt} = -\frac{1}{2\pi} \sum_1^{(n-1)} M_m \frac{\partial \log \rho_{nm}}{\partial x_n} + \frac{1}{2\pi} \sum_1^{n-1} \mp \gamma_m Q_m \frac{\partial \log \rho_{nm}}{\partial x_n} \mp \frac{\gamma_n \beta_n}{2\pi M_n}$$

Zwischen den Constanten M_m, und $\gamma_m Q_m$ besteht nun eine Beziehung, welche uns in den Stand setzt, diese Gleichungen noch einfacher zu schreiben. Es sei n_n die Normale an dem Wirbelgebiete (n). Damit die Grenzbedingung

$$\frac{\partial W_i}{\partial n_n} = \frac{\partial W_a}{\partial n_n}$$

erfüllt sei, muss

$$\sum_1^n \int \frac{(\zeta_m - 2\lambda \sin \theta)}{\rho_m} \frac{\partial \rho_m}{\partial n_n} d\omega_m = K \sum_1^n \frac{\partial \varphi_m}{\partial n_n} = K \sum_1^n \int \frac{\mp \gamma_m}{\rho_m} \frac{\partial \rho_m}{\partial n_n} d\omega_n$$

da $W_a = K \varphi_a$ und

$$\frac{\partial \rho_m}{\partial n_n} = \frac{(x - x_m)}{\rho_m} \cos(n_n x) + \frac{(y - y_m)}{\rho_m} \cos(n_n y)$$

ist, so folgt durch Vergleich

$$\sum_1^n \int \frac{(\zeta_m - 2\lambda \sin \theta)}{\rho_m^2} (x - x_m)\, d\omega_m = K \sum_1^n \mp \gamma_m \int \frac{(x - x_m)}{\rho_m^2} d\omega_m$$

$$\sum_1^n \int \frac{(\zeta_m - 2\lambda \sin \theta)}{\rho_m^2} (y - y_m)\, d\omega_m = K \sum_1^n \mp \gamma_m \int \frac{(y - y_m)}{\rho_m^2} d\omega_m$$

Nun denken wir uns in dem n'ten Wirbelgebiete ζ_n so bestimmt, dass

$$\int \frac{(\zeta_n - 2\lambda \sin \theta)(x - x_n)}{\rho_m^2} d\omega_n = \mp K \gamma_n \int \frac{(x - x_n)}{\rho_n^2} d\omega_n$$

$$\int \frac{(\zeta_n - 2\lambda \sin \theta)(y - y_n)}{\rho_m^2} d\omega_n = \mp K \gamma_n \int \frac{(y - y_n)}{\rho_m^2} d\omega_n$$

wo die Integration rechter Hand über das n'te Gebiet der Verticalströmung auszudehnen ist, so dass

$$\sum_1^{(n-1)} \int \frac{(\zeta_m - 2\lambda \sin \theta)(x - x_n)}{\rho_m^2} d\omega_m = K \sum_1^{n-1} \mp \gamma_m \int \frac{(x - x_m)}{\rho_m^2} d\omega_m$$

$$\sum_4^{(n-1)} \int \frac{(\zeta_m - 2\lambda \sin \theta)(y - y_m)}{\rho_m^2} d\omega_m = K \sum_1^{(n-1)} \mp \gamma_m \int \frac{(y - y_m)}{\rho_m^2} d\omega_m$$

Da hierin x, y einem Punkt an der Grenze des n'ten Wirbelgebietes angehören, so können wir bei der Annahme, die wir gemacht haben, x, y überall mit x_n, y_n vertauschen so dass

$$\sum_1^{(n-1)} \frac{(x_n - x_m)}{\rho_{n\,m}^2} M_m = K \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{(x_n - x_m)}{\rho_{n\,m}^2}$$

$$\sum_1^{(n-1)} \frac{(y_n - y_m)}{\rho_{n\,m}^2} M_m = K \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{(y_n - y_m)}{\rho_{n\,m}^2}$$

woraus dann folgt,

$$M_m = \mp \gamma_m Q_m K \tag{92$_a$}$$

Die Bewegungsgleichungen des Schwerpunktes werden mit Rücksicht auf diese Beziehung

$$\frac{dx_n}{dt} = \frac{K}{2\pi} \sum_1^{(n-1)} \mp \gamma_n Q_m \frac{\partial \log \rho_{n\,m}}{\partial y_n} + \frac{1}{2\pi} \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{\partial \log \rho_{n\,m}}{\partial x_n} \mp \frac{\gamma_n \alpha_n}{2\pi M_n}$$

$$\frac{dx_n}{dt} = -\frac{K}{2\pi} \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{d \log \rho_{n\,m}}{\partial x_n} + \frac{1}{2\pi} \sum_1^{(n-1)} \mp \gamma_m Q_m \frac{\partial \log \rho_{n\,m}}{\partial y_n} \mp \frac{\gamma_n \beta_n}{2\pi M_n}$$

Diese Gleichungen lassen sich noch mehr vereinfachen. Wir nennen das Produkt $\gamma_m Q_m$ die Masse* des Wirbelgebietes m, indem wir eine negative und positive Masse unterscheiden, und setzen der Abkürzung halber $\mp \gamma_m Q_m = \mu_m$ so dass (92$_b$)

$$\left.\begin{aligned}\frac{dx_n}{dt} &= \frac{K}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{n\,m}}{\partial y_n} + \frac{1}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{n\,m}}{\partial x_n} \mp \frac{\gamma_n \alpha_n}{2\pi M_n}\\ \frac{dy_n}{dt} &= -\frac{K}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{n\,m}}{\partial x_n} + \frac{1}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{n\,m}}{\partial y_n} \mp \frac{\gamma_n \beta_n}{2\pi M_n}\end{aligned}\right\} \tag{93}$$

und ferner

$$\phi = \frac{K}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k \log (\rho_{m\,k})$$

$$\psi = \frac{1}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k \log (\rho_{m\,k})$$

wo die Summation nur auf einfache Combination von m und κ zu beziehen ist. Es ist dann

---

* Berechtigung zu dieser Benennung liegt in dem Umstande, dass, da γ die Geschwindigkeit der Luft in Längeneinheit über der Erdoberfläche ist, γQ die Luftmasse bedeutet, welche in der Zeiteinheit durch *diesen* Querschnitt strömt. Da aber des Querschnitt des anticyklonalen Wirbelgebietes möglicherweise nicht mit dem Querschnitt des Gebietes der Verticalströmung zusammenfallt., und Q den Querschnitt des Wirbelgebietes bedeutet, so trifft diese Deutung des Produktes γQ für positives γ nicht zu; es ist möglicherweise grösser als die Luftmasse, welche in der Zeiteinheit durch jenen Querschnitt des Gebietes der Verticalströmung strömt.

$$\frac{\partial \Phi}{\partial x_n} = \frac{K}{2\pi}\left(\mu_1 \mu_n \frac{\partial \log \rho_{1n}}{\partial x_n} + \mu_2 \mu_n \frac{\partial \log \rho_{2n}}{\partial x_n} + etc\right) = \frac{K \mu_n}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{nm}}{\partial x_n}$$

$$\frac{\partial \Phi}{\partial y_n} = \frac{K}{2\pi}\, \mu_n \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{mm}}{\partial x_n}$$

$$\frac{\partial \psi}{\partial x_n} = \frac{\mu_n}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{mn}}{\partial x_n}$$

$$\frac{\partial \psi}{\partial y_n} = \frac{\mu_n}{2\pi} \sum_1^{(n-1)} \mu_m \frac{\partial \log \rho_{mn}}{\partial y_n}$$

Die Einführung dieser Ausdrücke in (93) ergiebt:

$$\mu_n \frac{dx_n}{dy_n} = \frac{\partial \Phi}{\partial y_n} + \frac{\partial \psi}{\partial x_n} \mp \frac{\gamma_n \alpha_n \mu_n}{2\pi M_n}$$

$$\mu_m \frac{dy_n}{dt} = -\frac{\partial \Phi}{\partial x_n} + \frac{\partial \psi}{\partial y_n} \mp \frac{\gamma_n \alpha_n \mu_n}{2\pi M_n}$$

Die Bewegungsgleichungen die Schwerpunktes für jedes einzelne Wirbelgebiet erhält man hieraus, indem man $n = 1, 2, 3, \ldots\ldots$ setzt, so dass das System der $2n$. Differentialgleichungen

$$\mu_1 \frac{dx_1}{dt} = \frac{\partial \Phi}{\partial y_1} + \frac{\partial \psi}{\partial x_1} \mp \frac{\gamma_1 \alpha_1 \mu_1}{2\pi M_1} \qquad \mu_2 \frac{dx_2}{dt} = \frac{\partial \Phi}{\partial y_2} + \frac{\partial \psi}{\partial x_2} \mp \frac{\gamma_2 \alpha_2 \mu_2}{2\pi M_2} \qquad \text{etc}$$
$$(94)$$
$$\mu_1 \frac{dy_1}{dt} = -\frac{\partial \Phi}{\partial x_1} + \frac{\partial \psi}{\partial y_1} + \frac{\gamma_1 \beta_1 \mu_1}{2\pi M_1} \qquad \mu_2 \frac{dy_2}{dt} = -\frac{\partial \Phi}{dt_2} + \frac{\partial \psi}{\partial y_2} \mp \frac{\gamma_2 \beta_2 \mu_2}{2\pi M_2} \qquad \text{etc.}$$

zu integriren ist, um die Bewegung eines jeden Wirbelgebietes zu finden.

Wenn gleich nicht gut möglich ist, diese Differentialgleichungen ganz allgemein zu integriren, so lassen sich jedoch unschwer einige Beziehungen ableiten, die wichtig sind. Addirt man die erste und zweite Horizontalreihe, so kommt

$$\sum_1^n \mu_m \frac{dx_m}{dt} = \sum_1^n \frac{\partial \Phi}{\partial y_m} + \sum_1^n \frac{\partial \psi}{\partial x_n} + \sum_1^n \mp \frac{\gamma_m \alpha_m \mu_m}{2\pi M_m}$$

$$\sum_1^n \mu_m \frac{dy_m}{dt} = -\sum_1^n \frac{\partial \Phi}{\partial x_m} + \sum_1^n \frac{\partial \psi}{\partial y_m} + \sum_1^n \mp \frac{\gamma_m \beta_m \mu_m}{2\pi M_m}$$

Es ist nun, weil ρ_{mk} symmetrisch ist in Bezug auf x_m, x_k und y_m y_k

$$\frac{\partial \log \rho_{mk}}{\partial x_m} = - \frac{\partial \log \rho_{mk}}{\partial x_k}$$

$$\frac{\partial \log \rho_{mk}}{\partial y_m} = - \frac{\partial \log \rho_{mk}}{\partial y_k}$$

Da aber in $\sum\limits_1^n \frac{\partial \Phi}{\partial x_m}$, und $\sum\limits_1^n \frac{\partial \Phi}{\partial y_m}$, $\log \rho_{mk}$ einmal nach x_m, und y_m das andere Mal nach x_k und y_k differentirt ist, und die Coefficienten μ_m μ_k in Bezug auf die Indices symmetrisch sind, so treten in $\sum\limits_1^n \frac{\partial \Phi}{\partial x_m}$, und $\sum\limits_1^n \frac{\partial \Phi}{\partial y_m}$ je zwei Glieder auf, welche bei gleichem absolutem Werthe entgegengesetzte Vorzeichen besitzen. Mithin ist

$$\sum_1^n \frac{\partial \psi}{\partial x_m} = 0 \qquad \sum_1^n \frac{\partial \Phi}{\partial y_m} = 0$$

und aus demselben Grund

$$\sum_1^n \frac{\partial \psi}{\partial x_m} = 0 \qquad \sum_1^n \frac{\partial \psi}{\partial y_m} = 0$$

folglich

$$\sum_1^n \mu_m \frac{dx_m}{dt} = \sum_1^n \mp \frac{\gamma_m \alpha_m \mu_m}{2\pi M_m}$$

$$\sum_1^n \mu_m \frac{dy_m}{dt} = \sum_1^n \mp \frac{\gamma_m \beta_m \mu_m}{2\pi M_m}$$

Da aber μ_m von t unabhängig ist, so kommt durch Integration

$$\sum_1^n \mu_m x_m = \sum_1^n \mp \frac{\gamma_m \alpha_m \mu_m}{2\pi M_m}, t + Const. \qquad \sum_1^n \mu_m y_m = \sum_1^n \mp \frac{\gamma_m \beta_m \mu_m}{2\pi M_m}, t + Const. \tag{94$_a$}$$

Wir fassen einen Punkt ξ, η in's Auge, der, wie folgt, definirt ist

$$\xi \sum_1^n \mu_m = \sum_1^n \mu_m x_m \qquad \eta \sum_1^n \mu_m = \sum_1^n \mu_\mu x_m$$

also ein Punkt, der nichts anderes ist, als der gemeinsame Schwerpunkt aller Wirbelgebiete, und in der Endlichkeit liegt, wenn $\sum\limits_1^n \mu_m$ nicht verschwindet. Die Gleichungen (94$_a$) verwandeln sich sodann in

$$\xi = \frac{\sum_1^n \mp \left(\frac{\gamma_m \alpha_m \mu_m}{2\pi M_m}\right)}{\sum_1^n \mu_m} . t + Const. \qquad \eta = \frac{\sum_1^n \mp \left(\frac{\gamma_m \beta_m \mu_m}{2\pi M_m}\right)}{\sum_1^n \mu_m} . t + Const.$$

Wenn demnach α_m und β_m für jedes Gebiet nicht verschwinden oder wenigstens $\sum_1^n \mp \frac{\gamma_m \alpha_m \mu_m}{2\pi M_m}$, $\sum_1^n \mp \frac{\gamma_m \beta_m \mu_m}{2\pi M_m}$ nicht verschwinden, so schreitet der gemeinsame Schwerpunkt aller Wirbelgebiete geradlinig und mit constanter Geschwindigkeit fort. Was die Richtung dieser Bewegung anbetrifft, so hängt sie offenbar davon ab, ob die Summe $\sum \mu_m$ für die cyklonalen Bewegungsformen oder die anticyklonalen oder was, wie aus der Beziehung (92_b) hervorgeht, dasselbe ist, ob die Produktensumme $\sum \gamma_m Q_m$ für die aufsteigende Strömung, oder die niedersteigende Strömung überwiegt, vorausgesetzt, dass α_m und β_m für jedes Wirbelgebiet positiv ist, was wiederum eine bestimmte Vertheilung der Rotationsgeschwindigkeit in jedem Wirbelgebiete voraussetzt. Es ist daher nicht möglich ohne specialisirende Annahmen über die Dimension der Querschnitte der Wirbelgebiete, über die Geschwindigkeiten der Verticalströmungen, und vor allem über die Vertheilung der Rotationsgeschwindigkeit in jedem Wirbelgebiete, über die Richtung der fortschreitenden Bewegung des Schwerpunktes aller Wirbelgebiete etwas Bestimmtes auszusagen.

Zu einer anderen allgemeinen Beziehung gelangt man auf folgende Weise. Multiplizirt man die erste Horizontalreihe der Gleichungen (94) mit $\frac{dy_1}{dt}$, $\frac{dy_2}{dt}$ etc, und die zweite Horizontalreihe mit $\frac{dx_1}{dt}$, $\frac{dx_2}{dt}$ etc, so kommt durch Addition

$$\sum_1^n \mu_m \frac{dx_m}{dt}\frac{dy_m}{dt} = \sum_1^n \frac{\partial \Phi}{\partial y_m}\frac{dy_m}{dt} + \sum_1^n \frac{\partial \psi}{\partial x_m}\frac{dy_m}{dt} + \sum A_m \frac{dy_m}{dt}$$

$$\sum_1^n \mu_m \frac{dy_m}{dt}\frac{dx_m}{dt} = \sum_1^n \frac{\partial \Phi}{\partial x_m}\frac{dx_m}{dt} + \sum_1^n \frac{\partial \psi}{\partial y_m}\frac{dx_m}{dt} + \sum B_m \frac{dx_m}{dt}$$

wo zur Abkürzung

$$\mp \frac{\gamma_m \alpha_m \mu_m}{2\pi M_m} = A_m \qquad \mp \frac{\gamma_m \beta_m \mu_m}{2\pi M_m} = B_m$$

gesetzt worden ist.

Die Subtraction ergiebt hieraus

$$0 = \sum_1^n \left(\frac{\partial \Phi}{\partial x_m} \frac{dx_n}{dt} + \frac{\partial \Phi}{\partial y_m} \frac{dy_m}{dt} \right) + \sum_1^n \left(\frac{\partial \psi}{\partial x_m} \frac{dy_m}{dt} - \frac{\partial \psi}{\partial y_n} \frac{dx_m}{dt} \right) + \sum \left(A_m \frac{dy_m}{dt} - B_m \frac{dx_m}{dt} \right)$$

Da nun ψ die Bedingung

$$\frac{\partial^2 \psi}{\partial x_m^2} + \frac{\partial^2 \psi}{\partial y_m^2} = 0$$

erfüllt, so ist

$$\frac{\partial \psi}{\partial x_m} dy_m - \frac{\partial \psi}{\partial y_m} dx_n$$

ein totales Differential. Die Integration ergiebt daher

$$const = \Phi + \sum_1^n \int \left(\frac{\partial \psi}{\partial x_m} dy_m - \frac{\partial \psi}{\partial y_m} dx_m \right) + \sum_1^n \left(A_m y_m - B_m x_m \right) \qquad (95)$$

Multiplizirt man die erste Horizontalreihe mit $\frac{dx_1}{dt}, \frac{dx_2}{dt}$, etc, und die zweite Horizontalreihe mit $\frac{\partial y_1}{dt}, \frac{dy_2}{dt}$, etc, so entsteht durch Addition

$$\sum_1^n \mu_m \left[\left(\frac{dx_m}{dt} \right)^2 + \left(\frac{dy_m}{dt} \right)^2 \right] = \sum_1^n \left(\frac{\partial \psi}{\partial x_m} \frac{dx_m}{dt} + \frac{\partial \psi}{\partial y_m} \frac{dy_m}{dt} \right)$$
$$+ \sum_1^n \left(\frac{\partial \Phi}{\partial y_m} \frac{dx_m}{dt} - \frac{\partial \Phi}{\partial x_m} \frac{dy_m}{dt} \right) + \sum_1^n \left(A_m \frac{dx_m}{dt} + B_m \frac{dy_m}{dt} \right)$$

Da nun Φ wieder die Gleichung

$$\frac{\partial^2 \Phi}{\partial x_m^2} + \frac{\partial^2 \Phi}{\partial y_m^2} = 0$$

befriedigt, so ist

$$\left(\frac{\partial \Phi}{\partial y_m} dx_m - \frac{\partial \Phi}{\partial x_m} dy_m \right)$$

ein totales Differential. Setzt man dieses $= d\, U_m$, so ist

$$\sum_1^n \mu_m \left[\left(\frac{dx_m}{dt}\right)^2+\left(\frac{dy_m}{dt}\right)^2\right]=\frac{d\psi}{dt}+\sum_1^n\frac{dU_m}{dt}+\frac{d}{dt}\sum_1^n(A_m x_n+B_n y_n)$$

wo

$$U_m=\int\left(\frac{\partial\Phi}{\partial y_m}\,dx_m-\frac{\partial\Phi}{\partial x_m}\,dy_m\right)$$

ist.

Da nun aber

$$\Phi=K\psi$$

so folgt

$$U_m=K\int\left(\frac{\partial\psi}{\partial y_m}\,dx_m-\frac{\partial\psi}{\partial x_m}\,dy_m\right)$$

Weil in Folge der Gleichung (95)

$$\sum_1^n\int\left(\frac{\partial\psi}{\partial y_m}\,dx_m-\frac{\partial\psi}{\partial x_m}\,dy_m\right)=-\,const+\sum_1^n(A_m y_m-B_m x_m)+K\psi$$

ist, so ist

$$\sum_1^n U_m=-\,const.\,K+K\sum_1^n\Big((A_m y_m-B_m x_m)+K^2\psi$$

Mithin folgt

$$\sum_1^n\mu_m\left[\left(\frac{dx_n}{dt}\right)^2+\left(\frac{dy_m}{dt}\right)^2\right]=(1+K^2)\frac{d\psi}{dt}+\frac{d}{dt}\sum_1^n\Big[A_m(x_m+Ky_m)$$
$$+B_n(y_m-Kx_m)\Big]$$

Noch zwei andere Beziehungen lassen sich durch Einführung der Polarcoordinaten ableiten. Man setze

$$x_1=\rho_1\cos\lambda_1 \qquad x_2=\rho_2\cos\lambda_2\ \ldots\ldots\ldots$$

$$y_1=\rho_1\sin\lambda_2 \qquad y_2=\rho_2\sin\lambda_2\ \ldots\ldots\ldots$$

$$\rho_1^2=x_1^2+y_1^2 \qquad \rho_2^2=x_2^2+y_2^2\ \ldots\ldots\ldots$$

Es ist dann, wenn man dies in die Gleichungen (94) einführt

$$\mu_m\frac{d\rho_m}{dt}\sin\lambda_m+\mu_m\rho_m\cos\lambda_m\frac{d\lambda_m}{dt}=-\frac{\partial p}{\partial\rho_m}\cos\lambda_m$$
$$+\frac{\sin\lambda_m^1}{\rho_m}\frac{\partial p}{\partial\lambda_m}+\frac{\partial\psi}{\partial\rho_m}\sin\lambda_m+\frac{\partial\psi}{\partial\lambda_m}\frac{\cos\lambda_m}{\rho_m}+A_m$$

$$\mu_m \frac{d\rho_m}{dt} \cos \chi_m - \mu_m \rho_m \sin \chi_m \frac{d\chi_m}{dt} = \frac{\partial \Phi}{\partial \rho_m} \sin \chi_m$$
$$+ \frac{\cos \chi_m}{\rho_m} \frac{\partial \Phi}{\partial \chi_m} + \frac{\partial \psi}{\partial \rho_m} \cos \chi_m - \frac{\partial \psi}{\partial \chi_m} \frac{\sin \chi_m}{\rho_m} + B_m.$$

Hieraus findet man durch Auflösung dieser Gleichungen nach $\frac{d\rho_m}{dt}$ und $\frac{d\chi_m}{dt}$

$$\mu_m \frac{d\rho_m}{dt} = \frac{\partial \Phi}{\partial \chi_m} \frac{1}{\rho_m} + \frac{\partial \psi}{\partial \rho_m} + A_m \sin \chi_m + B_m \cos \chi_m$$

$$\mu_m \rho_m \frac{d\chi_m}{dt} = -\frac{\partial \Phi}{\partial \rho_m} + \frac{\partial \psi}{\partial \chi_m} \frac{1}{\rho_m} + A_m \cos \chi_m - B_m \sin \chi_m$$

oder indem man beide Seiten mit ρ_m multiplizirt

$$\mu_m \rho_m \frac{d\rho_m}{dt} = \frac{\partial \Phi}{\partial r_m} + \frac{\partial \psi}{\partial \log \rho_m} + \rho_m (A_m \sin \chi_m + B_m \cos \chi_m)$$

$$\mu_m \rho_m^2 \frac{d\chi_m}{dt} = -\frac{\partial \Phi}{\partial \log \rho_m} + \frac{\partial \psi}{\partial \chi_m} + \rho_m (A_m \cos \chi_m - B \sin \chi_m)$$

Mithin folgt

$$\sum_1^n \mu_m \rho_m \frac{d\rho_m}{dt} = \sum_1^n \frac{\partial \Phi}{\partial \chi_m} + \sum_1^n \frac{\partial \psi}{\partial \log \rho_m} + \sum_1^n \rho_m (A_m \sin \chi_m + B_m \cos \chi_m)$$

$$\sum \mu_m \rho_m^2 \frac{d\chi_m}{dt} = -\sum_1^n \frac{\partial \Phi}{\partial \log \rho_m} + \sum_1^n \frac{\partial \psi}{\partial \chi_m} + \sum_1^n \rho_m (A_m \cos \chi_m - B_m \sin \chi_m)$$

Weil, wie leicht zu beweisen,

$$\sum_1^n \frac{\partial \Phi}{\partial \chi_m} = \sum_1^n \frac{\partial \psi}{\partial \chi_m} = 0$$

und

$$\sum_1^n \frac{\partial \Phi}{\partial \log \rho_m} = \frac{K}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k \qquad \sum_1^n \frac{\partial \psi}{\partial \log \rho_m} = \frac{1}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k$$

so erhalten wir die Beziehungen

$$\sum_1^n \mu_m \rho_m \frac{d\rho_m}{dt} = \frac{1}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k + \sum_1^n \rho_m (A_m \sin \chi_m + B_m \cos \chi_m) \tag{96}$$

$$\sum \mu_m \rho_m^2 \frac{d\chi_m}{dt} = -\frac{K}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_k + \sum_1^n \rho_m (A_m \cos \chi_m - B_m \sin \chi_m) \tag{97}$$

Wesentlich einfacher werden die abgeleiteten Beziehungen, wenn die Wirbelgebiete so symmetrisch gebildet sind und ζ_m selbst in dem einzelnen Wirbelgebiete so symmetrisch vertheilt ist, dass A_m und B_m für jedes Gebiet verschwindet, also dass der gemeinsame Schwerpunkt aller vorhandenen Wirbelgebiete sich nicht bewegt, *d. i.*

$$\sum_1^n \mu_m x_m = const \qquad \sum_1^n \mu_m y_m = const \tag{98}$$

wird. Es werden dann unter denselben Annahmen

$$const = \Phi + \sum_1^n \int \left(\frac{\partial \psi}{\partial x_m} dy_m - \frac{\partial \psi}{\partial y_m} dx_m \right) \tag{99}$$

$$\sum_1^n \mu_m \left[\left(\frac{dx_m}{dt} \right)^2 + \left(\frac{dy_m}{dt} \right)^2 \right] = (1+K^2) \frac{d\psi}{dt}$$

Dann wird die Gleichung (96) integrabel, so dass

$$\sum_1^n \mu_m \rho_m^2 = \frac{1}{\pi} \sum_1^n \sum_1^n \mu_m \mu_{n'} \, t + const \tag{100}$$

und die Gleichung (97) kann dann auch so geschrieben werden

$$\sum_1^n \mu_m \int \rho_m^2 d\chi = - \frac{K}{2\pi} \sum_1^n \sum_1^n \mu_m \mu_{n'} \, t + const. \tag{101}$$

eine Gleichung, welche nichts anderes ausspricht, als den Flächensatz, dass die algebraische Summe der von allen Leitstrahlen, welche je zwei Schwerpunkte mit einander verbinden, in der Zeiteinheit beschriebenen Flächen multiplizirt mit der Massen der Wirbelgebiete von der Zeit nicht abhängt.

Wenn nun in der Atmosphäre nicht mehr als drei Wirbelgebiete vorhanden sind, so lässt sich die Aufgabe dem Princip nach auf Quadraturen zurückführen, wie in dem Fall, wo keine Verticalströmung vorhanden ist, und auf die Wirbelbewegung der Flüssigkeit die Erdrotation nicht ablenkend wirkt, weil die Beziehungen (98)—(101) schon vier Integrale der Bewegungsgleichungen darstellen. Allein; da die Beziehung (99) nicht algebraisch und die Zurück-

führung der Aufgabe auf Quadraturen daher practisch nicht möglich ist, wollen wir uns lediglich auf eingehendere Betrachtung des Falls beschränken, wo nur zwei cylindrische Wirbelgebiete von sonst beliebigem Querschnitte vorhanden sind.

Wir nehmen zunächst an, dass die Summe $\mu_1 + \mu_2$ nicht verschwinde. Dann ergiebt sich aus der Gleichung (98) unmittelbar, als Coordinaten des gemeinsamen Schwerpunktes der beiden Wirbelgebiete

$$\xi = \frac{\mu_1 x_1 + \mu_2 x_2}{\mu_1 + \mu_2} \qquad \eta = \frac{\mu_1 y_1 + \mu_2 y_2}{\mu_1 + \mu_2} \tag{102}$$

ξ und η ändert sich nicht mit der Zeit.

Wir führen die Polarcoordinaten ein, so dass

$$x_1 = \rho_1 \cos \chi_1 \qquad y_1 = \rho_1 \sin \chi_1$$
$$x_2 = \rho_2 \cos \chi_2 \qquad y_2 = \rho_2 \sin \chi_2$$

so verwandelt sich (102) in

$$\xi \left(1 + \frac{\mu_2}{\mu_1}\right) = \rho_1 \cos \chi_1 + \frac{\mu_2}{\mu_1} \rho_2 \cos \chi_2$$
$$\eta \left(1 + \frac{\mu_2}{\mu_1}\right) = \rho_1 \sin \chi_1 + \frac{\mu_2}{\mu_1} \rho_2 \sin \chi_2$$

Bisher war der Anfangspunkt der Coordinatenaxe willkürlich. Verlegt man denselben jetzt in den Schwerpunkt der beiden Wirbelgebiete, so ist, da ξ und η endlich ist

$$\rho_1 \cos \chi_1 + \frac{\mu_2}{\mu_1} \rho_2 \cos \chi_2 = 0$$
$$\rho_1 \sin \chi_1 + \frac{\mu_2}{\mu_1} \rho_2 \sin \chi_2 = 0$$

Diese beiden Gleichungen sind nur dann gleichzeitig zu befriedigen, wenn

$$\chi_1 = \chi_2 \tag{103}$$

$$\rho_1 + \frac{\mu_2}{\mu_2} \rho_2 = 0 \qquad d.\,i.\ \rho_1 = -\frac{\mu_2}{\mu_1} \rho_2 \tag{104}$$

Nun ist nach (100)

$$\rho_1^2 + \frac{\mu_2}{\mu_1}\rho_2^2 = \frac{1}{\pi}\mu_2 t + Const.$$

Aus dieser und der Gleichung (104) ergeben sich

$$\rho_1^2 = \sqrt{\frac{\mu_2^2}{\pi(\mu_1+\mu_2)}\cdot t + Const.} = \sqrt{\frac{\mu_2^2}{\pi(\mu_1+\mu_2)}\cdot t + \rho_{01}^2} \qquad (105)$$

$$\rho_2 = -\sqrt{\frac{\mu_1^2}{\pi(\mu_1+\mu_2)}\cdot t + Const.} = -\sqrt{\frac{\mu_1^2}{\pi(\mu_1+\mu_2)}\cdot t + \rho_{02}^2} \qquad (106)$$

wo ρ_{01} ρ_{02} die Entfernungen der beiden Wirbelgebiete von dem gemeinsamen Schwerpunkte zur Zeit $t=0$ bedeuten. Die beiden Wirbelgebiete müssen sich demnach von gemeinsamen Schwerpunkt entweder entfernen, oder näheren, je nachdem $(\mu_1+\mu_2)$ positiv oder negativ ist.

Mit Rücksicht auf (104) folgt aus (101)

$$\mu_1\int\rho_1^2\, d\chi_1 + \frac{\mu_2\mu_1^2}{\mu_2^2}\int\rho_1^2\, d\chi_1 = -\frac{K}{2\pi}\mu_1\mu_2 t + const.$$

d. i.

$$\int\rho_1^2\, d\chi_1 = -\frac{K\mu_2^2}{2\pi(\mu_1+\mu_2)}\cdot t + const.$$

Differentirt man dies nach χ_1 und eliminirt ρ_1 aus (105), so folgt

$$-\frac{K\mu_2^2}{2\pi(\mu_1+\mu_2)}\frac{dt}{d\chi_1} = \left(\frac{\mu_2^2}{\pi(\mu_1+\mu_2)}t + const.\right)$$

Hieraus findet man durch Integration

$$\chi_1 = Const. - \frac{K}{2}\cdot log\left(\frac{\mu_2^2}{\pi(\mu_1+\mu_2)}\cdot t + \rho_{01}^2\right)$$

oder indem man die willkürliche Constante ändert, erhält man

$$\chi_1 = Const. - \frac{K}{2} log\left[\frac{1}{(\mu_1+\mu_2)\pi}\left(\frac{\mu_2}{\rho_0}\right)^2 t + 1\right]$$

Setzt man ferner in (101) $\rho_1 = -\frac{\mu_2}{\mu^1}\rho_2$ ein, so folgt wieder

$$\chi_2 = const - \frac{K}{2} log\left[\frac{1}{(\mu_1+\mu_2)\pi}\left(\frac{\mu_1}{\rho_{02}}\right)^2 t + 1\right]$$

Da nun $\rho_1^2 = \left(\frac{\mu_2}{\mu_1}\right)^2\rho_2^2$, mithin auch $\left(\frac{\rho_{10}}{\mu_2}\right)^2 = \left(\frac{\rho_{10}}{\mu_1}\right)^2$ ist, so folgt in der That

$$\chi_1 = \chi_2$$

d. h. die Gerade, welche durch den gemeinsamen Schwerpunkt der beiden Wirbelgebiete hindurch gehend ihre Schwerpunkte mit einander verbindet, rotirt um den gemeinsamen Schwerpunkt mit der Winkelgeschwindigkeit

$$\frac{d\chi_1}{dt} = -\frac{K}{2\left[t + \pi \frac{\rho_{01}^2}{\mu_2^2}(\mu_1 + \mu_2)\right]} \tag{107}$$

Ist demnach $(\mu_1 + \mu_2)$ positiv, so nimmt die Rotationsgeschwindigkeit allmälig ab, — ist $(\mu_1 + \mu_2)$ dagegen negativ, so wächst sie nach und nach und wird unendlich gross nach dem Verfluss der Zeit $\pi\left(\frac{\rho_{01}}{\mu_2}\right)^2(\mu_1 + \mu_2)$, wo dann so wohl ρ_1 als ρ_2 unendlich klein ist. Da χ in dem Sinn wachsend genommen werden muss, in welchem eine Gerade, um den rechten Winkel gedreht werden muss, um aus einer Lage parallel der positiven x Achse in die Lage parallel der positiven y Achse zu gelangen, *d. h.* in dem Sinne der Erdrotation, da ja die positiv x Achse gegen Süden, und die positive y Achse gegen Osten gekehrt ist, so drehen sich beide Wirbelgebiete auf der nördlichen Hemisphäre *mit* der Sonne, da $\frac{d\chi_1}{dt}$ negativ ist, wenn $(\mu_1 + \mu_2)$ positiv ist und wenn $(\mu_1 + \mu_2)$ hingegen negativ ist, *gegen* die Sonne, da $\frac{d\chi_1}{dt}$ dann positiv ist, während sie sich gleichzeitig entfernen respect. näheren, mit der fortwährend abnehmenden, respect. zunehmenden Geschwindigkeit

$$\frac{d\rho_1}{dt} = \frac{1}{2\pi}\frac{\mu_2^2}{(\mu_1 + \mu_2)}\frac{1}{\sqrt{\frac{\mu_2^2}{\pi(\mu_1 + \mu_2)}t + \rho_{01}^2}}$$

$$\frac{d\rho_2}{dt} = -\frac{1}{2\pi}\frac{\mu_1^2}{(\mu_1 + \mu_2)}\frac{1}{\sqrt{\frac{\mu_1^2}{\pi(\mu_1 + \mu_2)}t + \rho_{02}^2}} \tag{107$_b$}$$

je nachdem $(\mu_1+\mu_2)$ positiv oder negativ ist. Auf der südlichen Hemisphäre ändert $\frac{d\chi_1}{dt}$ ihr Vorzeichen; sie wird positiv, da $K=\frac{2\lambda \sin\theta}{\kappa}$ für $-\theta$ negativ wird, wo es auf der nördlichen Hemisphäre negativ ist, und umgekehrt. Es drehen sich daher auf der südlichen Hemisphäre die beiden Wirbelgebiete um den gemeinsamen Schwerpunkt *gegen* die Sonne, wenn $(\mu_1+\mu_2)$ positiv und wenn aber $(\mu_1+\mu_2)$ negativ ist, *mit* der Sonne. Die Rotationsrichtung der beiden Wirbelgebiete wird also die entgegengesetzte wie auf der nördlichen Hemisphäre, während die Richtung der radialen Bewegung dieselbe bleibt. Da, wenn die beiden Wirbelgebiete von vertical aufsteigender Strömung gebildet, und daher von cyklonalen Bewegungsformen erfüllt sind, $\mu_1+\mu_2$ negativ zu setzen ist, und da ferner diese Summe positiv ist, wenn die beiden Wirbelgebiete von vertical niedersteigender Strömung gebildet sind, und die Luftbewegung in ihnen daher im anticyklonalen Sinne vor sich geht, so gelangen wir zum folgenden Satze. *Bilden sich irgendwo in der Erdatmosphäre zwei Cyklonen, so nähren sie sich gegenseitig, indem sie im cyklonalen Sinne um einen bestimmten ruhenden Punkt rotiren, während zwei Anticyklonen sich hingegen von einander entfernen, indem sie im anticyklonalen Sinne um einen bestimmten Punkt rotiren.*

Die Taf. XXVI mag den Verlauf der Bewegungsbahnen der beiden Wirbelgebiete für nördliche Hemisphäre veranschaulischen und zwar in dem speciellen Fall

$$\pm\mu_1=\pm\mu_2$$

$$arct\, K=85^\circ$$

Die Bahncurven sind logarithmische Spiralen. Denn es ist

$$\mu_1\left(1+\frac{\mu_1}{\mu_2}\right)\int\rho_1^2\, d\chi_1=-\frac{K}{2\pi}\mu_1\mu_2 t$$

Da aus (105) folgt

$$\mu_1\left(1+\frac{\mu_1}{\mu_2}\right)(\rho_1^2-\rho_{01}^2)=\frac{\mu_1\mu_2 t}{\pi}$$

so wird die obenstehende Gleichung

$$\int \rho_1^2\, d\,\chi_1 = -\frac{K}{2}(\rho_1^2-\rho_{01}^2)$$

Die Differentiation dieses ergiebt

$$\rho_1^2 = -K\,\rho_1 \frac{d\rho_1}{d\chi_1}$$

Hieraus folgt durch Integration

$$\rho_1 = C_1\, e^{-\frac{1}{K}\chi_1}$$

Auf ganz dieselbe Weise findet man

$$\rho_2 = C_2\, e^{-\frac{1}{K}\chi_1}$$

wobei es noch bemerkt werden mag, dass χ_1 bei anticyklonalen Bewegungsformen negativ gesetzt werden muss.

Wir haben bisher vorausgesetzt, dass μ_1, μ_2 einerlei Vorzeichen besitzen. Haben diese Grössen verschiedene Vorzeichen, so dass das eine Wirbelgebiet cyklonal, und das andere anticyklonal ist, so tritt entweder eine cyklonale, oder anticyklonale Drehung der beiden Wirbelgebiete um den gemeinsamen Schwerpunkt ein, je nach dem Sinne, in welchem die eine von den beiden Grössen μ_1, μ_2 die andere überwiegt. Der Schwerpunkt der beiden Wirbelgebiete liegt dabei auf der Verlängerung der Gerade, welche die Schwerpunkte der beiden Wirbelgebiete mit einander verbindet. Indem die beiden Wirbelgebiete sich allmälig näheren oder entfernen, beschreiben sie um diesen Punkt Spiralbahnen, welche entweder cyklonal oder anticyklonal gewunden ist, je nachdem die Masse des cyklonalen Wirbelgebietes diejenige des anticyklonalen Wirbelgebietes überwiegt, oder das Umgekehrte stattfindet.

Haben die Produkte μ_1 und μ_2 gleichen aber entgegengesetzten Werth, so dass $\mu_1 + \mu_2$ verschwindet, was auch geschehen kann, so rückt der gemeinsame Schwerpunkt der beiden Wirbelgebiete in die Unendlichkeit. So wohl ρ_1 als ρ_2 wird dann unendlich gross, mithin auch ρ_{01}, ρ_{02}.

Um auch diesen besonderen Fall zu erledigen, bemerken wir, dass, wenn wir uns die Grösse $(\mu_1 + \mu_2)$ zunächst unendlich klein denken, ξ, η, mithin auch ρ_1, ρ_2, vermöge der Gleichungen (102) unendlich gross von der Ordnung $\frac{1}{(\mu_1 + \mu_2)}$ werden, wenn $(x_1 - x_2)$ und $(y_1 - y_2)$ endlich ist, was der Fall ist, wenn der Abstand der beiden Wirbelgebiete ein endlicher ist. Wir denken uns nun den Anfangspunkt der Coordinaten in die Unendlichkeit verrückt, und bilden die Differenz $\rho_1 - \rho_2$ aus (105) (106). Es ist, da vermöge (104) ρ_1 dasselbe Vorzeichen haben muss wie ρ_2,

$$\rho_1 - \rho_2 = \rho_{01}\sqrt{\frac{\mu_2^2}{\pi(\mu_1 + \mu_2)\rho_{01}^2} \cdot t + 1\Big)} - \rho_{02}\sqrt{\frac{\mu_1^2}{\pi(\mu_1 + \mu_2)\rho_{01}^2} \cdot t + 1\Big)}$$

Dies ist endlich und zwar gleich dem ursprünglichen Abstand der Schwerpunkte der beiden Wirbelgebiete.— Denn; da $(\mu_1 + \mu_2)\,\rho_{01}^2$ $(\mu_1 + \mu_2)\,\rho_{02}^2$ der obigen Bemerkung zu Folge unendlich gross ist, wie $\frac{1}{(\mu_1 + \mu_2)}$, so wird bei verschwindendem $(\mu_1 + \mu_2)$

$$\rho_1 - \rho_2 = \rho_{01} - \rho_{02} \tag{107$_a$}$$

wie es oben behauptet wurde.

Der Abstand der Schwerpunkte der beiden Wirbelgebiete, welchen wir mit r bezeichnen wollen, ändert sich also nicht mit der Zeit. Die Winkelgeschwindigkeit, mit welcher sich die beiden Wirbelgebiete um den unendlich fernen Schwerpunkt dreht, ist dabei unendlich klein; die Geschwindigkeit aber, mit welcher die beiden Wirbelgebiete bei unverändertem gegenseitigem Abstande in der Atmo-

sphäre fortschreiten, ist endlich. Es ist nämlich, wie die Gleichung (107) auch so geschrieben werden kann

$$\frac{d\chi_1}{dt} = -\frac{K\mu_2^2}{2\pi\rho_{01}^2(\mu_1+\mu_2)} \frac{1}{\left(1 + \frac{\mu_2^2}{\pi\rho_{01}^2(\mu_1+\mu_2)}\cdot t\right)}$$

Die Grösse $\frac{\mu_2^2}{\pi\rho_{01}^2(\mu_1+\mu_2)}$ ist nun unendlich klein wie $(\mu_1 + \mu_2)$. Es kann daher bei verschwindendem $(\mu_1 + \mu_2)$ gesetzt werden

$$\frac{1}{\left(1 + \frac{\mu_2^2}{\pi\rho_{01}^2(\mu+\mu_2)}\cdot t\right)} = 1 - \frac{\mu_2^2}{\pi\rho_{01}^2(\mu_1+\mu_2)}\cdot t$$

so dass

$$\frac{d\chi_1}{dt} = -\frac{K\mu_2^2}{2\pi\rho_{01}^2(\mu+\mu_2)} + \frac{K}{2\pi^2}\frac{\mu_2^4}{\rho_{01}^4(\mu_1+\mu_2)^2}\cdot t$$

oder mit Vernachlässigung unendlich kleiner Grösse höherer Ordnung

$$\frac{d\chi_1}{dt} = -\frac{K\mu_2^2}{2\pi\rho_{01}^2(\mu_1+\mu_2)}$$

woraus dann als Fortschreitungsgeschwindigkeit folgt

$$\rho_{01}\frac{d\chi_1}{dt} = -\frac{K\mu_2^2}{2\pi\rho_{01}(\mu_1+\mu_2)}$$

Auf ganz dieselbe Weise findet man

$$\rho_{02}\frac{d\chi_2}{dt} = -\frac{K\mu_1^2}{2\pi\rho_{02}(\mu_1+\mu_2)}$$

Die Drehungsgeschwindigkeit ist also constant, und endlich da $\lim[\rho_{01}(\mu_1+\mu_2)]$, $\lim[\rho_{02}(\mu_1+\mu_2)]$ endlich und constant sind. Wir setzen

$$lim[\rho_{01}(\mu_1+\mu_2)] = lim[\rho_{02}(\mu_1+\mu_2)] = C.$$

Dass diese beiden Grössen gegen einen und denselben Grenzwerth convergiren, ersicht man daraus, dass die Einführung der Beziehung

$\rho_{01} = r + \rho_{02}$, in $\lim[\rho_{01}(\mu_1+\mu_2)]$ sofort $\lim[\rho_{02}(\mu_1+\mu_2)]$ zur Folge hat, da $\lim r(\mu_1+\mu_2) = 0$ ist. Wir erhalten somit, da $\mu_1 = -\mu_2$ ist

$$\rho_{01}\frac{dX_1}{dt} = \rho_{02}\frac{dX_2}{dt} = -\frac{K\mu_1^2}{2\pi C},$$

so dass

$$\int(\rho_{02}+\rho_{01})\frac{dX_1}{dt}\,dt = -\frac{K\mu_1^2}{\pi C}\cdot t. \tag{108}$$

Die Gleichungen (107_b) können für unseren Fall auch so geschrieben werden

$$\frac{d\rho_1}{dt} = \frac{1}{2\pi}\frac{\mu_2^2}{(\mu_1+\mu_2)\rho_{01}}\left[1-\frac{\mu_2^2 t}{2\pi(\mu_1+\mu_2)\rho_{01}^3}\right]$$

$$\frac{d\rho_2}{dt} = \frac{1}{2\pi}\frac{\mu_1^2}{(\mu_1+\mu_2)\rho_{02}}\left[1-\frac{\mu_1^2 t}{2\pi(\mu_1+\mu_2)\rho_{02}^3}\right]$$

oder mit Vernachlässigung von unendlich kleiner Grösse höherer Ordnung

$$\frac{d\rho_1}{dt} = \frac{d\rho_2}{dt} = \frac{\mu_1^2}{2\pi C}.$$

Die beiden Wirbelgebiete können sich demnach mit constanter Geschwindigkeit parallel der Gerade bewegen, welche ihre Schwerpunkte mit einander verbindet, ohne ihren Abstand von einander zu ändern.

Es ist leicht C zu bestimmen. Die Gleichung (101) verwandelt sich für unseren besonderen Fall

$$\mu_1\int(\rho_1^2-\rho_2^2)\frac{dX_1}{dt}\,dt = +\frac{K\mu_1^2}{2\pi}\cdot t. \tag{109}$$

Es ist nach (105) (106)

$$\rho_1^2 - \rho_2^2 = \rho_{01}^2 - \rho_{02}^2 = (\rho_{01}-\rho_{02})(\rho_{01}+\rho_{02})$$

oder mit Rücksicht auf (107_a)

$$\rho_1^2 - \rho_2^2 = r(\rho_{01}+\rho_{02})$$

so folgt

$$\mu_1 r \int (\rho_{01} + \rho_{02}) \frac{d\chi_1}{dt} dt = \frac{K\mu_1^2}{2\pi} t.$$

Durch Vergleich dieses mit (109) ergiebt sich, μ_1 als positiv vorausgesetzt

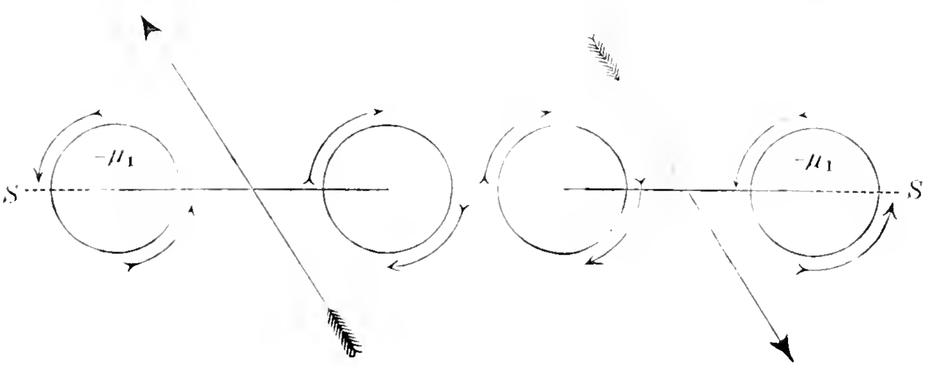
$$C = -2 r \mu_1$$

Wir erhalten somit als Componenten der Geschwindigkeit

$$\rho_{01} \frac{d\chi_1}{dt} = \rho_{02} \frac{d\chi_2}{dt} = + \frac{K\mu_1}{4\pi r}$$

$$\frac{d\rho_1}{dt} = \frac{d\rho_2}{dt} = - \frac{\mu_1}{4\pi r}$$

Die beiden Wirbelgebiete bewegen sich demnach geradlinig und zwar in einer Richtung, welche mit der Verbindungsgerade ihrer Schwerpunkte einen Winkel einschliesst, dessen trigonometrische Tangente $= K$ ist und zwar im cyklonalen Sinne abgelenkt von der zur Verbindungsgerade der beiden Wirbelgebiete senkrechten Richtung, da $\rho_{10} \frac{d\chi_1}{dt}$ für die nördliche Hemisphäre positiv, und für die südliche dagegen negativ ist. Die beigefügte Figur veranschaulicht die Fortschreitungsrichtung der Wirbelgebiete in dem in Rede stehenden Fall für die nördliche Hemisphäre, wo S die Richtung bedeutet, in der der gemeinsame Schwerpunkt in unendlicher Ferne liegt.



Es verdient ferner der Fall eine besondere Betrachtung, wo die eine von den beiden Grössen μ_1, μ_2 gegen die andere unendlich gross wird; sei es nun, weil die Geschwindigkeit der Verticalströmung in dem einen Gebiete gegen diejenige in dem anderen Gebiete unendlich gross ist, oder, weil der Querschnitt des einen gegen denjenigen des anderen Gebietes unendlich gross ist; ein Fall, der uns in den Stand setzt, ungefähr zu beurtheilen, welchen Einfluss ein Wirbelgebiet von sehr grosser Dimension auf ein in seiner Nähe befindliches Wirbelgebiet von geringerer Dimension ausübt.

Wir nehmen an; μ_2 sei unendlich gross gegen μ_1, so dass $\frac{\mu_2}{\mu_1}$ unendlich klein ist. Die Gleichung (102) wird unter diesem Umstande

$$\xi = x_2 \qquad \eta = y_2$$

Das Wirbelgebiet 2 bewegt sich gar nicht. Verlegt man den Coordinatenanfang in diesen ruhenden Punkt in dem Wirbelgebiet 2 so verschwindet ρ_2. Die Gleichungen (100) (101) reduciren sich auf

$$\mu_1 \rho_1^2 = \frac{1}{\pi} \mu_1 \mu_2 t + Const.$$

$$\mu_1 \int \rho_1^2 \, d\chi_1 = -\frac{K}{2\pi} \mu_1 \mu_2 t + Const.$$

Nennt man den Werth von ρ_1 für $t = 0$ wieder ρ_{01}, so folgt aus der ersten Gleichung sofort

$$\rho_1 = \sqrt{\frac{\mu_2 t}{\pi} + \rho_{01}^2}$$

und aus der zweiten mit Rücksicht auf dieses

$$\int \left(\frac{\mu_2 t}{\pi} + \rho_{01}^2 \right) d\chi_1 = -\frac{K}{2\pi} \mu_2 t + Const.$$

Differentirt man dieses nach χ_1, so kommt.

$$\frac{d\chi_1}{dt} = -\frac{K \mu_2}{2\pi \left(\frac{\mu_2 t}{\pi} + \rho_{01}^2 \right)}$$

Hieraus folgt durch Integration

$$\chi_1 = const - \frac{K}{2} log\left(\rho_{01}^2 + \frac{\mu_2 t}{\pi}\right)$$

Bei der Bewegung des Wirbelgebietes 1 in dem vorliegenden Fall kommt also gar nichts darauf an, ob dasselbe cyklonalen, oder anticyklonalen Wirbel enthält, sie wird lediglich bestimmt durch den Rotationscharakter des dominirenden Wirbelgebietes. Ist dieses cyklonal, so nährt sich das Wirbelgebiet 1 dem Wirbelgebiet 2 mit stetig wachsender Geschwindigkeit, indem es gleichzeitig dasselbe mit stetig wachsender Geschwindigkeit umkreist. Ist hingegen das Wirbelgebiet 2 anticyklonal, so entfernt sich das Wirbelgebiet 1 von demselben mit stetig abnehmender Geschwindigkeit, es gleichzeitig mit stetig abnehmender Geschwindigkeit umkreisend.

Die Bahncurve, welche das Wirbelgebiet 1 um das Wirbelgebiet 2 beschreibt, findet man leicht. Es ist nämlich

$$\rho_1^2 - \rho_{01}^2 = \frac{\mu_2 t}{\pi}$$

$$\int \rho_1^2 \, d\chi_1 = - \frac{K \mu_2 t}{2\pi}$$

Hieraus folgt durch Elimination von t

$$\int \rho_1^2 \, d\chi_1 = - \frac{K}{2}(\rho_1^2 - \rho_{01}^2)$$

Die Differentiation ergiebt

$$\rho_1 = - K \frac{d\rho_1}{d\chi_1}$$

wobei χ negativ in Rechnung gebracht werden muss, wenn μ_2 positiv ist. Das Integral hiervon ist

$$\rho_1 = C e^{-\frac{1}{K}\chi_1}$$

Das Wirbelgebiet 1 beschreibt also eine logarithmische Spirale um das superpondirende Wirbelgebiet, welche entweder nach Innen oder nach Aussen gewunden ist, je nachdem das unbewegliche Wirbelgebiet cyklonale oder anticyklonale Bewegungsformen besitzt.

§ XI. *Veränderung der Windrichtung, der Windstärke und des Luftdrucks, für einen gegebenen äusseren Punkt bei zwiefacher Wirbelbildung.*

Es lässt nun somit im Allgemeinen überschauen, wie die Isodynamen also auch Isobaren mit der Zeit von Ort zu Ort fortrücken, indem sie gleichzeitig ihre Gestalten ändern, wenn zwei Wirbelgebiete in der Atmosphäre sich gebildet haben.

Wenn beide Wirbelgebiete cyklonale Bewegungsformen haben, so dass die Linie, welche den Centraltheil der Wirbelgebiete mit einander verbindet, sich im cyklonalen Sinne um einen unbeweglich zwischen den beiden Wirbelgebieten liegenden Punkt dreht und gleichzeitig sich zusammenzieht, so muss sich das Curvensystem der Isodynamen, [welches in diesem Fall erstens aus einer Schaar jedes Wirbelgebiet umschliessender Curven, zweitens aus einer lemniskatenähnlichen Curve, und drittens wieder aus einer Schaar geschlossener Curven bestehen, welche beide Wirbelgebiete einschliessen] mit der Zeit so ändern, dass die Linie, welche die Punkte der Druckminima mit einander verbindet, sich allmälig um einen festen Punkt im cyklonalen Sinne dreht, während sie sich gleichzeitig zusammen zieht, so lange, bis das Curvensystem sich in ein einziges System concentrischer Kreise verwandelt. Wenn die Bewegungsformen in beiden Wirbelgebieten anticyklonal sind, also dass die Verbindungslinie der Centraltheile der Wirbelgebiete um einem gewissen festen Punkt dreht, indem sie sich fortwahrend ausdehnt, so wandern die Isodynamen entlang der Erdoberfläche im anticyklonalen Sinne um

einen gewissen festen Punkt, welcher auf der Verbindungslinie zwischen den beiden Wirbelgebieten liegt, und breiten sich immer weiter aus, so lange, bis sie sich in zwei Systeme geschlossener Curven aufgelöst haben.

Haben die beiden Wirbelgebiete gleiche aber entgengesetzte Massen, so wandern die Isodynamen entlang der Erdoberfläche geradelinig mit constanter Geschwindigkeit im Sinne der cyklonalen Bewegung, ohne ihre Gestalten zu ändern, da die Verbindungslinie der Centraltheile der beiden Wirbelgebiete sich mit der Zeit nicht ändert. Wenn die beiden Gebiete hingegen entgegengesetzte, aber ungleiche Massen, haben, so dass die Gerade, welche die Centraltheile der beiden Wirbelgebiete mit einander verbindet, indem sie in *dem* Sinne, wie die eine Masse, die andere überwiegt, um einen auf ihrer Verlängerung liegenden Punkt umkreist, mit der Zeit sich ausdehnt oder zusammenzieht, so breiten sich die Isodynamen in die Unendlichkeit aus, oder ziehen sich allmälig zusammen in ein System geschlossener Curven, indem das Curvensystem um einen auf der Verbindungslinie der Centraltheile der Wirbelgebiete liegenden Punkt rotirt im anticyklonalen oder cyklonalen Sinne, je nachdem die Masse des anticyklonalen Gebiete diejenige des cyklonalen Gebietes überwiegt, oder umgekehrt.

Ist schliesslich die Masse eines Wirbelgebietes unendlich gross gegen diejenige des anderen Wirbelgebietes, so dass das eine Wirbelgebiet sich gar nicht bewegt, so drehen sich die Isodynamen allmälig um das unbewegliche Wirbelgebiete im cyklonalen oder anticyklonalen Sinne, während sie sich nach und nach in ein System geschlossener Curven zusammenziehen, oder sich in die Unendlichkeit ausbreiten, je nachdem das superpondirende Wirbelgebiet cyklonal oder anticyklonal ist.

Dem entsprechend änderen die Windbahnen ihre Lage gegen die Coordinatenaxen, wie ihre Gestalten, unaufhörlich mit der

Zeit; der Wind, welcher über einem Ort weht, wechselt seine Richtung unausgesetzt und zwar auf eine solche Weise, dass, ausgenommen den Ort, über welchen Eins der Wirbelgebiete gerade hinwegschreitet, die Windrichtung immer mit der Normale der Isodyame, welche gerade durch den gegebenen Ort geht, den constanten Winkel *arct.* K einschliesst. Wie die Windrichtung verändert sich so wohl die Windstärke, als der Luftdruck, in einem gegebenen Ort fortwährend mit der Zeit, und zwar so, dass die Vertheilung der Maxima und Minima der Zeit nach durchaus verschieden ausfallen muss, je nach der Lage des gegebenen Ortes in Bezug auf den gemeinsamen Schwerpunkt der beiden Wirbelgebiete.

So verwickelt die Gesetze auch sein mögen, gemäss denen die Veränderung dieser drei anemometrischen Faktoren in einem gegebenen Orte von sich geht, wenn zwei Wirbelgebiete in der Nähe desselben sich entwickelt haben, so lassen sich Ausdrücke dafür ableiten, und ohne jede Schwierigkeit, wenn die Wirbelgebiete entweder kreisförmig begrenzt oder bei beliebig gestalteiem Querschnitte unendlich klein sind gegen ihren Abstand vom gegebenen Ort.

Es sei ω das Azimuth der Windrichtung zur Zeit t, in *der* Richtung gezählt, in der eine Gerade aus einer Lage parallel der positiven x Achse (parallel dem Meridian) um 90° gedreht werden muss, um in *die* Lage parallel der positiven y Achse (parallel dem Parallelkreis) zu gelangen. Es seien x, y ferner die Coordinaten des gegebenen Ortes in Bezug auf ein Axensystem, dessen Anfangspunkt noch passend gewählt werden kann. Die Gleichung der Windbahn, welche zur Zeit t durch diesen Ort geht, ist in unserem speciellen Fall, wenn wir uns die beiden Wirbelgebiete kreisförmig begrenzt oder den Querschnitt der Wirbelgebiete gegen die Entfernung des Ortes von denselben sehr klein denken

$$K\mu_1 \log \rho_1' + K\mu_2 \log \rho_2' + \mu_1 \, arct. \left(\frac{y-y_1}{x-x_1}\right) + \mu_2 \, arct. \left(\frac{y-y_2}{x-x_2}\right) = const.$$

oder, indem wir $\frac{\mu_1}{\mu_2} = m$ setzen, und über die willkürliche Constante passend verfügen

$$K \, m \log \rho_1' + K \log \rho_2' + m \, arct\left(\frac{y-y_1}{x-x_1}\right) + arct\left(\frac{y-y_2}{x-x_2}\right) = const.$$

wo

$$\rho_1' = \sqrt{(x-x_1)^2 + (y-y_1)^2} \qquad \rho_2' = \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

ist

Nun hat die an dieser Curve gelegte Tangente ω zum Azimuth. Wir erhalten somit durch Differentiation der Windbahngleichung

$$tag\,\omega = -\frac{K[(m\,\rho_2'^2(x-x_1) + \rho_1'^2)] - m(y-y_1)\rho_2'^2 - (y-y_2)\rho_1'^2}{K[(m\,\rho_2'^2(y-y_1) + \rho_1'^2)] + m(x-x_1)\rho_2'^2 + (x-x_2)\rho_1'^2} \quad (110)$$

Behufs weiterer Entwickelung wollen wir den Anfangspunkt des Coordinatensystems in den gemeinsamen Schwerpunkt der beiden Wirbelgebiete legen und setzen

$$\begin{array}{lll} x = \rho \cos \chi & y = \rho \sin \chi & \rho^2 = x^2 + y^2 \\ x_1 = \rho_1 \cos \chi_1 & y_1 = \rho_1 \sin \chi_1 & \rho_1^2 = x_1^2 + y_1^2 \\ x_2 = \rho_2 \cos \chi_2 & y_2 = \rho_2 \sin \chi_2 & \rho_2^2 = x_2^2 + y_2^2 \end{array}$$

Führt man dies in die Gleichung (110) ein und berücksichtigt dabei die Beziehungen

$$m\,\rho_1 = -\rho_2 \qquad \chi_1 = \chi$$

so findet man

$$tag\,\omega = -\frac{U}{V} \quad (110_a)$$

wo zur Abkürzung gesetzt worden ist

$$\left.\begin{aligned} U &= \rho(K\cos\chi - \sin\chi)[(m+1)\rho^2 + (1+m^3)\rho_1^2 + 2(m-1)\rho\rho_1\cos(\chi-\chi_1)] \\ &\qquad - m\rho_1(K\cos\chi_1 - \sin\chi_1)[(m^2-1)\rho_1^2 + 2(m+1)\rho\rho_1\cos(\chi-\chi_1)] \end{aligned}\right\} \quad (111)$$

$$\begin{aligned} V &= \rho(K\sin\chi + \cos\chi)[(m+1)\rho^2 + (1+m^2)\rho_1^2 + 2(m-1)\rho\rho_1\cos(\chi-\chi_1)] \\ &\qquad - m\rho_1(K\sin\chi_1 + \cos\chi_1)[(m^2-1)\rho_1^2 + 2(m+1)\rho\rho_1\cos(\chi-\chi_1)] \end{aligned}$$

Die Grössen ρ_1 und χ_1 sind schon als Functionen von der Zeit t bekannt; es ist

$$\rho_1^2 = \rho_{01}^2\,(1+\varepsilon t) \qquad\qquad \chi_1 = \chi_0 + \frac{K}{2}\log\,(1+\varepsilon t)$$

wo $\dfrac{\mu_2^2}{\pi\rho_{01}^2(\mu_1+\mu_2)} = \varepsilon$ gesetzt worden ist. Die Substitution dieser Ausdrücke in (111) ergiebt

$$\begin{aligned} U = \rho(K\cos\chi - \sin\chi) &\Big[(m+1)\rho^2 + (1+\varepsilon t)(1+m^3)\rho_{01}^2 \\ &+ 2(m-1)\sqrt{(1+\varepsilon t)}\,\rho\rho_{01}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log\,(1+\varepsilon t)\Big)\Big] \\ &- m\rho_{01}\sqrt{(1+\varepsilon t)}\Big[K\cos\Big(\chi_0 + \frac{K}{2}\log(1+\varepsilon t)\Big) - \sin\Big(\chi_0 + \frac{K}{2}\log(1+\varepsilon t)\Big)\Big] \\ &\Big[(m^2-1)(1+\varepsilon t)\rho_{01}^2 + 2(m+1)\sqrt{(1+\varepsilon t)}\,\rho\rho_{01}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log\,(1+\varepsilon t)\Big)\Big] \end{aligned}$$

$$\begin{aligned} V = \rho(K\sin\chi + \cos\chi) &\Big[(m+1)\rho^2 + (1+\varepsilon t)(1+m^2)\rho_{01}^2 \\ &+ 2(m-1)\sqrt{(1+\varepsilon t)}\,\rho\rho_{01}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log\,(1+\varepsilon t)\Big)\Big] \\ &- m\rho_{01}\sqrt{(1+\varepsilon t)}\Big[K\cos\Big(\chi_0 + \frac{K}{2}\log(1+\varepsilon t)\Big) + \cos\Big(\chi_0 + \frac{K}{2}\log(1+\varepsilon t)\Big)\Big] \\ &\Big[(m^2-1)(1+\varepsilon t)\rho_{01}^2 + 2(m+1)\sqrt{(1+\varepsilon t)}\,\rho\rho_{01}\cos\Big(\chi - \chi_1 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big] \end{aligned}$$

wodurch $\tan\,\omega$ als Function von t dargestellt worden ist.

Es ist ebenso leicht einen Ausdruck abzuleiten für die Geschwindigkeit, mit welcher der Wind in einem gegebenen Orte zu gegebener Zeit weht. Nennt man die resultirende Geschwindig-

keit des Windes F, so ist für einen Ort, der ausserhalb der Wirbelgebiete liegt

$$F^2 = (1+K^2)\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right]$$

Setzt man hierin die für zwei kreisförmige oder unendlich kleine Wirbelgebiete giltige Lösung

$$\varphi = \mu_1 \log \rho_1' + \mu_2 \log \rho_2'$$

so kommt

$$F^2 = \frac{(1+K^2)}{\rho_1'^2 \rho_2'^2}\Big[\mu_1^2 \rho_2'^4 + \mu_2^2 \rho_1'^2 + 2\mu_1\mu_2\Big(y^2 + x^2 - y(y_1+y_2) - x(x_1+x_2) + y_1 y_2 + x_1 x_2\Big)\Big]$$

Dieses lässt sich nun leicht durch Einführung der Polarcoordinaten mit Rücksicht auf die Beziehungen $\rho_2 = -m\,\rho_1$ und $\chi_1 = \chi_2$ umformen in

$$F^2 = \frac{\mu_2^2(1+K^2)(1+m)^2[\rho^2 + (m-1)^2\rho_1^2 + 2(m-1)\rho\rho_1 \cos(\chi-\chi_1)]}{[\rho^2 + \rho_1^2 - 2\rho\rho_1 \cos(\chi-\chi_1)][\rho^2 + m^2\rho_1^2 + 2\,m\,\rho\rho_1 \cos(\chi-\chi_1)]}$$

Die Substitution der oben angegebenen Ausdrücke für ρ_1 und χ_1 ergiebt den verlangten Ausdruck

$$F^2 = \frac{\mu_2^2(1+m)^2(1+K^2)\Big[\rho^2 + (m-1)^2(1+\varepsilon t)\rho_{01}^2 + 2(m-1)\sqrt{(1+\varepsilon t)}\,\rho\rho_{01}\cos\Big(\chi-\chi_0-\frac{K}{1}\log(1+\varepsilon t)\Big)\Big]}{\Big[\rho^2 + \rho_{01}^2(1+\varepsilon t) - 2\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi-\chi_0-\frac{K}{2}\log(1+\varepsilon t)\Big)\Big]\Big[\rho^2 + m^2\rho_{01}^2(1+\varepsilon t) + 2\,m\,\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi-\chi_0-\frac{K}{2}\log(1+\varepsilon t)\Big)\Big]} \qquad (112)$$

wodurch die resultirende Windgeschwindigkeit in einem gegebenen Punkt der Erdoberfläche als Function von t dargestellt worden ist. Die Gleichungen (110_a) (112) setzen uns in den Stand, sowohl die Windrichtung als die Windgeschwindigkeit für verschiedene Orte,

und Zeiten zu ermitteln, so bald die anfängliche Lage der beiden Wirbelgebiete, ihre Massen und ihr gegenseitiger Abstand zu *einer* Zeit gegeben sind. Da sie aber zu complicirt sind, als dass sie ohne Weitschweifigkeit eine allgemeine Discussion gestatteten, so wollen wir uns damit begnügen, den Richtungswechselund die Veränderung der Windgeschwindigkeit für einige besondere Fälle näher zu verfolgen.

Wenn ρ unendlich gross gegen ρ_{01} und ρ_{02} ist, d. h, wenn der Beobachtungsort unendlich weit vom Schwerpunkt der beiden Wirbelgebiete liegt, so dass $\frac{\rho_{01}}{\rho}\sqrt{(1+\varepsilon t)}$ für endliche t unendlich klein ist, so hat man die einfache Gleichung

$$tag\,\omega = -\frac{(K\,cos\,\chi - sin\,\chi)}{(K\,sin\,\chi + cos\,\chi)}$$

oder mit Rücksicht auf die Gleichung $tag\,i = K$, wo i den Deviationswinkel bedeutet

$$\omega = \chi - i$$

Die Windrichtung ist demnach constant. Liegt daher der betreffende Ort *z. B* im ersten Quadrante, und sind die beiden unendlich fernen Wirbelgebiete cyklonal, so weht dort *SW*, *SSW*, *S*, *SSO*, *SO*, oder je nach dem der Ort in Bezug auf den Schwerpunkt der Wirbelgebiete mehr südlich oder östlich liegt, und zwar nur schwach, da unter diesem Umstande die Windstärke

$$V^2 = \frac{\mu_2^2\,(1+m)^2\,(1+K^2)}{\rho^2}$$

um so kleiner ist, je grösser ρ ist.

Liegt hingegen der Beobachtungsort im Schwerpunkt der beiden Wirbelgebiete, so dass $\rho = 0$ ist, so hat man für diesen Fall,

$$tag\,\omega = -\frac{K\,cos\left[\chi_0 + \frac{K}{2}\,log\,(1+\varepsilon t)\right] - sin\left[\chi_0 + \frac{K}{2}\,log\,(1+\varepsilon t)\right]}{K\,sin\left[\chi_0 + \frac{K}{2}\,log\,(1+\varepsilon t)\right] + cos\left[\chi_0 + \frac{K}{2}\,log\,(1+\varepsilon t)\right]}$$

d. h.

$$\omega = \chi_0 + \frac{K}{2} \log (1 + \varepsilon t) - i$$

Der Wind geht demnach allmälig durch alle Himmelsstriche und zwar entweder im cyklonalen Sinne immer langsamer oder anticyklonalen Sinne immer schneller, je nachdem die beiden Wirbelgebiete anticyklonal oder cyklonal sind oder je nachdem das superpondirende Wirbelgebiet anticyklonal oder cyklonal ist. Dabei nimmt die Windgeschwindigkeit im ersteren Fall unaufhörlich ab, und im letzteren Fall dagegen unaufhörlich zu; denn der Ausdruck für die resultirende Geschwindigkeit verwandelt sich in unserem besonderen Fall in

$$F^2 = \frac{\mu_2^2 (1+m)^2 (1+K^2)(m-1)^2}{m^2 \rho_2^2 (1+\varepsilon t)}$$

F^2 wächst sonach mit der Zeit oder nimmt ab, je nachdem $\varepsilon < 0$ oder > 0 ist, vorausgesetzt dass $m \gtrless 1$ ist. Ist nun $m = 1$ *d. h.* sind die Massen der beiden Wirbelgebiete gleichsinnig und einander gleich, so verschwindet F^2; es entsteht um den Schwerpunkt ein Gebiet der Windstillen.

Wir fassen ferner einen Fall in's Auge, wo

$$m = 1 \qquad \chi_0 = 0 \qquad \chi = 0$$

ist, womit gesagt ist, dass die beiden Wirbelgebiete gleiche und gleichsinnige Massen, und zur Zeit $t = 0$ *die* Lage haben, dass die Verbindungslinie ihrer Schwerpunkte parallel dem Meridian gerichtet ist und dass der Beobachtungsort südlich von Schwerpunkt der beiden Wirbelgebiete liegt. Die Gleichungen (110_a) (112) verwandeln sich für diesen Fall nach leichter Umformung in

$$\tan \omega = -\frac{[K\rho^2 - \rho_{01}^2 (1+\varepsilon t)[K \cos K \log (1+\varepsilon t) - \sin K \log (1+\varepsilon t)]]}{[\rho^2 + \rho_{01}^2 (1+\varepsilon t)[K \sin K \log (1+\varepsilon t) - \cos K \log (1+\varepsilon t)]]}$$

$$F^2 = \frac{4 \mu_2^2 (1+K^2) \rho^2}{\rho^4 + \rho_{01}^4 (1+\varepsilon t)^2 - 2\rho^2 \rho_{01}^2 (1+\varepsilon t) \cos K \log (1+\varepsilon t)}$$

Wir nehmen zuerst an, dass $\varepsilon > 0$ sei. *d. h.* dass die beiden Wirbelgebiete anticyklonal seien. Dann sind wieder zwei Fälle zu unterscheiden, ob $\rho \leqq \rho_{01}$ oder $\rho > \rho_{01}$ ist. Im ersteren Fall, wo der Beobachtungsort innerhalb der Bewegungsbahn der Wirbelgebiete liegt, kann sowohl der Zähler als der Nenner in dem Ausdruck für tag ω verschwinden. Es weht daher in dem betreffenden Ort jedesmal *S.* oder *N.*, so oft

$$\frac{K\rho^2}{\rho_{01}^2(1+\varepsilon t)} = K\,cos\,K\,log\,(1+\varepsilon t) - sin\,K.\,log(1+\varepsilon t)$$

wird, und jedesmal *O.* oder *W.*, so oft

$$\frac{\rho^2}{\rho_{01}^2(1+\varepsilon t)} = cos\,K\,log\,(1+\varepsilon t) - K\,sin\,K\,log\,(1+\varepsilon t)$$

wird, was immer für reelle Werthe von t stattfinden kann, da auch die linker Hand stehende Grösse ein positiver echter Bruch ist. Der Wind geht daher in diesem Orte durch alle Himmelsstriche und die Windfahne dreht sich dabei immer langsamer im cyklonalen Sinne, während die Windgeschwindigkeit, abgesehen von den Schwankungen, deren Periode immer grösser wird, allmälig abnimmt. Ist dagegen $\rho > \rho_{01}$ *d. h.* liegt der Ort ausserhalb des Bewegungsbahn der Wirbelgebiete, so kann tagω nie $= 0$, noch $= \pm \infty$ werden, so lange $\rho > \rho_{01}(1+\varepsilon t)$ ist und die Windrichtung schwankt nur zwichen *N.* und *O.*, so lange bis das eine Wirbelgebiet über den gegebenen Ort oder südlich davon hinwegschreitet, so dass der Ort dann zwischen dem Schwerpunkt und dem Wirbelgebiet zu liegen kommt, und der Wind dann wieder durch alle Striche weht.

Wenn nun $\varepsilon < 0$ ist, *d. h.* wenn die beiden Wirbelgebiete cyklonal sind, so verhält sich die Sache etwas anderes. Die obenstehenden Gleichungen werden in diesem Fall

$$tag\,\omega = \frac{K\rho^2 - \rho_{01}^2(1-\varepsilon t)[K\,cos\,K\,log\,(1-\varepsilon t) - sin\,K\,log\,(1-\varepsilon t)]}{\rho^2 + \rho_{01}^2(1-\varepsilon t)[K\,sin\,K\,log\,(1-\varepsilon t) - cos\,K\,log\,(1-\varepsilon t)]}$$

$$F^2 = \frac{4\mu_2^2(1+K^2)\,\rho^2}{\rho^4+\rho_{01}^4(1-\varepsilon t)^2-2\rho\rho_{01}^2(1-\varepsilon t)\cos K\log(1-\varepsilon t)}$$

So lange $\rho^2 < \rho_{01}^2(1-\varepsilon t)$, so verschwindet $tag\ \omega$, so oft

$$\frac{K\rho^2}{\rho_{01}^2(1-\varepsilon t)} = K\cos K\log(1-\varepsilon t) - \sin K\log(1-\varepsilon t)$$

ist und wird $= \pm\infty$, so oft

$$\frac{\rho^2}{\rho_{01}(1-\varepsilon t)} = \cos K\log(1-\varepsilon t) - K\sin K\log(1-\varepsilon t)$$

wird. So lange weht der Wind demnach von allen Himmelsstrichen und die Richtungsänderung geschieht anticyklonal und zwar immer schneller, wobei Maxima und Minima der Windstärke immer rascher auf einander folgen, während sie selbst immer mehr zunimmt.

Ist ρ endlich $> \rho_{01}$ *d. h.* liegt der Ort jenseits der Bewegungsbahn der Wirbelgebiete, so schwankt die Windrichtung nur zwischen *S.* und *W.* alle zwischenliegende Striche hindurch, während die Schwankungen der Windstärke immer schneller erfolgen, wobei die Windstärke unaufhörlich abnimmt, bis sie für den betreffenden Ort einen constanten Werth erreicht hat.

Es bleibt nur noch der Fall zu untersuchen übrig, wo die Massen der beiden Wirbelgebiete gleich aber entgegengesetzt sind, wo also der Schwerpunkt der beiden Wirbelgebiete in die Unendlichkeit rückt. Wie wir oben gesehen haben, bewegen sich die beiden Wirbelgebiete in diesem Fall geradelinig mit constanter Geschwindigkeit

$$\sqrt{\left(\frac{d\rho_1}{dt}\right)^2+\left(\rho_{01}\frac{dX_1}{dt}\right)^2} = \frac{\mu_1}{4\pi r}\sqrt{1+K^2}$$

und zwar in *der* Richtung, in der die Luft zwischen den beiden Wirbelgebieten strömt. Bezeichnen wir diese Geschwindigkeit mit R, und beziehen die Schwerpunkte der Wirbelgebiete auf ein Coordinatensystem, dessen x Achse wieder gegen *S.* und dessen positive y Achse

gegen O. gerichtet, and dessen Anfangspunkt in dem Punkt auf der Erdoberfläche liegen soll, dessen Windverhältniss wir untersuchen wollen, so dass die Gleichungen (110_a) (112) sich in diesem Fall verwandeln in

$$tag\ \omega = -\frac{\left[(x_2^2+y_2^2)(Kx_1-y_1)+(x_1^2+y_1^2)(y_2-Kx_2)\right]}{\left[(x_2^2+y_2^2)(Ky_1+x_1)-(x_2^1+y_1^2)(x_2+Ky_2)\right.} \qquad (113)$$

$$F^2 = \frac{(1+K^2)\ \mu_1^2\,(x_1^2+y_1^2+x_2^2+y_2^2-2\,y_1\,y_2-2\,x_1\,x_2)}{(x_1^2+y_1^2)(x_2^2+y_2^2)} \qquad (114)$$

Es seien $\alpha_1\ \beta_1$, $\alpha_2\ \beta_2$ die Coordinaten der Schwerpunkte der beidem Wirbelgebiete zur Zeit $t=o$, und ψ sei der Winkel, welchen die Fortschreitungsrichtung der beiden Wirbelgebiete mit der positiven x Achse einschliesst. Wenn wir setzen

$$\begin{array}{ll} x_1 = Rt.\ cos\ \psi + \alpha_1 & x_2 = Rt.\ cos\ \psi + \alpha_2 \\ y_1 = Rt.\ sin\ \psi + \beta_1 & y_2 = Rt.\ sin\ \psi + \beta_2 \end{array} \qquad (115)$$

so ist die Bedingung von der Unveränderlichkeit des Abstandes der beiden Wirbelgebiete

$$(x_1-x_2)^2+(y_1-y_2)^2=(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2=r^2$$

erfüllt. Die Substitution der Ausdrüche (115) in (113) (114) ergiebt nach leichter Umformung.

$$tag\ \omega = -\frac{U}{V}$$

wo

$$\begin{aligned} U = {} & (R^2t^2+2Rt[\alpha_2 cos\,\psi+\beta_2 sin\psi]+\alpha_2^2+\beta_2^2)(K[Rt\,cos\,\psi+\alpha_1]-Rt\,sin\,\psi-\beta_1) \\ & -(R^2t^2+2Rt[\alpha_1 cos\,\psi+\beta_1 sin\psi]+\alpha_1^2+\beta_1^2)(K[Rt\,cos\,\psi+\alpha_2]-Rt\,sin\,\psi-\beta_2) \end{aligned}$$

$$\begin{aligned} V = {} & (R^2t^2+2Rt[\alpha_2 cos\,\psi+\beta_2 sin\psi]+\alpha_2^2+\beta_2^2)(K[Rt\,sin\,\psi+\beta_1]+Rt\,cos\,\psi+\alpha_1) \\ & -(R^2t^2+2Rt[\alpha_1 cos\,\psi+\beta_1 sin\psi]+\alpha_1^2+\beta_2^2)(K[Rt\,sin\,\psi+\beta_2]+Rt\,cos\,\psi+\alpha_2) \end{aligned}$$

ist.

$$F^2 = \frac{\mu_1^2(1+K^2)\,r^2}{(R^2t^2+2Rt[\alpha_1 cos\psi+\beta_1 sin\psi]+\alpha_1^2+\beta_1^2)(R^2t^2+2Rt[\alpha_2 cos\,1+\beta_2 sin\psi]+\alpha_2^2\beta_2^2]}$$

wodurch das Azimuth des Windes und die Windgeschwindigkeit als Function der Zeit dargestellt worden ist.

Der Winkel ψ ist dabei auf gewisse Weise von der Richtung der Verbindungslinie der beiden Wirbelgebiete abhängig. Zieht man zu dieser Gerade eine Senkrechte, so macht, wie wir schon oben gesehen haben, die Richtung der Fortschreitungsgeschwindigkeit mit dieser Senkrechte einen Winkel, dessen trigonometrische Tangente

$$= K$$

ist. Bezeichnet man diesen mit i und den Winkel, welche die zur Verbindungslinie der beiden Wirbelgebiete senkrechte Linie mit der positiven x Achse einschliesst mit ψ, so hat man

$$\psi = \varphi + i \qquad \text{und} \quad cot \varphi = \frac{\beta_1 - \beta_2}{\alpha_1 - \alpha_2}$$

Mithin

$$\psi = arcot. \left(\frac{\beta_1 - \beta_2}{\alpha_1 - \alpha_2}\right) + i$$

wobei der rechter Hand stehende Quotient im absoluten Sinne genommen werden kann, wenn man nur fest setzt, dass φ in *der* Richtung gezählt wird, in der man das cyklonale Wirbelgebiet erblickt, wenn man nach der Richtung der Fortschreitung hinsieht.

Um einen einfachen Fall beispielsweise zu betrachten, nehmen wir an, dass

$$\alpha_1 = \alpha_2 = 0 \qquad \beta_1 = - \beta_2 = \beta.$$

d. h. dass der Beobachtungsort zur Zeit $t = 0$ gerade in der Mitte der Verbindungslinie der beiden Wirbelgebiete liege. Unsere Gleichungen lassen sich in diesem einfachen Fall mit Rücksicht auf die Beziehung $tag\, i = K$ umformen in

$$tag\, \omega = \frac{(R^2 t^2 + \beta^2)}{(\beta^2 - (2K - 1) R^2 t^2)}$$

$$V^2 = \frac{\mu_1^2 (1+K^2) r^2}{[(R^2 t^2 + \beta^2)^2 - 4 R^2 t^2 \beta^2 \sin^2 i]}$$

Die Windgeschwindigkeit ist demnach, wie leicht vorauszusehen war, am grössten zur Zeit $t = 0$, und nimmt, indem die beiden Wirbelgebiete fortschreiten, unaufhörlich ab. Wie der Wind seine Richtung mit der Zeit wechselt, das hängt, wie man sieht, davon ab, ob $K > \frac{1}{2}$ oder $< \frac{1}{2}$ ist. Ist das erstere der Fall, so weht in dem betreffenden Orte anfangs ein NW. mit dem Azimuth 45°, und die Windrichtung geht dann durch NWW. in reinen W über, und zwar nach dem Verlaufe der endlichen Zeit $t = \frac{\beta}{R\sqrt{2K-1}}$, von welchem Zeitpunkt ab der Wind rasch SSW. SW. wird, und sich dann immer langsamer einer bestimmten südwestlichen Richtung nährt, deren Azimuth durch die Gleichung

$$tag\ \omega = -\frac{1}{(2K-1)}$$

bestimmt ist. Wenn aber $K < \frac{1}{2}$ ist, so findet ein solcher Windwechsel nicht statt; der Wind bleibt fortwährend zwischen N. und W. und nährt sich einer bestimmten nordwestlichen Richtung, deren Azimuth durch die Gleichung

$$tag\ \omega = \frac{1}{1-2K}$$

bestimmt ist.

Der in einem gegebenen Ort herrschende Druck lässt sich auch eben so leicht als Function von der Zeit darstellen. Aus den Gleichungen

$$\Phi = -\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right)(\mu_1 \log \rho_1 + \mu_2 \log \rho_2')$$

$$\frac{p}{\mu} + \frac{1}{2}(u + v^2) - G = \Phi$$

folgt durch Einführung der Polarcoordinaten

$$p = \mu G - \mu\left(\kappa + \frac{4\lambda^2 \sin^2\theta}{\kappa}\right) \log\Big\{[\rho^2 + \rho_1^2 - 2\rho\rho_1 \cos(\chi - \chi_1)]^m$$

$$[\rho^2 + m^2\rho_1^2 + 2m\rho\rho_1 \cos(\chi - \chi_1)]\Big\} - \frac{\mu\mu_2^2(1+m)^2(1+K^2)}{2} *$$

$$* \frac{[\rho^2 + (m-1)^2\rho_1^2 + 2(m-1)\rho\rho_1 \cos(\chi - \chi_1)]}{[\rho^2 + \rho_1^2 - 2\rho\rho_1 \cos(\chi - \chi_1)][\rho^2 + m^2\rho_1^2 + 2m\rho\rho_1 \cos(\chi - \chi_1)]}$$

Hieraus folgt weiter durch die Einführung der Ausdrücke für ρ_1 und χ_1

$$p = \mu G - \mu\mu_2\kappa(1+K^2) \log\Big\{\Big[\rho^2 + \rho_{01}^2(1+\varepsilon t) - 2\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big]^m$$

$$\Big[\rho^2 + m^2\rho_{01}^2(1+\varepsilon t) + 2m\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big]\Big\} - \frac{\mu\mu_2^2(1+m)^2(1+K^2)}{2} *$$

$$* \frac{\Big[\rho^2 + (m-1)^2\rho_{01}^2(1+\varepsilon t) + 2(m-1)\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big]}{\Big[\rho^2 + \rho_{01}^2(1+\varepsilon t) - 2\rho\rho_{01}\sqrt{(1+\varepsilon t)}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big]} *$$

$$* \overline{\Big[\rho^2 + m^2\rho_{01}^2(1+\varepsilon t) + 2m\rho\rho_{01}\cos\Big(\chi - \chi_0 - \frac{K}{2}\log(1+\varepsilon t)\Big)\Big]}$$

ein Ausdruck für die Veränderlichkeit des in einem gegebenen Ort herrschenden Drucks mit der Zeit, wenn zwei Wirbelgebiete sich in der Nähe des Ortes gebildet haben.

Da nun eine allgemeine Discussion dieses complizirten Ausdrucks schwierig ist, wegen der grossen Anzahl der Parameter und der dadurch bedingten möglichen Fälle, wollen wir uns damit begnügen, einige besondere Fälle näher zu betrachten.

Es sei zunächst $\rho = 0$, *d. h.* der Ort liege gerade im Schwerpunkte der beiden Wirbelgebiete, so wird in diesem Fall

$$p = \mu G - \mu\mu_2\kappa(1+K^2)\log[m^2(1+\varepsilon t)^{m+1}\rho_{01}^{2m+2}]$$

$$- \frac{\mu\mu_2^2(1+m)^2(1+K^2)}{2m^2} \frac{(m-1)^2}{\rho_{01}^2(1+\varepsilon t)}$$

oder

$$p = \mu G - \mu \mu_2 \kappa (1 + K^2)[(m+1) \log (1 + \varepsilon t) - 2 \log m \rho_{01}^{m+1}]$$
$$\frac{\mu \mu_2^2 (1+m)^2 (1 + K^2) (m-1)^2}{2 m^2 \rho_{01}^2 (1 + \varepsilon t)}$$

Hierbei sind vier Fälle zu unterscheiden, ob die beiden Wirbelgebiete anticyklonal sind oder cyklonal, oder auch, ob das superpondirende Wirbelgebiet anticyklonal oder cyklonal ist. Wenn wir das erste annehmen, und festsetzten, dass $\varepsilon > 0$ ist, so sehen wir, indem wir den ersten Differentialquotienten

$$\frac{dp}{dt} = \frac{\mu \mu_2 (1+m)(1+K^2) . \varepsilon .}{(1 + \varepsilon t)} \left(\frac{\mu_2}{\rho_{01}^2} \frac{(m+1)(m-1)^2}{(1+\varepsilon t)} - \kappa \right)$$

betrachten, dass, falls $\frac{\mu_2}{\rho_{01}^2} (m+1 \; (m-1)^2 > \kappa$ ist, der Druck zuerst zunimmt, und dann immer langsamer abnimmt. Die Zeit, wo der Maximaldruck eintritt ist

$$= \frac{\pi}{\kappa} \left[\mu_2 (1+m)(m-1)^2 - \kappa \rho_{01}^2 \right]$$

Sind aber die beiden Wirbelgebiete cyklonal, so dass μ_2, wie ε negativ zu setzen ist, so nimmt der Druck unter denselben Umständen nur unaufhörlich ab, und zwar immer rascher, da $(1 - \varepsilon t)$ mit wachsender t abnimmt.

Betrachten wollen wir noch die Druckveränderung in dem besonderen Fall, wo

$$X_0 = 0 \qquad X = 0 \qquad m = 1$$

ist. Der Ausdruck für den Druck vereinfacht sich unter diesen Umständen zu

$$p = \mu G - \mu \mu^2 \kappa (1 + K^2) \log [\rho^4 + \rho_{01}^4 (1 + \varepsilon t)^2 - 2 \rho_{01}^2 \rho^2 (1 + \varepsilon t) \cos K \log (1 + \varepsilon t)]$$
$$- 2 \mu \mu_2 (1 + K^2) \rho^2 \left[\frac{1}{\rho^4 + \rho_{01}^4 (1 + \varepsilon t)^2 - 2 \rho_{01} \rho^2 (1 + \varepsilon t) \cos K \log (1 + \varepsilon t)} \right]$$

Die Veränderungen des Drucks sind demnach theils periodische theils stetige; der Druck schwankt zwischen Minimum und Maximum,

bald schneller, bald langsamer, indem Maxima und Minima entweder unaufhörlich abnehmen, oder leise zu einem beistimmten Grenzwerthe wachsen, je nachdem die beiden Wirbelgebiete anticyklonal oder cyklonal sind.

Bildet man den ersten Differentialquotienten, und setzt zur Abkürzung

$$\frac{\rho_{01}}{\rho} = \alpha \qquad \frac{\mu_2}{\rho_{01}^2} = \beta \qquad 2\,\varepsilon\,(1+K^2)\,\mu\,\mu_2\,\alpha^2 = \gamma.$$

so kommt

$$\frac{dp}{dt} = \gamma\,\frac{[\alpha^2\beta - \kappa(1+\alpha)^4(1+\varepsilon t)^2 - 2\,\alpha^2(1+\varepsilon t)\cos K\log(1+\varepsilon t)]}{[1+\alpha^4(1+\varepsilon t)^2 - 2\,\alpha^2(1+\varepsilon t)\cos K\log(1+\varepsilon t)]^2} *$$

$$* \frac{[\alpha^2(1+\varepsilon t) - \cos K\log(1+\varepsilon t) + K\sin\log(1+\varepsilon t)]}{} \tag{116}$$

Setzt man die beiden Factoren im Zähler $= 0$, und löst nach $\alpha^2(1+\varepsilon t)$ auf, so dass

$$\alpha^2(1+\varepsilon t) = \cos K\log(1+\varepsilon t) - \sqrt{\frac{\alpha^2\beta}{\kappa} - 1 + \cos^2 K\log(1+\varepsilon t)}$$

$$\alpha^2(1+\varepsilon t) = \cos K\log(1+\varepsilon t) - K\sin K\log(1+\varepsilon t)$$

oder indem man setzt, unter der Voransetzung des positiven ε

$$K\log(1+\varepsilon t) = \vartheta$$

$$\alpha^2 e^{\frac{t}{K}} = \cos\vartheta + \sqrt{\left(\frac{\alpha^2\beta}{\kappa} - 1\right) + \cos^2\vartheta} = \cos\vartheta + \sqrt{\frac{\alpha^2\beta}{\kappa} - \sin^2\vartheta}$$

$$\alpha^2 e^{\frac{\vartheta}{K}} = \cos\vartheta - K\sin\vartheta. \tag{117}$$

Ob diese transscendenten Gleichungen reelle Wurzeln haben, das hängt von den Werthen der hierin auftretenden Parameter ab. Wir wollen, um die Discussion zu erleichtern, die Constante $K = 1$ setzen dann sind vier mögliche Fälle denkbar

$$\frac{\alpha^2\beta}{\kappa} > 1 \qquad \alpha^2 > 1$$

$$\frac{\alpha^2\beta}{\kappa} > 1 \qquad \alpha^2 < 1$$

$$\frac{\alpha^2\beta}{\kappa} < 1 \qquad \alpha^2 > 1$$

$$\frac{\alpha^2\beta}{\kappa} < 1 \qquad \alpha^2 < 1$$

Während in den beiden letzteren Fällen die erste Gleichung nicht stattfindet, da der Wurzelausdruck im allgemeinen imaginär wird, kann dieselbe in den beiden letzteren Fällen wohl durch einen reellen Werth von ϑ befriedigt werden, und zwar durch einen einzigen zwischen 0 und $\frac{\pi}{2}$ liegenden Werth, wie man sich leicht davon überzeugen kann durch graphische Darstellung der beiden Curven

$$y = \sqrt{\frac{\alpha^2\beta}{\kappa} - \sin^2\vartheta} \qquad y = \alpha^2 e^{\vartheta} - \cos\vartheta$$

Die zweite Gleichung wird auch nur dann durch ein reelles ϑ erfüllbar, wenn $\alpha^2 < 1$ ist, und zwar durch einen einzigen zwischen 0 und $\frac{\pi}{4}$ liegenden Werth. Es folgt hieraus: haben die beiden Wirbelgebiete anticyklonale Bewegungsformen, so kann der Luftdruck für den Ort von der vorausgesetzten Lage in Bezug auf den Schwerpunkt der beiden Wirbelgebiete höchstens einmal Maximum und ein Minimum sein, wenn der betreffende Ort ausserhalb der Fortschreitungsbahn der beiden Wirbelgebiete liegt, und zwar so dass

$$\alpha^2 e^{\vartheta} \quad \cos\vartheta \quad \sin\vartheta$$

den Zeitpunkt bestimmt, wo das Druckminimum eintritt, während

$$\alpha^2 e^{\vartheta} = \cos\vartheta + \sqrt{\frac{\alpha^2\beta}{\kappa} - \sin^2\vartheta}.$$

den Zeitpunkt angiebt, wo in dem betreffenden Ort der Druck Maximum wird.

Note on the Specific volumes of Aromatic Compounds.

By

Jōji Sakurai, F.C.S.

Professor of Chemistry, Imperial University.

It is well known that the observed specific volumes of benzene and its derivatives are less than those calculated with the use of Kopp's constants which, when applied to saturated fatty compounds, give results fairly agreeing with the observed values.

Other values for carbon and hydrogen to be applied to benzene (and its derivatives?) have therefore been calculated. Lother Meyer regards the hydrogen of benzene as having the value 5.* instead of 5.5, while the carbon is regarded as having the value 11, as in saturated fatty compounds. Löschmidt assumes that half the carbon atoms in benzene have the value 11, and the remainder the value 14, and that hydrogen has the constant value 3.5.

Each of these sets of values, I consider, has been arbitrarily deduced from the observed specific volume of benzene, and therefore, when calculated back, they naturally give results agreeing with the latter.

* I have not been able to refer to Meyer's original paper. Thorpe (J. chem. Soc. Trans. 1880, 381) states: "Lother Meyer makes H=3 ...." but this I regard as a misprint for H=5. This misprint is reproduced in Watt's Dict. III Suppl. p. 2126, and again in Mr. Kuhara's paper to be referred to later on.

Meyer's calculation.	Löschmidt's calculation.	observed.
$C_6 = 6 \times 11 = 66$	$C_3 = 3 \times 14 = 42$	
$H_6 = 6 \times \ \ 5 = \underline{30}$	$C_3 = 3 \times 11 = 33$	
96	$H_6 = 6 \times 3.5 = \underline{21}$	
	96	95.9

Now, Meyer's and Löschmidt's constants are only applicable to benzene, and cannot be regarded as giving results agreeing with the observed values, when applied to the homologues of benzene. For,

(1) If we regard these constants to be applied to the side-chains as well, then the calculated values of the homologues of benzene would be less than the observed values; for, while benzene possesses abnormally low specific volume, its homologues show the constant increase of 22 in the volume for an increment of $C H_2$. The hydrogen in the side-chain cannot, therefore, possess the value 5, and still less the value 3.5.

(2) If we regard the above constants as applicable to the benzene nucleus only, and the carbon and hydrogen in the side-chain as possessing their normal values, then the use of Meyer's constants means an advantage of 2.5 units over Kopp's values for one atom of hydrogen replaced by $C H_3$, $C_2 H_5$, &c; for $C_6 H_5$=91 instead of 93.5. For di-substitution products the advantage is 2.0, for tri-substitution products 1.5, and so on, until we shall find in hexamethyl benzene, for example, a body possessing normal specific volume. This, however, does not seem to be the case. The use of Löschmidt's constants for only the benzene-nucleus leads to a singular result. For mono-substitution products the advantage over Kopp's constants is 1 unit, as $C_6 H_5$=92.5 instead of 93.5; but for di-substitution products the *disadvantage* is 1 unit, for tri-substitution products the *disadvantage* is 2 units, and so on.

Meyer's and Löschmidt's constants are, therefore, only applicable to benzene, and not to its homologues with any strictness.

I have calculated another value for carbon which, I admit, is as arbitrary as Meyer's or Löschmidt's, but which has the merit of being applicable to aromatic hydro-carbons in general. I regard each of the six atoms of carbon in benzene-nucleus as having the value 10.5, while the hydrogen of the nucleus as well as the carbon and hydrogen of the side-chains possess their normal values, viz. C=11 and H=5.5.

This consideration is, of course, derived from the observation that the replacement of hydrogen in benzene by $C H_3$, $C_2 H_5$, &c causes the same increase in volume as in saturated fatty compounds.

The following comparison will make the above statement clear, the numbers under 'observed' being those obtained by R. Schiff.* Kopp's determinations of benzene, cymene, and naphthalene are also added. The ten atoms of carbon, in the last hydrocarbon, constituting the two benzene nuclei are each of them regarded as having the value 10.5.

	Observed.		Calculated.
	Kopp.	Schiff.	C = 10.5 & 11, H = 5.5
Benzene $C_6 H_6$	95.8	95.94	96
Toluene $C_6 H_5 . CH_3$	—	117.97	118
Xylene $C_6 H_4 . (CH_3)_2$	—	139.74	140
Ethyl benzene $C_6 H_5 . C_2 H_5$	—	138.93	140
N. propyl benzene $C_6 H_5 . C_3 H_7{}^a$	—	161.82	162
P. Ethyl toluene $C_6 H_4 . CH_3 . C_2 H_5$	—	161.94	162
Mesitylene $C_6 H_3 . (CH_3)_3$	—	162.41	162
Cymene $C_6 H_4 . CH_3 . C_3 H_7{}^a$	183.5	184.46	184
Naphthalene $C_{10} H_8$	149.2	—	149

* Annalen 220,71

The value 10.5 for nucleus-carbon thus adapts itself well either for benzene or its homologues ; and in the deducing of such special values for carbon or hydrogen or for both, the aromatic *hydrocarbons* are certainly the most suitable, as they consist of carbon and hydrogen alone and do not contain oxygen or nitrogen, the presence of which introduces elements of uncertainty.

The above value of carbon may, however, be employed in the same manner in calculating the specific volumes of simpler aromatic compounds containing oxygen, the two values given to this element by Kopp being adopted in the calculation. Thus—

	Observed. (Kopp.)	Calculated. C = 10.5 & 11, H = 5.5. O = 12.2 & 7.8.
Phenol $C_6 H_5 . O'H$	103.6	103.8
Benzoic aldehyde $C_6 H_5 . CHO''$	118.4	119.2
Benzoic acid $C_6 H_5 . CO'O''H$	126.9	127.0
Ethyl benzoate $C_6 H_5 . CO'O''C_2 H_5$	172.4—174.8	171.0
Benzylic alcohol $C_6 H_5 . CH_2 O'H$	123.7	125.8

Mr. Kuhara, in Vol. II. Pt. IV of this journal, calculated the specific volumes for the above bodies, using Löschmidt's constants and on the supposition that 3 atoms of carbon have the value 14, the rest 11, and that hydrogen, whether of the nucleus or of the side-chain, possesses the constant value 3.5. He will find that his numbers are not more concordant with the experimental values than are those above calculated. If, moreover, he tried his method of calculation upon the aromatic *hydrocarbons*, bodies which, as I already pointed out, are certainly best suited to test the accuracy or utility of the constants for carbon and hydrogen—he will get very different results, as I have already broadly indicated and as the following table will show in greater detail :—

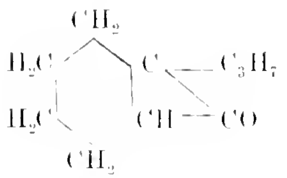
	Observed. Kopp.	Observed. Schiff.	Calculated. C=14 & 11, H=3.5.
Benzene C_6H_6	95.8	95.94	96
Toluene $C_6H_5.CH_3$	—	117.97	114
Xylene $C_6H_4.(CH_3)_2$	—	139.74	132
Ethyl benzene $C_6H_5.C_2H_5$	—	138.93	132
N. Propyl benzene $C_6H_5.C_3H_7{}^a$	—	161.82	150
P. Ethyl toluene $C_6H_4.CH_3.C_2H_5$	—	161.94	150
Mesitylene $C_6H_3.(CH_3)_3$		162.41	150
Cymene $C_6H_4.CH_3.C_3H_7{}^a$	183.5	184.46	168
Naphthalene $C_{10}H_8$	149.2	—	147 or 153*

The above table clearly shows the fault of the method of calculation employed by Mr. Kuhara, which when applied to simpler aromatic compounds containing oxygen gives results (by chance?) fairly agreeing with the experimental values, but which when applied to the aromatic hydrocarbons gives results entirely at variance with those experimentally determined.

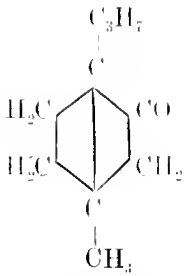
Using the same method of calculation and on the supposition that oxygen is ketonic, Mr. Kuhara finds that the observed specific volume of camphor agrees almost exactly with the calculated value, and draws therefore a probable conclusion that its constitution is correctly represented by one of the following formulae :

* Mr. Kuhara makes naphthalene = 150, but it seems to me that the two possible ways of calculation consistent with those adopted for other bodies are :

(1)	(2)
$C_3 = 3 \times 14 = 42$	$C_5 = 5 \times 14 = 70$
$C_7 = 7 \times 11 = 77$	$C_5 = 5 \times 11 = 55$
$H_8 = 8 \times 3.5 = 28$	$H_8 = 8 \times 3.5 = 28$
147	153



(Kachler.)



(Kanonnikow.)

The agreement between the observed and the calculated values for the specific volume of camphor, viz. 187.42 and 187.2 respectively, is certainly very remarkable, but one does not feel very much inclined to accept any conclusion based upon a method of calculation apparently open to so grave a criticism as I have indicated.

With our present imperfect state of knowledge regarding the relation between specific volumes of compounds and their 'constitution,' especially in the case of aromatic compounds, we can neither make a free nor a safe application of it in the discussion of the probable constitution of such bodies as camphor and borneol.

It is probable, for instance, that the specific volume of carbon may vary not only according as it exists in the benzene nucleus or in the side-chains, but also according as it is combined with 1, 2, 3, or 4 other carbon atoms, and again according to the nature of other atoms directly combined with it. Schiff, indeed, has pointed out that the specific volume of carbon may vary from 8 to 13, and that of oxygen from 5.6 to 19.

The difference observed between the specific volumes of

(1) Para chlorotoluene......$C_6H_4.CH_3.Cl$ = 134.91

(2) Benzylic chloride ......$C_6H_5.CH_2.Cl$ = 133.47

may, for example, be due to either one or all of the following differences :

1°. Chlorine in (1) is in direct combination with carbon of the nucleus, that in (2) with one in the side-chain.

2°. There are four (CH) groups in (1), five in (2); in other words, there are two side-chains in (1), and only one in (2).

3°. The carbon to which chlorine is combined in (1) is in its turn directly combined with two other atoms of carbon, while that to which chlorine is combined in (2) is only combined with one other carbon atom.

Such considerations as these make us hesitate very much before making a free application of the very imperfect knowledge we possess at present regarding the relation between specific volume data and the constitution of chemical compounds.

It is true that the specific volume of camphor calculated with the use of constants, C = 10.5 & 11, H = 5.5, O = 12.2 or 7.8, does not agree with the experimental value, but this is simply because we are ignorant of the law which connects together the variation of the specific volume of an element with its mode of combination. It is possible, for example, that the specific volume of carbon directly combined with four other atoms of carbon may be considerably lower than 10.5 or 11.* and the abnormally low specific volume of camphor, as actually observed, may be due to the existence in it of two or more such carbon atoms. The six formulae of camphor quoted by Mr. Kuhara represent it as containing respectively 0, 0, 2, 1, 2 and 1 such carbon atoms, and it may possibly turn out that either the formula (3) or the formula (5) correctly represents its constitution. Further theoretical speculations upon these points are more than

* It may be remarked that the atomic volume of diamond calculated in the usual manner is $\frac{12}{3.53} = 3.4$.

useless at present, and I therefore do no more than broadly make the above suggestions for future consideration.

明治廿六年七月一日印刷
明治廿六年七月三日發行

編纂兼發行者　帝國大學

東京市日本橋區兜町一番地
製紙分社
印刷者　星野錫

東京市日本橋區通三丁目
十四番地
賣捌所　丸善商社書店

2722